Recommendations about the Big Ideas in Statistics Education: A Retrospective from Curriculum and Research[1]

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Summary

Five decades of research and curriculum development on the teaching and learning of statistics have produced many recommendations from both researchers and national organizations on the statistical education of our students. Within the last ten years work by both statisticians and statistics educators has focused on a collection of big ideas that are the most important concepts and processes to develop the statistical thinking of our students, our work force, and the lifelong statistical literacy of our citizens. In this paper I look back at the roots of big ideas in statistics education and identify what I believe are the two most important overarching ideas for the statistical education of our students as they progress from the elementary years into tertiary. The paper discusses research on student thinking about big ideas in statistics and presents recommendations for the future of teaching and research in statistics education.

Key Words: Statistics education, distribution, inference, variability, expectation, sampling, statistical investigation processes.

Resumen

Cinco décadas de investigación y desarrollo curricular sobre la enseñanza y el aprendizaje de la estadística han producido muchas recomendaciones de investigadores y organizaciones nacionales sobre la educación estadística de nuestros estudiantes. En los últimos diez años, el trabajo de estadísticos y educadores en estadística se ha centrado en una colección de grandes ideas que son los conceptos y procesos más importantes para desarrollar el pensamiento estadístico de nuestros estudiantes, nuestra fuerza laboral y la alfabetización estadística de nuestros ciudadanos. En este artículo repaso las raíces de las grandes ideas en la educación estadística e identifico las que creo que son las dos más importantes para la educación estadística de nuestros estudiantes a medida que progresan desde los años de primaria hasta la educación superior. El documento analiza la investigación sobre el pensamiento de los estudiantes acerca de grandes ideas en estadística y

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[1] Este trabajo corresponde a una conferencia paralela dictada por el autor en la XV CIAEM, celebrada en Medellín, Colombia, del 5 al 10 de mayo de 2019.
[2] El resumen y las palabras clave en español fueron agregados por los editores.

Recibido por los editores el 13 de junio de 2019 y aceptado el 28 de julio de 2019.
Cuadernos de Investigación y Formación en Educación Matemática. 2019. Año 14. Número 18. pp 44–58. Costa Rica

presenta recomendaciones para el futuro de la enseñanza y la investigación en educación estadística.

Palabras clave: Educación estadística, distribución, inferencia, variabilidad, expectativa, muestreo, procesos de investigación estadística.

1. Introduction

Curriculum and Practice

Prior to the 1960's, there was almost no statistics included in the school curricula of many nations of the world. In their review What is Statistics Education? Zeiffler, Garfield, and Fry (2018) point to early recommendations from the 1960's in which several curriculum projects in the UK recommended the inclusion of probability and statistics in schools for students ages 11 – 16. In 1967, the American Statistical Association (ASA) and the National Council of Teachers of Mathematics created the Joint Committee on the Curriculum in Statistics and Probability in the U.S. and Canada. In the early 1970's the Joint Committee spearheaded the publication of some of the first materials for teaching statistics in schools, such as *Statistics: A Guide to the Unknown* (Tanur, Mosteller, Kruskal, Link, Pieters, & Rising, 1972), and *Statistics by Example* (Mosteller, Kruskal, Link, Pieters, & Rising, 1973). To this day the Joint Committee continues to sponsor and promote statistics education and the professional development of teachers with curriculum materials such as *The Quantitative Literacy Project* (Ganadesikan et. al., 1995) and recommendations for the teaching and learning of statistics that appeared in the GAISE documents, *Guidelines for Assessment and Instruction in Statistics Education* (Franklin, Kader, Mewborne, Moreno, Peck, Perry, & Schaeffer, 2007).

Research

Early attempts to include statistics in the education of school age students prompted research into the teaching and learning of statistics, which began in the 1970's (particularly in the UK, Germany, Israel, and the U.S. For details, see Shaughnessy, 1992). This growing international interest in teaching and research in statistics education eventually gave birth to the *First International Conference on Teaching Statistics* in Sheffield, England in 1982, ICOTS I. Since then an ICOTS has convened every four years. ICOTS X was held in Hiroshima in 2018, and ICOTS XI will take place in Rosario, Argentina in 2022, after which an ICOTS conference will have been held on every continent and in 11 different countries.

This paper presents a retrospective analysis of the development of the most important ideas in statistics education from two different viewpoints: First from the perspective of curriculum recommendations and then from a research lens.

