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Mathematics and Physics Study and Research Paths within two groups of pre-service teacher Education

Parcours d'études et de recherche en mathématiques et en physique au sein de deux groupes de formation initiale des enseignants

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Abstract

In this paper, we present results of an inquiry based teaching implementation carried out on a teacher training course in the University. The framework of the Anthropological Theory of Didactics (ATD) is adopted, and a co-disciplinary Study and Research Path (SRP) whose generative question requires studying physics and mathematics together is carried out by N=25 training teachers of Mathematics at University. Some conclusions concerning the conditions, restrictions and relevance of introducing the RSC in teachers training courses at the university are performed.

Keywords: Study and Research Path (SRP), Trainee teachers.

Résumé

Dans ce travail nous présentons des résultats d'un enseignement développé dans un cours de formation d’enseignants de mathématiques à l'Université. On adopte le cadre théorique de la Théorie Anthropologique du Didactique (TAD) pour développer un Parcours

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d'Étude et Recherche (PER) avec N=25 enseignants de Mathématiques en formation à l'université. La question génératrice demande l’étude conjointe des mathématiques et de la physique. Nous faisons quelques remarques liées aux conditions, restrictions et à l'importance des PER dans la formation des enseignants à l’université.

Mots-clés : Parcours d'étude et de recherche (SRP), Enseignants stagiaires.

The training of mathematics teachers has been the subject of numerous investigations in the field of Mathematical Education (Cirade, 2006; Chevallard & Cirade, 2009; Gómez, 2007; Llinares, Valls & Roig, 2008; Godino, 2009; Ribeiro, Carrillo & Monteiro, 2010; Font, 2011; Ruiz-Olarría, Sierra, Bosch & Gascón, 2014). These authors emphasize the importance that the training of the mathematics teacher includes knowledge that exceeds the mathematical contents that the teacher should teach. In this line, the notion of Pedagogical Content Knowledge (PCK) developed by Shulman (1987), which specifically in mathematics, originates Mathematical Knowledge for Teaching (MKT), as an essential mathematical knowledge in teacher training (Ball, 2000, Ball, Lubienski & Mewborn, 2001; Hill, Ball & Schilling, 2008).

According to the Anthropological Theory of the Didactic (ATD), that is the framework of this work, the training of mathematics teachers requires professional knowledge whose construction and development is the responsibility of the community of researchers in didactics of Mathematics, in close collaboration with the teaching profession (Chevallard & Cirade, 2009). The didactic phenomenon called monumentalism is characteristic of the paradigm of the "visit of the works" and has been described by Chevallard (2001, 2013). This paradigm focuses on the teaching of answers rather than questions, ignoring the fact that knowledge always arises as a response to a question, which if hidden, leads to the presentation of scholastic knowledge that lack motives and reasons for being. Knowledge is signalled as if it were an historical monument, which at most is seen and venerated. To overcome the monumentalism prevailing in educational systems, the ATD has proposed the Paradigm of Research and Questioning the World (Chevallard, 2013) advocating an epistemological and didactic revolution of the teaching of mathematics and school disciplines (CHEVALLARD, 2013), where knowledge should be taught by its usefulness or potential uses in life.

However, to opt for such paradigm, it is necessary to train future teachers in a different way, to provide them with the necessary equipment to develop a teaching based on questions. It is very difficult to put into practice a teaching based on questioning and inquiry, since it requires systems of teacher training that are appropriate and are not generally available. In this paper, we describe the results obtained in two courses of mathematics teachers in training at the university, when addressing a question that places future teachers in an investigation and questioning situation.

To learn what an SRP is, and which kind of teaching is involved in, the trainee teachers (TT) must deeply experience a genuine SRP. The starting point of the SRP is the question Q_0: *Why did the Movediza stone in Tandil fall?* Which, to be answered – in a provisional and unfinished way- needs the study of Physics and Mathematics jointly.

The rationale of the paper is to describe the trainee teachers' activities and their difficulties when they must experience an SRP and to face a strong question.

Questions

Which was the role of the students and the teacher during the SRP?

Which mathematical and physical contents were studied along the SRP?

Which mathematical and physical models were developed by the students during the SRP?

Which were the most relevant constraints to develop the SRP in this level?

Methodology

This work involves a qualitative and exploratory research that aims to carry out a research and study course as it is proposed by the ATD, in a mathematics teacher training course at the University. The SRP was implemented in a state university, in the city of

Tandil, Argentina, in a discipline which is part of the didactic studies within the Mathematics Teaching Training Course, in which two of the researchers are also teachers. There were two implementations, where N=12 and N=13 students from the last year (4th), aged 21-33 took part in it.

