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A General Scheme for a Heterogeneous Manifold of Transitions

Un schéma général pour un ensemble hétérogène de transitions

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Abstract

A general praxeological scheme is applied to express relations between praxeological blocks depending on the goal of the analysis and the specific institutional setting within which a mathematical praxeology is considered. Besides its heuristic function, the scheme provides a framework for context dependent categorizations of praxeologies. The poster exemplarily illustrates the application of the scheme to two different contexts: measures supporting students in their first year of study; the use of mathematics in engineering sciences.

Keywords: ATD; categorization of praxeologies; mathematic students; mathematics in engineering

Résumé

Un schéma praxéologique général est utilisé pour décrire les relations entre des blocs praxéologiques en fonction de l’objectif de l’analyse et de l’environnement institutionnel dans lequel on considère les praxéologies mathématiques. À côté de cette fonction heuristique, le schéma propose un cadre pour la catégorisation des praxéologies à partir de deux contextes différents : des mesures de support pour les étudiants universitaires de première année, l’utilisation des mathématiques dans les sciences de l’ingénieur.

Mots-clés : TAD, Catégorisation des praxéologies; Étudiants en mathématiques; Mathématiques en ingénierie

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A General Scheme for a Heterogeneous Manifold of Transitions

The anthropological theory of the didactic (ATD) aims at a precise description of knowledge and its epistemic constitution (Chevallard, 1992, 1999; Winslow, Barquero, Vleeschouwer & Hardy, 2014). This theoretical framework allows explicating institutional specificities of knowledge and related practices. A basic concept of ATD are praxeologies, which are represented in so called "4T-models (T, τ, θ, Θ)" consisting of a practical and a theoretical block. The practical block (know how, "doing") includes the type of task (T) and the relevant solving techniques (τ). The theoretical block (knowledge block, discourse necessary for interpreting and justifying the practical block, "spoken environment") covers the technology (θ) explaining and justifying the used technique and the theory (Θ) justifying the underlying technology. Praxeologies give descriptions of mathematics by reference models that are activity oriented (techniques, technologies). The interconnectedness of knowledge is modelled in ATD by means of local and regional mathematical organizations that allow contrasting and integrating practical and epistemological aspects in view of different institutional contexts.

A General Scheme

Following Winslow & Grønbæk (2014) we refer to the notion $R_I(x, o)$ introduced by Chevallard (1991) to indicate the relation of a position x (roles of persons such as teachers and students) within an institution *I* to a praxeology *o*. Examples for institutions are in our case: school, university, engineering course, mathematical lectures, lectures about pedagogical content knowledge, etc. Transitions between relations are in the following represented by an arrow. Of course, transitions can itself be described in terms of 4T-models. But a discussion of this point lies beyond the scope of this contribution.

Then a general scheme suitable for describing and analysing a large and heterogeneous family of transitions is given as follows:

$$R_{S_1}(s_1, o) \rightarrow R_{S_2}(s_2, \omega[X(o)]) \; with \; X \in \wp\{\tau, \theta, \Theta\},$$

where *o* represents a praxeology within an institution S_1, $\wp\{\tau, \theta, \Theta\}$ the power set of $\{\tau, \theta, \Theta\}$ (the set consisting of technique, technology and theory of o) and ω a praxeology within an institution S_2 in view of one or several blocks of the praxeology o. The latter generality in the scheme allows to express that techniques, technologies or theories of *o* might be differently relevant in the relation of a position x_2 within the institution S_2 to a (perhaps new) praxeology ω.

We consider the scheme amongst others as a heuristic tool for the characterization and categorization of institutionalized and possibly problematic (from an individual and/or an institutional point of view) relations. It allows focusing on relations between praxeologies and their blocks as well as on specific task constructions or course designs etc. We claim that such a scheme particularly allows to focus on important knowledge and goal related institutional aspects, which have to be addressed, if one intends to optimize teaching and learning conditions.