Curriculum Documents 1: NCTM Standards for Statistics Education of K-12 Students

Starting with the *Agenda for Action* document (NCTM, 1980), through the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) and subsequently with *Principles and Standards for School Mathematics* (PSSM) (NCTM, 2000), the National Council of Teachers of Mathematics has long advocated for teaching statistics in Grades

K–12 in the United States and Canada. The 1989 standards were NCTM's first foray into establishing goals in statistics for school mathematics. In 1989, NCTM made the following recommendations:

For grades K – 4:

- Formulate and solve problems that involve collecting, describing and analyzing data
- Construct, read and interpret displays of data
- Explore concepts of chance

For grades 5 – 8:

- Systematically collect, organize and describe data
- Construct, read and interpret tables, charts, and graphs
- Make inferences and convincing arguments, and evaluate the arguments of others based on data analysis
- Develop an appreciation for statistical methods as powerful means for decision making

For grades 9 – 12:

- Construct and draw inferences from charts, tables and graphs that summarize data from real world situations
- Use curve fitting to predict data
- Understand and apply measures of central tendency, variability, and correlation
- Understand sampling and recognize its role in statistical claims
- Design a statistical experiment to study a problem
- Analyze the effects of data transformations on measure of center and variability
- Test hypotheses using appropriate statistics

The 1989 standards recommended starting with data analysis in grades K–8, but took quite a jump in depth and abstraction in the grades 9–12 recommendations by including statistical design, mathematical transformations of parameters, and hypothesis testing. The 1989 standards are predominantly a list of content, concepts and procedures that students should know and be able to do, though the process of making inferences was included for grades 5–12. Ten years later in *Principles and Standards for School Mathematics* (*PSSM*), NCTM's standards for statistics were organized under four broad processes in grade bands K–2, 3–5, 6–8, and 9–12. PSSM recommended that instructional programs PreK-12 should enable all students to:

- Formulate questions that can be addressed with data and collect, organize and display relevant data to answer them
- Select and use appropriate statistical methods to analyze data
- Develop and evaluate inferences and predictions based on data
- Understand and apply basic concepts of probability

These four broad processes remain the same throughout all four grade bands in PSSM, but grow in depth throughout the grades. For example, the trajectory for *Developing and evaluating inferences and predictions based on data* across the grades progresses through the grade bands:

PreK–2.	Discuss events related to students' experiences as likely or unlikely
Grades 3–5.	Propose and justify conclusions and predictions based on data and design studies to further investigate conclusions or predictions
Grades 6–8.	Use observations about differences between two or more samples to make conjectures about the populations from which the samples were taken. Make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots and approximate lines of fit.
Grades 9–12.	Use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions. Understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis of informal inference.

Notable in the PSSM standards when compared to the earlier NCTM standards is the growing emphasis on making and testing data-based conjectures and the introduction of the term "distribution".

Curriculum Documents 2: The Central Role of Variability—Recommendations from the American Statistical Association

During the 1980's statistics education in schools concentrated on measures of center and neglected the important role that variability plays in statistics. Mathematics curricula introduced statistics to students primarily through calculating modes, medians, and means. In his position paper on statistics content and pedagogy, president David Moore of the ASA (1997) emphasized the crucial role that variability plays in statistics education. Without variability, statistics would not even exist. The writings of Moore and others sounded a clarion call for mathematics education to rethink what the big ideas in statistics education really are. Subsequently statistics education researchers began to concentrate more on investigating students reasoning about variability. (E.g., Shaughnessy, Watson, Moritz & Reading, 1999; Melitou, 2002; Toruk & Watson, 2000; Watson, Kelly, Callingham & Shaughnessy, 2003; Watson & Kelly, 2004; Reading & Shaughnessy, 2004). The GAISE documents (*Guidelines for Assessment and Instruction in Statistics Education* (Franklin et al, 2007) appropriately pointed to the central role of variability in its four-step statistical investigation process (Figure 1)

I. Identify a Statistical Question—*Anticipate Variability*
Formulate questions that can be answered with data

II. Collect Data—*Acknowledge Variability*
Design for differences and plan to collect appropriate data

III. Analyze Data—*Account for Variability by Using Distributions*
Select appropriate graphical and numerical methods to analyze the data

IV. Interpret Results—*Allow for Variability as you look beyond the data*
Interpret the analysis and relate it back to the original question

Figure 1. The statistical investigation process

Like PSSM, the GAISE documents also emphasized statistical processes as the organizing principles for teaching statistics in K–12. GAISE describes three levels (A, B, and C) of sophistication and growth for each of the four components of the statistical investigation cycle. The three levels roughly correspond to recommendations for teaching statistics in grades K–4, 5–8 and 9–12 respectively.

