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The external didactic transposition of mathematics at university level: dilemmas and challenges

La transposition didactique externe des mathématiques au niveau universitaire: dilemmes et défis

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Résumé

Nous développons l'idée que la recherche en didactique des mathématiques au niveau universitaire devrait étudier d'une manière plus systématique la transposition didactique externe. Après avoir présenté sommairement la recherche qui semble aller dans cette direction, nous commentons pourquoi et comment cette ligne pourrait se poursuivre dans la recherche en TAD sur l'enseignement universitaire.

Mots -clés : Théorie anthropologique de la didactique, Transposition didactique externe, Mathématiques au niveau universitaire.

Abstract

We develop the idea that research on university mathematics education needs to more systematically address the external didactic transposition. After outlining existing research that goes more or less in this direction, we comment on the why and how this direction could be pursued by ATD research on UME.

Keywords: Anthropological theory of the didactic, External didactic transposition, Mathematics at the university level.

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University mathematics education (UME) designates a heterogeneous set of didactic practices, occurring in a variety of higher education institutions. These institutions carry a number of conditions which are not shared by secondary and primary education, such as:

• The proximity of scholarly knowledge and teaching practice, both at the institutional and personal level (teachers are often also researchers);

• The professional and scholarly aims and potentials of instructional processes (some students will become researchers or take on other professions related to scholarly knowledge);

• Students engage in a higher education programme based on a personal choice, often linked to plans for a future career;

• Knowledge is commonly delivered in 'modules' (courses, project units, etc.) which may be mandatory or optional, and can be passed more or less independently;

• The teaching institution is itself responsible of the elaboration of curricula and syllabi.

It should also be noted that mathematics as a discipline takes on two specific roles in higher education programmes, which may in practice be quite different:

• As the *core discipline* within programmes of pure or applied mathematics;

• As an *auxiliary* or more or less *integrated* discipline within a large number of other specialties, such as engineering, natural science or business programmes.

This paper begins from the assumption that the basic features of didactic transposition still makes sense for the case of UME, and includes two major steps (Chevallard, 1991, 35):

• *External didactic transposition* (EDT), where scholarly knowledge (whether purely mathematical, or mathematics integrated within other specialties) is selected, adapted and declared as “matter to be taught”, resulting in documents such as syllabi, textbooks and other resources and regulations for teaching.

• *Internal didactic transposition* (IDT), through which the “matter to be taught” is transposed to fit the demands of actual teaching (lectures, assignments, evaluations etc.), with due adaptation to the more or less specific conditions and constraints of the institution and its members.

One can interpret almost all existing research on UME as primarily concerned with IDT, which considers the “knowledge to be taught” (along with the institutional conditions and constraints) as more or less given. Concluding a synthesis of 20 years of European research on university mathematics education, the authors wrote:

> … we should mention also the potential, but currently quite limited, impact [of research on UME] at the level of innovation of curricula and policy. Some of the more global problems identified by [research on UME], such as transition problems between secondary and tertiary institutions (…), the isolation of mathematics courses in non-mathematics majors (…) or the need for more systematic and research based UME teacher development and UME practice innovation (…), clearly call for research and impact at this level (Winsløw, Gueudet, Hochmut & Nardi, to appear).

Besides citing some of the major challenges identified through papers on UME presented over 20 years at the CERME congress, this quote implies a need for systematic research into EDT at university. The naïve interest of this endeavour is to achieve a better understanding of how the contents and other ‘given’ aspects of UME came to be what they currently are, as well as how they are (or could be) changed. But what would such research look at exactly? What results could it produce, with what effects?

Our motivation for considering these questions is an apparent paradox: in spite of the proximity of scientific research already mentioned, the work demanded of students in university mathematics courses often appears very far from research. Already Klein

(1908, 250) warned against university mathematics study "building only on what one reads in books" while ignoring the development of "vivid conceptions" and "independent judgment". Burton (2004, p. 198) conducted a systematic study on mathematicians' beliefs and practices as researchers, and also reflected on the contrast with common forms of mathematics teaching: "the gap between mathematicians' views of mathematical knowing and that encountered by learners is monstrous" (p. 198). Madsen & Winsløw (2009) found that mathematicians tend to see at most indirect connections between their tasks as teachers and as researchers, and generally consider that the accumulation of specialised knowledge makes it impossible for students to encounter genuine research at the undergraduate level. Similarly, Barquero, Bosch, and Gascón (2013) pointed out that the teaching of mathematics to students in non-mathematics programmes often fails to engage students with genuine problems and models from the disciplines they study.