Application contexts

The poster focuses on the heuristic function of the scheme and demonstrate its potential usefulness for characterizing and categorization praxeological issues in two different contexts:

a) measures supporting students in their first year of study;

b) the use of mathematics in engineering sciences.

Ad a) Measures for supporting students in their first year of study are investigated in the WiGeMath project (*Wirkung und Gelingensbedingungen von Unterstützungsmaßnahmen für mathematikbezogenes Lernen in der Studieneingangsphase*; Effects and success conditions of mathematics learning support in the introductory study phase), which is a joint research project of the Universities of

Hannover and Paderborn (Colberg et al., 2016; Liebendörfer et al.) led by Biehler, Hochmuth and Schaper. The aim of the WiGeMath project is to develop and exemplify a taxonomy that categorizes features and goals of projects of mathematics learning support and to use this taxonomy to evaluate different support measures at German universities. In the WiGeMath project, different supporting measures are subsumed under one of four categories, namely bridging courses, mathematics support centres, support measures that parallel courses and redesigned lectures. With design challenges for bridging courses in mind a more specific scheme as above was used in (Biehler & Hochmuth, 2017).

For the poster results from the project are picked up and reanalyzed applying the above general scheme as lens. It illustrates in particular the following three basic foci of supporting students in transitions:

- Techniques: Improving skills in applying techniques stemming from current or past attained praxelogies without further developing technology ($X = \{\tau, \theta\}$), for example:
    - determining solutions to quadratic equations by using square roots;
    - calculating proper decimal fractions as approximations using hand calculators.
- Technology: Improving and extending technological knowledge concerning past attained praxelogies ($X = \{\theta\}$), for example:
    - knowing, that square roots are treated symbolically and that they can only be approximated by finite decimal fractions.
- Theory: Improving or reflecting theoretical and technological aspects of past attained praxelogies ($X = \{\theta, \Theta\}$), for example:

- constructing the set of reals from the known“ rationals in a mathematical exact construction;

- starting with the axioms of $\mathbb{R}$ and identifying of natural and rational numbers in this new axiomatically defined object.

Ad b) Here, S_1 typically represents a Higher Mathematics course for engineering students (HM), S_2 a course on System and Signal Theory for electrical engineering students (SST). For further details we refer to (Schreiber & Hochmuth, 2015; Peters, Hochmuth & Schreiber, 2017). One can generally observe that basic techniques and notions from HM are relevant for SST, but details concerning the justification of assertions taught in HM are not only nearly irrelevant but often misleading. For the poster, again, we point to a three typical situations in SST:

• HM-technique but SST-justification ($X = \{\tau\}$);

• SST-technique relying on HM-techniques and HM- as well as SST- justifications and –theory ($X = \{\tau, \theta, \Theta\}$);

• SST-justification relying on HM-technique and –justification ($X = \{\tau, \theta\}$).

Outlook (Work in Progress)

The psychological lens of qualitative learning jumps has been introduced by Holzkamp (1993, pp. 239) within the framework of a learning theory that is based on the subject scientific approach (Holzkamp, 1985; see also Tolman (1991) for an English written introduction) and aims (besides others) to provide individuals with analytic tools for the self-reflection of problematic experiences and situations to reveal their inherent dependencies and circumstances. The focus lies on institutionalized teaching-learning situations. Ideas from the *Theory of Didactic Situations* as well as from the notions *Didactic Moments* and *Study & Research Courses* might also be taken into account. It turns out, that depending on the teaching-learning situation steps of qualitative learning jumps can be related to specific types of the general scheme. A suitable mathematical context is provided by the convergence of function sequences differentiating pointwise, uniform as well as $L^p(0 < p \leq \infty)$-convergence, linear and nonlinear approximation and smoothness modules.

The poster

The poster explains the scheme and presents for both application contexts examples demonstrating the heuristic function of the scheme and its potential for characterizing and categorization issues.

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