Curriculum Documents 3: Recommendations from NCTM's *Essential Understandings of Statistics*

As the implementation of statistics has grown in school mathematics programs, many of our mathematics teachers have found themselves in the position of having to teach statistics concepts when they have little or no preparation in statistics themselves. In order to provide some professional development and assist middle and secondary school mathematics teachers in adding statistics to their teaching repertoire, the National Council of Teachers of Mathematics included statistics in their series *Essential Understandings* in school mathematics. The *Essential Understandings* books cover algebra, geometry, number and operation, proportional reasoning, mathematical reasoning and statistics. Both *Essential Understanding of Statistics Grades 6–8* (Kader & Jacobbe, 2013) and *Essential Understanding of Statistics Grade 9–12* (Peck, Gould, & Miller, 2013) identify some big ideas in statistics that all teachers should know and be able to teach at their respective grade levels. In the grade 6–8 book, Kader & Jacobbe identify four big ideas for teaching statistics to middle school students:

- Variability in Data and Distributions
- Comparing Distributions
- Associations between Two Variables
- Samples and Populations

The concept of a distribution plays a prominent role in all four of these recommendations if one considers that bivariate distributions of data form the basis for exploring associations between variables. In the grade 9–12 book, Peck et al identify the following essential themes that form the foundation of the big ideas in statistics:

1. Data consists of structure and variability
2. Distributions describe variability
3. Hypothesis tests answer the question, "Do I think this could have happened by chance?"
4. The way data are collected matters
5. Evaluating an estimator involves considering bias, precision, and sampling method

For Peck et al these five big organizational ideas are interrelated. Hypothesis testing is the basis for making decisions under uncertainty based on the limitations of the data provided. The data upon which statistical decisions are made are only as good as the care with which they are produced, so that attention to sources of bias and precision in estimating parameters such as measures of center and variation is critical.

2. The two BIGGEST ideas in statistics education

Suppose that you were asked to pick two ideas in statistics that you thought were the most important ones for our students to learn and our citizens to be competent in understanding. What would be your choice? The most important goal for statistics education is to enable our students and citizens to understand that decision-making under uncertainty is based upon samples of data. We rarely have access to complete information about an entire population under consideration when making statistical decisions or estimating the likelihood of events. Statistics does not rely on mathematical proof or deterministic reasoning using axioms. Rather, statistics involves making decisions based on data generated under conditions of uncertainty. Given that the *pre-eminent goal of statistics education is to understand decision making under uncertainty*, I claim that the two biggest ideas in statistics education are *distribution* and *inference*. These two ideas are the heart and soul of statistical decision making. I base this conclusion partly on the analysis above of the trajectory and development of curriculum recommendations throughout the history of statistics education, but also upon some recent research in statistics education that gives added support to the claim that *distribution* and *inference* are indeed the two biggest overarching ideas in statistics education.

Recommendations from Research: The big ideas in statistics

Beginning in the 1990's, researchers began to investigate students' understandings of big statistics concepts from a developmental perspective. Research on student understanding of concepts such as average and variability has proposed trajectories of student reasoning, levels of student understanding that become deeper over time. Furthermore, concepts such as expectation and variation are components of bigger ideas such as distribution and inference. Examples of reasoning trajectories from research on some big ideas in statistics are discussed below.

Expectation.

The term expectation encompasses research on measures of center such as mean, median, and mode as well as considerations of expected clumping in the data. Mokros & Russell (1995) proposed one of the first trajectories of students' understanding of average. Using interview tasks that involved "messy situations" from everyday familiar contexts with grades 4, 6, and 8 students Mokros & Russell identified five different ways that students think about average: Average as *mode* (mosts), average as *algorithm*, average as *reasonable*, average as *midpoint*, and average as *balance point*. Watson & Moritz (2000) interviewed about a hundred students in grades 3, 5, & 7 and found their conceptions of average moved from telling idiosyncratic stories about average to thinking of average as `mosts or middles', and eventually to understanding that an average is a representative for summarizing a data set. Reflecting on the research on students' conceptions of expectation, Konold & Pollatsek (2002) proposed that students' think of average in various ways: average as *typical value*, average as *fair share*, average as *data reducer*, or average as *signal amid noise*. It is clear from the research on students reasoning about expectation that students possess a rich collection of conceptions about expectation, which teachers can build upon. (For a more detailed discussion about research on students' conceptions of average, see for example Shaughnessy, 2007).

Variation.