In the CERME context mentioned above, we do find numerous small scale innovations in UME which attempt to offer students more "research like" activities than lecture attendance and exercise solving. However, these innovations almost all concern the IDT, while many of them take note of obstacles from the "context" (such as an overloaded syllabus, exam practices etc.). Many of these obstacles are in fact products of the EDT. Research should go beyond noting them as just boundary conditions.

Unlike other school institutions, universities are to some extent themselves in charge of the EDT (from the construction of educational programmes to the production of teaching material). And this transposition work depends not only on specialized academic knowledge but also on university-internal interests and policies, external interests (such as target professions and their needs), and on further external constraints such as resources, regulations and policies decided outside of the institutions themselves.

The aim of this paper is to outline some directions and ideas for research on EDT in UME and to nourish an important methodological debate within the ATD community.

Theoretical questions and framework

To delimit our object of study and better define its scope, we propose a first rough terminology to talk about different university teaching entities of different size. Providing a common language is not only necessary: it also shows, by its current absence, how little attention is paid to these entities. We shall name three main units that are characterized by different extension in time:

- A *period* is an event of teaching or study which is defined by the institution and takes place within at most a day (e.g. a lecture, a tutorial, a lab session etc.).
- A *module* is a theoretical description of a set of study and possibly teaching activities which take place within at most one year, and which the student must complete as a whole, whereupon the institution will deliver some official recognition to confirm completion, usually after some form of assessment (a specific part of the module). A module may for instance describe a course unit such as 'Calculus 3', or an independent project or assignment.
- A *study programme* is a collection of modules which, once they are all completed, entitles the student to a degree that is often used to name the programme (e.g. bachelor, master, PhD, etc.). The length can vary from one to several years.

Certainly, the above notions reflect the 'programming of learning' which Verret (1975, 146) considers one of the characteristic features of the 'bureaucratic transmission' of knowledge delivered by contemporary mass universities. They are thus pragmatic categories for research into current practices.

We shall consider also an intermediate level: within a programme, modules may be structured in *sequences* of modules (a collection of modules concerned with the same knowledge sector, in the sense of ATD, and often to be taken in a certain order – e.g. Calculus 1-2-3). Our terminology is summarized in Table 1, which indicates the approximate relationship with the levels of didactic co-determination (CHEVALLARD, 2002), along with examples of these levels in the case of a classical study programme on mathematics.

Table 1:

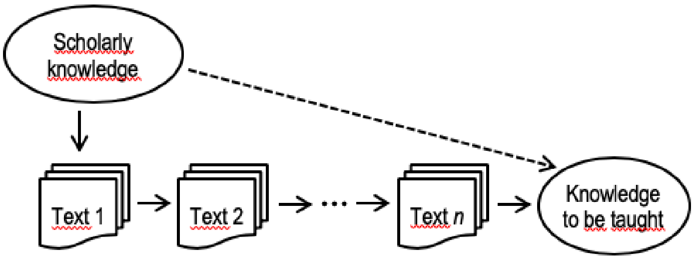
Some first terminology

Unit:	Duration:	Level of didactic co-determination
Study Programme	≥ 1 year	Discipline (1 or more) - e.g. Mathematics
Sequence of modules	> 1 module	Domain - e.g. Calculus
Module	≤ 1 year	Sector - e.g. vector fields
Period or small sequence of periods	≤ 1 day	Theme or Subject – e.g. curve integrals, gradient theorem

A possible way to explain what could cause the between teaching and scholarly knowledge (current research) in universities, is that EDT is not a simple, one-step transformation of fresh, scholarly knowledge to principles and presentations of knowledge to be taught. EDT relies crucially on what Chevallard (1991, 61) calls the *textualisations of knowledge*, which detaches the 'synthesis' of research productions from the original context of production, and its problems. What is even more important: the resulting texts are soon rewritten and combined with other previous texts, and in other ways generate a whole posterity of supposedly refined and improved texts (including present day textbooks, Wikipedia entries, etc.). For instance, the hundreds of textbooks on Calculus written in English over the past 50 years probably draw more on previous textbooks than on scholarly sources. The same could be the case for legal descriptions of modules, study programmes and so on: they are continuously rewritten. A large part of ongoing EDT may thus be *solely concerned with other products of EDT*, not direct transpositions of scholarly knowledge (see fig. 1). Thus, the relation between knowledge to be taught and the scholarly institution and its praxeologies may be more indirect, even in the case of universities. For instance, the ongoing transposition might be informed in some way by a development in the scientific field, without necessarily relating to the actual sources of the knowledge to be taught (cf. Fig. 1, dotted arrow).

Figure 1

A refined model of external didactic transposition.