Three developmental frameworks for students reasoning about variability have contributed to identifying a trajectory for students understanding of variability. (Langrall, Makar, Nilsson & Shaughnessy, 2017, p. 494, the NCTM *Compendium of Research in Mathematics Education*). Ben-Zvi (2004) noticed students begin by recognizing variability across various data values. Later students use variability to compare groups, then they are able to combine measures of spread and center in comparing groups, and eventually they consider variability as a construct that occurs both within and between distributions of data. Watson, Callingham, & Kelly (2007) describe a progression of student thinking that encompasses both expectation and variability. Reid & Reading (2008) describe a hierarchy of student reasoning about variation ranging from no consideration of variation, to recognition of variation within a group, to recognition of variation between groups that can lead to inference. In an analysis of the research on students' conceptions of variability, Shaughnessy (2007) outlined eight types of conceptions of variability that have been identified by research:

1. Variability in *particular values* in a data set
2. Variability as *change over time*
3. Variability as the *whole range* of a data set
4. Variability as the *likely range* of a sample
5. Variability as *distance from some fixed point*
6. Variability as *sum of residuals*

7. Variability as *covariation or association*

8. Variability as *distribution*

The first four types of variability in this list involve an exploratory data analysis perspective, while the last four types refer primarily to ways to measure variability. The terms variability and variation are sometimes used almost interchangeably, however some authors (e.g., Reading and Shaughnessy, 2004) prefer to use the term variability for the tendency for a characteristic to change, while the word variation is a measurement characteristic. The first four types refer to variability, while the last four involve some type of measurement of change. Research on type 4, variability as the *likely range of a sample*, has led to research about students' conceptions of sampling distributions. A closer look at some research tasks and student responses to the tasks may provide some insight into why distribution is one of the two biggest ideas in statistics education.

The Candy Sampling Task

100 candies, 20 yellow, 50 red, and 30 blue are put in a jar and mixed together. A student pulls 10 candies from the mixture, counts the number of reds, and writes that number on the board. Then the student puts the candies back in the bowl and mixes them all up again.

Four more students also draw a sample of 10 candies, and write their number of reds on the board. What numbers would you predict for the number of reds in each of those five samples of 10 candies? Write your predictions in the spaces below.

________ ________ ________ ________ ________

Why do you think those would be the numbers of reds in the five samples?

Fig 2. The candy sampling task.

This task and variations of it were given to hundreds of students in grades 4, 5, 6, 9, & 12 in Australia, New Zealand, and the United States (Shaughnessy, Watson, Moritz, & Reading, 1999). The task was used to determine what students perceived as the *likely range* of values that would occur in a repeated sampling scenario. Student responses fell into clusters that were deemed *narrow, wide, high, low*, and *reasonable*. For example, some students said they'd expect the results of the sample to be 6, 7, 5, 8, 9 reds because `there are a lot of reds in the jar.' This is typical of a *high* response as all the sample predictions are above the expected value of 5 red. High responses focused on `mosts', and ignored the proportion of reds in the mixture. In *low* responses like 3, 4 ,3, 5, 2 students felt that the other two colors would overwhelm red, so there would be fewer reds than might be expected. Students who predicted *wide*, like 1, 5, 7, 10, 2, did so because they claimed that `anything can happen.' On the other hand, some students predicted results like 5, 5, 6, 5, 6, or even 5, 5, 5, 5, 5 because `that's what is supposed to happen.' Such *narrow* predictions put too much weight on the theoretical probability of obtaining 5 red candies on any one pull, and neglected potential variability in repeated samples. Overall, some students attended to centers too much, some to variability too much, while other students did rely on both expectation and variation to predict a *reasonable* range of numbers of reds in the outcomes, such as 3, 7, 5, 6, 4. Research on tasks like the candy sampling task have led to research into students' conceptions about distributions, in particular their conceptions about sampling distributions. The candy sampling task also surfaces the tension that can arise between attending to expectation or attending to variability in data, especially when students are asked to make predictions for samples from a known population (Watson 2009; Watson & Kelly, 2004).

Distribution.

Comparisons across several hypothesized developmental frameworks for the concept of distribution are provided in Langrall et al (2017, p. 494). Each of these frameworks acknowledges that the concept of distribution encompasses multiple aspects such as shape, variability, and expectation, and that integration of all of these aspects is required for students to reason about and make inferences from distributions. Reading and Reid's framework (2006) for understanding distributions starts with students acknowledging a single parameter of a distribution (e.g., center), then several parameters, next integrating centers and spreads, and finally proceeding to a second cycle of reasoning which involves students' growth in making inferences from distributions. A framework for distributional thinking proposed by Noll & Shaughnessy (2012) was based on their research on students' conceptions of sampling distributions. According to Noll & Shaughnessy, students' development of the concept of distribution involves the gradual integration of shape, centers, and spread. Students first notice individual aspects of a distribution, then learn to make predictions for sampling distributions by relying on both expectation and variation. Noll & Shaughnessy propose that students' reasoning about sampling distributions progresses through levels from *additive* to *proportional* and finally to *distributional* reasoning. (See Figure 3).

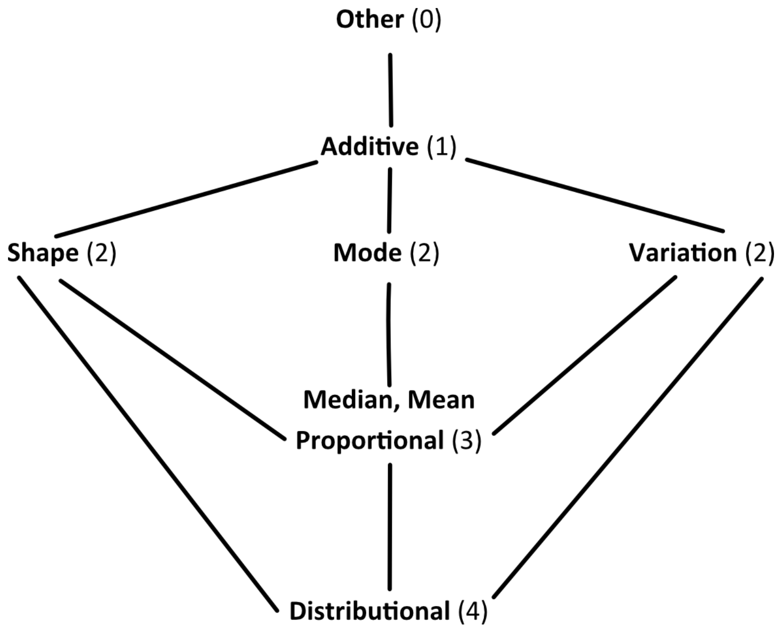


Fig 3. Lattice of student reasoning about sampling distributions.

The lattice in Figure 3 points first to a development of conceptions of expectation from `more' to `most' (mode) to `means and medians' (which involve proportional reasoning), and finally to reasoning about distributions in which students' thinking integrates the aspects of shape, expectation, and variation. Students' responses to tasks like the *Prediction Task* and the *Mystery Mixture* task (see below) led Noll & Shaughnessy to propose this developmental reasoning lattice for distributional thinking.

Working in small groups, students in a class pull samples of 10 candies from a jar that has 1000 candies. They pull 50 samples of ten. The jar has 250 yellow and 750 red candies in it. Each time they put the sample back and remix the jar.

Consider the number of reds in each handful. Where would you expect 95% of the handfuls of ten candies to be?

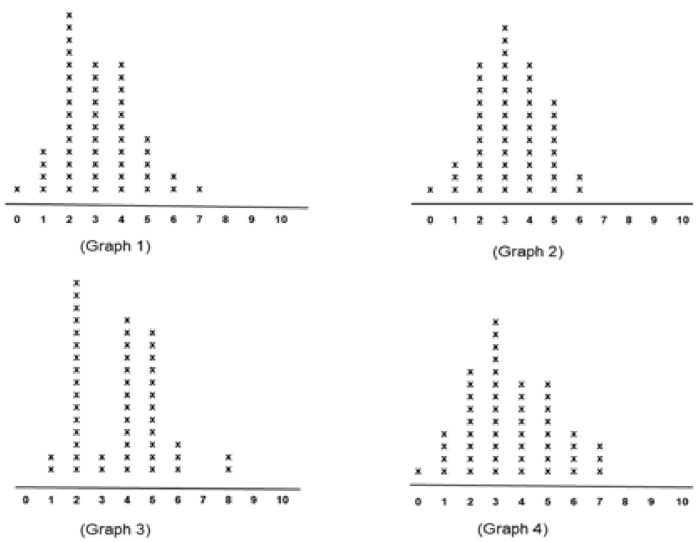
From __________ # reds To _______________# reds (Fill in the blank spaces).

Why do you think that?

Complete the frequency chart below to show what you think the numbers of reds in 50 trial handfuls might look like. (Note: students were provided labeled graph paper to plot their predicted sampling distribution).

Fig 4. The Prediction Task.

The four frequency graphs below came from a class in which four groups of students drew 50 samples of ten candies to estimate the proportion of reds in a mystery mixture of 1000 red and yellow candies in a large jar.



a) What do you think the mixture in the jar might be?

b) Explain why you think this.

Fig 5. The Mystery Mixture Task.