We can now formulate, with a bit more precision, the general research questions which we consider in this paper:

1. What are the main processes and rationales in the elaboration and change of UME study programmes, sequences and modules?

2. What institutional mechanisms contribute to the process? What is the noosphere in this case? What conditions and constraints appear?

In particular, how does scholarly knowledge (for instance, university teachers' own research activity) affect the construction of UME programmes and modules at the level of institutions? What is the respective role of 'didactic syntheses' of knowledge (e.g. textbooks, articles) and of problems (pursued by scholars, at present or in the past)?

Studying the *process* of EDT has two major implications, which can raise important methodological challenges for researchers. The first one is an essential specificity of the French tradition in didactics: being aware of the fact that a transposition takes place, and studying its historical variations, reveals that the pieces of knowledge involved in an instructional process are not evident or transparent. They are the fruit of a process where many decisions are taken by several agents. This process can happen in different ways and bring about different final products. Research should question and analyse the quality and the specificities of these products. Moreover, the study of EDT also reveals that what is taken as the core of the teaching and learning process depends on the institutions involved in the whole process and on their way of conceiving knowledge (the institutional epistemologies). Even the way of delimiting, labelling and describing the 'target knowledge' of a teaching and learning process may have important implications.

The second implication is a direct consequence of the first. It refers to the *units of analysis* that researchers consider when analysing a given teaching and learning process. Because instructional processes depend on what is elaborated as bodies of knowledge to be taught and leant (and not only in the way it is taught and learnt), researchers should consider the origin of these bodies and its process of production. And, of course, the institutional time and space dependence of the EDT process should be included in the empirical evidence to support the analysis. It is not enough to take into account what teachers and students do in a classroom (and outside it), it is also necessary to consider the steps followed by the knowledge since it is produced in a scholarly institution till it becomes an instructional target: what pieces of knowledge (and know-how) are selected and why, where these pieces come from, who produced it and what for, etc.

Elaborating suitable methodological tools to take all this social, historical, epistemological and even political institutional evidence into account requires a wide range of expertise, including (but certainly not limited to) that of scholars, i.e. advanced mathematics. This might explain, in part, the rarity of research which undertakes a critical study of EDT in the UME context.

Preliminary answers from previous research

We now outline some of the (not very numerous) studies which could inspire and contribute to the pursuit of the research questions and which hold ideas or results that could be taken further with the theoretical affordances of ATD.

Evidently, university teachers and researchers of mathematics are central agents whose shared views and knowledge are of great impact on the questions raised. Grigutsch and Törner (1998) investigated the global views on mathematics held by German mathematicians and found that they privilege a view of mathematics as a process of 'discovery and understanding', rather than as a 'collection of complete knowledge' (amongst a total of five positions identified by factor analysis on a questionnaire with multiple items which respondents could indicate their agreement with). They note that "the reality of mathematics teaching in practice is not identical with this view - lectures are the imparting of complete knowledge with a high degree of formal exactness", which aligns with the 'gap' later found by Burton (cf. above). Many opinion pieces by individual mathematicians argue that university teaching should somehow be changed to reflect the 'creative side' of mathematics and not (just) the 'created side':

> In the end we won't be able to cover all the mathematics that every student will need in later life, so we need to think about how best to teach students so that they are capable of finding out new, to them, mathematics if and when they require it (Holton, 2005, 306).

In a similar vein, based on interviews with a large number of interviews with mathematics students and graduates, Petocz and Reid (2012, 90) argued for a "curriculum that is outward looking focuses on the use of mathematics as a way of thinking, an approach to life, an inclusive tool to investigate—and even change—the world".

Naturally such general ideas do not settle what study programmes actually should consist of, and can be considered general philosophies about the discipline as a whole, which are quite similar to the "competence approach" to curriculum writing (e.g. NISS, 2018).

Historical studies and accounts could also furnish important insights into the above questions, both as regards the origins of the knowledge to be taught in scholarly developments, and in terms of how the curriculum itself has developed.