In the Prediction task students are told ahead of time the proportions of the colors in the parent population and then asked to predict what a sampling distribution for sample proportions will look like. As with the Candy Task, most students' responses fell into the categories *wide, narrow, and reasonable* for both their predicted range for the # of red in handfuls, and in the graphs that they constructed for 50 sample proportions.

However, in the Mystery Mixture task students are provided multiple sampling distributions from an unknown population and asked to use them to infer what the parent population is. Students who predicted 200-250 reds in the mystery mixture reasoned using a `mosts' point of view, and were heavily influenced by distributions 1 & 3 which have a mode of 2 reds in handfuls. Students who looked for `balance points' of the graphs reasoned proportionally and tended to predict around 300 reds in the mixture. Still other students noticed that the graphs tended to be skewed to the right, and they integrated shape with center and spread into their reasoning and inferred that the mixture probably contained more than 300, perhaps even 350 - 400 reds. In both the Prediction task and the Mystery Mixture task students had opportunities to focus solely on one of the aspects of the sampling distributions (shape, centers, variability) or to integrate them into their predictions and inferences.

Inference.

The Mystery Mixture task asks students to make an inference from several sampling distributions of a sample statistic. There is no formal hypothesis testing involved, students are simply asked what they believe the composition of the parent population is and why they believe it. Researchers and curriculum developers refer to this type of inference as `informal inference'. Informal inference has its roots in exploratory data analysis, often in the exploration of data that have been produced from simulations. Students can estimate likelihoods from samples of data without resorting to a test statistic or working with a probability distribution. Cobb (2007) argued that introductory statistics courses should start with inference early on prior to any hypothesis testing that resorts to theoretical distributions. Since the logic of formal statistical inference has always been a difficult stumbling block for students, educators have been experimenting with various approaches to introducing inference that could avoid some of the cognitive complexity and pitfalls in formal inference. Rossman (2008) promotes simulations via randomization tests as a transparent informal approach to statistical inference. Zeiffler, Garfield, Del Mas, & Reading, (2008) define informal statistical inference as students using informal statistical knowledge about observed samples to support inferences about unknown populations. Makar and Rubin (2018) point out that there is general agreement that the important characteristics of informal inference are: i) a claim is made that goes beyond the data at hand; ii) the data are used as evidence to support the claim; iii) the claim involves the articulation of uncertainty; iv) decisions or inferences are based upon aggregates in the data, variability, or shape, i.e., aspects of distributions of data; v) contextual knowledge plays a role in the analysis and inference. Inference is one of the two biggest ideas in statistics because students, even at a very young age, can begin to make inferences based on data that they have collected on a statistical question. The statistical investigation cycle outlined in the GAISE document—pose a question, collect data, analyze the data, make conclusions—is even more powerful for students when it includes making inferences in the analysis and conclusions phases.

3. Recommendations for future research and teaching on the big ideas in statistics

The future of research in statistics education

Many of the big conceptual ideas in statistics such as distribution, expectation, variation, and randomness identified above reside predominantly in the third stage of the GAISE statistical investigation cycle, analyzing the data. However, there are other important processes in the statistical investigation cycle, such as posing a statistical question, generating appropriate data to answer a statistical question and communicating the results and conclusions. These other statistical processes must also be included in the statistical education of our students and in the professional development of our classroom teachers. Unlike some big concepts such as expectation, variability, and distribution there has not been much research on developing students' ability to pose statistical questions or their ability to communicate and defend their data-based conclusions. The first and last phases of the GAISE statistical investigation cycle have not yet been adequately researched.

More research is needed to validate the developmental frameworks that have already been proposed for expectation, variation, and distribution. Meanwhile the next `big idea' in research appears to be inference, in particular informal inference. A special issue of the *Statistics Education Research Journal* was dedicated to articles about informal inference, particularly within statistical modeling contexts (*SERJ* 16 (3), November 2017). Various definitions of informal inference have been proposed and some of the components of informal inference have been identified. However, a developmental framework for students' reasoning about inference analogous to those for variability and distribution does not yet exist. Case & Jacobbe (2018) report a framework for understanding students' difficulties when making inferences from simulations. However, much more research is needed about how inferential reasoning develops starting with young children and up through the grades in order to identify potential levels of student reasoning about inference.