At the level of study programmes, Tucker (2013) offers an overview of how undergraduate programmes developed in the USA in the 20th century. He distinguishes three major periods: before 1950, 1950-1970, and after 1970. It appears that until 1950, the components of undergraduate mathematics programs remained relatively stable, and held few modules on more advanced domains than calculus. Calculus here means the study of functions in one and several variable, focusing on calculation techniques while relying on very informal theory (cf. WINSLØW, 2015, for the praxeological interpretation of Calculus vs. Analysis). Tucker notes (p. 697) that *in the late 1950's, there still were many able students going to graduate school from mathematics programs that did not teach the Riemann integral*. As in other Western countries, the cold war in the 1950's led to a sharp increase in funding and student numbers for mathematics degrees. In the following years (up to 1970), the study programmes were profoundly changed, with the introduction of advanced mandatory modules (such as abstract algebra and topology) which are common today. Around the same time, research replaces teaching as the main priority for many professors at universities offering doctorates (but not in 'colleges' offering only undergraduate programmes, lasting from 2 to 4 years). Famous universities such as Berkeley develop a deliberate policy of attracting the best researchers by offering them positions with "no undergraduate teaching and a light graduate teaching load" (p. 700). And indeed, the assortment of more advanced modules in the undergraduate mathematics programme paves the way for graduate studies. Tucker notes that since 1970, study programmes have not changed dramatically as far as the module structure is concerned.

A similar, but more extensive survey was done in the recent thesis of Huntington (2015). Related accounts are available in other countries. They offer at least initial hypotheses about our research questions, concerning the timewise origin of the current inventory of modules in mathematics programmes all over the world, and the idea of a relative stability of programmes over the past 40 years, when it comes to the modules offered.

Stability is much less pronounced when it comes to the detailed contents and other requirements for modules and sequences. It is interesting to consider in more details how "new" modules become stable, and again a historical point of view can be useful. For instance, courses on linear algebra, discrete mathematics and computing were introduced in most universities in the 1950's (HUNTINGTON, 2015, 94), with some mutual dependence. Dorier (2000, 56-61) explains how linear algebra became teachable through the production of didactic syntheses like Birkhoff and MacLanes' *Modern Algebra* (1941), as the culmination of a relatively long process of maturation and clarification in the scholarly field. Indeed, abstract linear algebra was among the novelties introduced into mathematics programmes in the 1950's, both in Europe and in the USA. Pursuing the study of textbooks and module descriptions from these first courses would indeed represent an excellent case of the model in Figure 1. We might also investigate further the causes of subsequent developments of the linear algebra modules. For instance, as noted by Dorier (2000, 61), there seems to be a tendency that the strict axiomatic approach has been 'softened' in later years, in response to the needs of students who have not been exposed to axiomatic mathematics in upper secondary education, as they had been in many countries up to around 1980.

A similar – if not entirely parallel – development may be seen in the area of Calculus, but it appears to have been somewhat more controversial in the United States, where the so-called *Calculus Reform* is the subject of numerous studies and opinion papers (cf. Robert and Speer, 2001). An interesting special case, studied in depth by Lavicza (2007), is the extent to which computer algebra systems are required (EDT) or just optionally chosen (IDT) as devices for the teaching in the Calculus or Linear Algebra sequences.

Conclusions: future ATD research on EDT in RUME

This paper highlights a gap in UME research regarding the process of EDT. We believe that for didactic research to make non-trivial and long term impact on UME, it is of crucial importance to consider this process as a research object in itself. The fact that study programmes have varied very little in the last 30-40 years, makes the necessity of its study less visible. In the case of changes – or crises – the EDT becomes more apparent, as for the Calculus Reform in the US (within sequences of modules). However, acknowledging the existence of EDT is not enough. We need methodological tools to investigate the phenomena involved. Previous ATD research on EDT (e.g. Chevallard, 1991), as well as the studies considered in section 3, provide us with some directions, which we finish by outlining.

The first direction is to study the variation of study programmes over time and across institutions, based on relevant documents. The analysis of curriculum documents, along with interviews with those directly involved in the drafting of study programmes, should focus on the didactic technology and theory that prevail in the *noosphere*, and which hinders or furthers change.

The second one refers to the 'texts' that constitute the main products of EDT at the level of sequences, modules and chapters, such as textbooks and syllabi. Tracing the evolution of these texts over a longer period of time can shed light on the way mathematical contents are (re)structured, on the parts that remain untouched, on the sources used in the process, on the balance between problems and synthesis (BOSCH & WINSLØW, 2015), etc. Prefaces and other commenting documents (reviews, outlines), as well as official recommendations, such as Tuning (2007), can be an important source for analysing the specific didactic technology and theory which prevails in a given domain.

Thirdly, the scholars' prevailing epistemology about the mathematics to be taught, including its relation to mathematical research, appears to be an important object for research. Even if much of it is implicit, interviews with teachers could be designed to uncover the rationales and other factors that support or hinder changes in EDT. And as these decisions are collective and founded in institutional beliefs, several interviews may be needed. Interview studies are still quite rare in ATD (an example can be found in MADSEN & WINSLØW, 2009) and will require separate methodological attention for the above purposes.

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