The future of teaching statistics

Teaching is a social process; it involves countless interactions between students and their instructors. Any recommendations for teaching statistics must include considerations about the teacher as well as the students. Our teachers are on the front line of statistics education, and many of the teachers in our current work force still do not have sufficient experience with statistics. Most of them are mathematics teachers. As Cobb & Moore (**1997**) pointed out, mathematics and statistics are very different disciplines. Mathematics is grounded in certainty and deductive reasoning based on axioms and previously established results. Statistics on the other hand lives in the realm of uncertainty; statistical results are couched in terms of likelihoods, probabilities, or confidence intervals. Mathematics and statistics are epistemologically and philosophically different from one another. Our teachers need experiences themselves in carrying out statistical investigations—perhaps in conjunction with their students—they need to immerse themselves in thinking about and drawing conclusions from data. Teachers need to develop both their content knowledge and their pedagogical content knowledge of statistics. The NCTM *Essential Understandings* series in statistics provides content knowledge support for middle and secondary school teachers of statistics (Kader & Jacobbe, 2013; Peck, Gould & Miller, 2013). The ASA GAISE documents also provide support for developing teachers' pedagogical content knowledge in statistics. Both publications provide many examples of tasks that can be implemented in the classroom using the statistical investigation cycle.

As for our future teachers of statistics, the ASA recently developed and published The *Statistical Education of Teachers* (*SET*) which lays out recommendations for the statistical education of all perspective elementary, middle, and secondary teachers (Franklin, Bargagliotti, Case, Kader, Scheaffer, & Spangler, 2015). SET recommends both coursework and statistical modeling experiences for all teachers. Coursework for all teachers should begin with a data analytic approach. Additional recommendations for middle and secondary school teachers include coursework in statistical methods and statistical modeling. The ASA has taken a very futuristic view in the SET document, projecting that the need for statistics education will continue to grow throughout the world. Teachers will need to know much more about statistics and statistical thinking to prepare students and citizens for our data-intense world.

What does our walk through the history and research on the Big Ideas in statistics suggest for the teaching and learning of statistics for our students? We should START with the big ideas. Introduce the concept of a statistical question, one that requires data to be answered. Make sure that students understand the difference between mathematics and statistics; they are two different disciplines involving two different types of reasoning. Give students opportunities early on to make informal inferences about distributions of data. Ask students, "What do you notice? What do you wonder about?" Get students involved in generating sampling distributions from repeated samples from both known and unknown populations, making informal inferences from the samples they have obtained. First have students recognize and attend to important aspects of distributions such as shape, centers, and variability, later include ways of measuring expectation and variation. Use the research on developmental frameworks for expectation, variability and distribution to help guide instruction. Get students in the habit of posing their own statistical questions while using the statistical investigation cycle from GAISE. Make sure that attention to variability is foremost throughout the statistical investigation cycle. Most of all, empower students to be competent and confident with the big concepts and processes of statistics, and with the nature of statistical argumentation.

Referencias y bibliografía

Ben-Zvi, D. (2004) Reasoning about variability in comparing distributions. *Statistics Education Research Journal, 3*(2), 42-63.

Case, C. & Jacobbe, T. (2018). *Statistics Education Research Journal, 17*(2), 9-29. Retrieved from **https://iase-web.org/documents/SERJ/SERJ17(2)_Case.pdf**

Cobb, G. (2007). The introductory statistics course. A Ptolemaic curriculum? *Technology Innovations in Statistics Education, 1*(1), Article 1. Retrieved from **http://escholarship.org/uc/item/6hb3k0nz**

Cobb, G. & Moore, D. (1997). Mathematics, statistics, and teaching. *The American Mathematical Monthly,104*(9), 801-823.

Franklin, C., Kader, G., Mewborne, D., Moreno, J., Peck, R., Perry, M., & Scheaffer, R. (2007). *Guidelines for Assessment and Instruction in Statistics Education*. Alexandria, VA: ASA.

Franklin, C.A., Barbabliotti, A. E., Case, C. A., Kader, G. D., Scheaffer, R. L., & Spangler, D. A. (2015). *The Statistical Education of Teachers*.

Ganadesikan, M., Scheaffer, R. L., Landwehr, J. M., Watkins, A. E., Barbella, P. Kepner, J., Newman, C. M., Obremski, T. E., & Swift, J. (1995). *Quantitative Literacy Series*. New York: Pearson Learning (Dale Seymour Publications).

Kader, G. & Jacobbe, T. (2013). *Developing Essential Understanding of Statistics Grades 6-8*. Reston, VA: NCTM

Konold, C., & Pollatsek, A. (2002). Data analysis as a search for signals in noisy processes. *Journal for Research in Mathematics Education, 33*(4), 259-289.

Langrall, C. W., Makar, K., Nilsson, P., & Shaughnessy, J. M. (2017). Teaching and learning probability and statistics: An integrated perspective. In J. Cai (Ed.), *Compendium for Research in Mathematics Education*. Reston, VA: NCTM.

Makar, K., & Rubin. A. (2018). Learning about statistical inference. In D. Ben-Zvi, K. Makar, & J. Garfield (Eds.). *International Handbook of Research in Statistics Education* (pp. 261-294). Cham, Switzerland: Springer International.

Melitou, M. (2002). Conceptions of variation. A literature review. *Statisics Education Research Journal, 1*(1), 46-52.

Mokros, J., & Russell, S. J. (1995). Children's concepts of average and representativeness. *Journal for Research in Mathematics Education, 26*(1), 20-39.

Moore, D. (1997). New pedagogy and new content: The case of statistics. *International Statistics Review, 65*(2), 123-165.

Mosteller, F., Kruskal, W. H., Link, R. F., Pieters, R. S., & Rising, G. R. (1973). *Statistics by Example*. Reading, MA: Addison-Wesley.

National Council of Teachers of Mathematics. (1980). *An Agenda for Action*. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (1989). *Curriculum and Evaluation Standards for School Mathematics. Reston*, VA: NCTM.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for school Mathematics. Reston*, VA: NCTM.

Noll, J., & Shaughnessy J. M. (2012). Aspects of students' reasoning about variation in empirical sampling distributions. *Journal for Research in Mathematics Education, 43*(5), 509-556.

Peck, R., Gould, R., & Miller, S. J. (2013). *Developing Essential Understanding of Statistics Grades 9-12*. Reston, VA: NCTM.

Reading C., & Reid, J. (2006). An emerging hierarchy of reasoning about distribution: From a variation perspective. *Statistics Education Research Journal, 5*(2), 46-68.

Reading, C., & Shaughnessy, J. M. (2004). Reasoning about variation. In D. Ben-Zvi & J. Garfield (Eds.). *The Challenge of Developing Statistical Literacy, reasoning and thinking* (pp. 201-226). Dordrecht, The Netherlands: Kluwer.

Reid, J., & Reading, C. (2008). Measuring the development of students' consideration of variation. *Statistics Education Research Journal, 7*(1), 40-59.

Rossman, A. (2008). Reasoning about informal statistical inference: One statistician's point of view. *Statistics Education Research Journal, 7*(2), 5-19.

Shaughnessy, J. M. (1992). Research in probability and statistics: Reflections and directions. In D. Grouws (Ed.). *Handbook of Research on Mathematics Teaching and Learning* (pp. 465-494). New York, NY: Macmillan.

Shaughnessy, J. M. (2007). Research on statistics learning and reasoning. In F. Lester (Ed.). *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 957-1009). Reston, VA: NCTM.

Shaughnessy, J. M., Watson, J., Moritz, J. & Reading, C. (1999. April). School mathematics students' Acknowledgement of statistical variation: There's more to life than centers. Paper presented at the research pre-session of the 77th annual meeting of the National Council of Teachers of Mathematics, San Francisco, CA.

Tanur, J., Mosteller, F., Kruskal, W. H., Link, Pieters, R. S., & Rising, G. R. (1972). *Statistics: A Guide to the Unknown*. San Francisco, CA: Holden-Day.

Toruk, R., & Watson, J. M. (2000). Development of the concept of statistical variation: An exploratory study. Mathematics Education Research Journal. 9, 60-82.

Watson, J. M., (2009). The influence of variation and expectation on developing awareness of distribution. *Statistics Education Research Journal, 8*(1), 32-61.

Watson, J, M., & Kelly, B. A. (2004). Expectation versus variation: Students' decision making in a chance environment. *Canadian Journal of Science, Mathematics, and Technology Education, 4*, 371-396.

Watson, J. M., & Moritz, J. B. (2000). The longitudinal development of understanding average. *Mathematical Thinking and Learning, 2*, 11-50.

Watson, J. M., Callingham, R. A., & Kelly, B. A. (2007). Students appreciation of expectation and variation as a foundation for statistical inference. *Mathematical Thinking and Learning, 9*(2), 83 – 130.

Watson, J. M., Kelly, B. A., Callingham, R. A, & Shaughnessy, J. M. (2003). The measurement of school Students' understanding of statistical variation. *International Journal of Mathematical Education in Science and Technology, 34*(1), 1-29.

Zeiffler, A., Garfield, J., Del Mas, R., & Reading, C. (2008). A framework to support research on informal inferential reasoning. *Statistics Education Research Journal, 7*(2), 40-58.

Zeiffler, A., Garfield, J., & Fry, E. (2018). What is statistics education? In D. Ben-Zvi, K. Makar, and J. Garfield (Eds.). *International Handbook of Research in Statistics Education*, 37-70.