It is important to notice that these students had not studied physics before at the university but had a relatively strong mathematical formation. In addition, the students had studied the ATD in two previous Didactics courses; however, they report difficulties to understand what an SRP is, and how it works? In this respect, we propose to design, implement and analyse a physics and mathematics co-disciplinary SRP, adapted to the institution in which it is developed.

The SRP was carried out in a total of 7 weekly hours provided in two lessons per week. In both implementations, which we will identify as I1 and I2, respectively, three work groups were organized with approximately 4 members each.

In a SRP, the generative question Q_0 has to be pointed out by the teacher, and this was made in the first lesson. Then, the students started their research in the library, by selecting some texts, documents etc. as possible $R^{\lozenge}_i$. In every class, each group presented and discussed with the teacher and the other groups their findings and possible ways to face Q_0. In the second class, many derived questions Q_i were made explicit by the students, and the community of study selected the questions Q_i to be studied as well as their related knowledge O_k. The regular dynamic during the SRP was characterized by the roles of the teacher and students described in the previous section of this text.

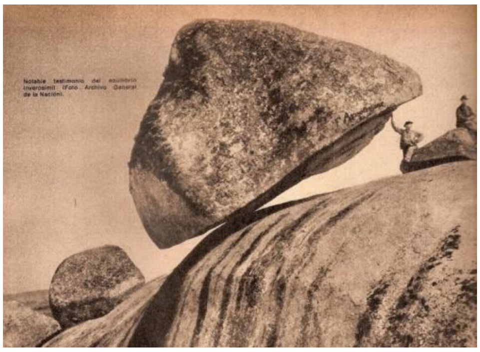
Recordings of each class were obtained and the student productions were digitalized and returned in the subsequent class. The teacher wrote a class diary writing down the tasks performed by the study group. The remaining researchers of the team performed non-participant observation during classes. The data analysis was performed by using the categories provided by the Developed Herbartian model (CHEVALLARD, 2013).

The Epistemological model of reference (EMR) and the SRP

As we mentioned, the starting question Q_0 is: *Why did the Movediza Stone in Tandil fall down?* This enormous basalt stone has remained the city's landmark, providing it with a distinctive feature. Many local people and national celebrities visited the place to closely observe the natural monument. It was a 248-ton rock, sitting on the top of a 300-meter-high hill (above sea level), which presented very small oscillations when disturbed in a specific spot, (Figure 1).

Figure 1

Photography of the Movediza Stone 9: Photo Archivo General de la Nación Argentin), available in: http://bibliocicop.blogspot.com.ar/2012/02/piedra-movediza-100-anos-de-su-caida.html)



Unexpectedly, on February 28, 1912, the stone fell down the cliff and fractured into three pieces, filling the town with dismay by the loss of their symbol. For over 100 years, the event produced all kinds of conjectures and legends for the causes of the fall. Within the two groups where the SRP was performed, there existed a certain curiosity and interest in finding a scientific answer to this question. Once in contact with the available information, the question evolved into: *What are the conjectures about the causes the Movediza Stone fall, and which is the most likely from a scientific viewpoint?* Assuming that the fall can be explained by means of the Mechanical Resonance phenomenon, several questions Q_i emerged which are linked to the physical and mathematical knowledge necessary to answer Q_0.

If we consider that the stone was an oscillating system, the study can be carried out within the Mechanic Oscillations topic, starting from the ideal spring or the pendulum. In this case, frictionless systems are used, in which the only force in action is the restoring force depending (for small amplitude oscillations) in a linear way on the deviation respect to the equilibrium position. This model is known as simple harmonic oscillator whose motion, via Newton equations, is described by a second-order linear differential equation.

Progressively, the system becomes more complex. If friction-produced damping is considered, it provides a new term to the differential equation connected to the first derivative of the position (speed). Finally, it is possible to study systems that apart from being damped, are under the influence of an external force, and therefore called driven systems. In the case that the external force is periodic and its frequency is approximately equal (the order of the approximation will be clarified later) to the natural (free of external forces) frequency of the oscillating system, a maximum in the oscillation amplitude is produced, generating the phenomenon known as mechanical resonance.

By increasing the complexity of the model, it is possible to consider a suspended rotating body, instead of a punctual mass. This leads to the study of the torque and the moment of inertia of an oscillating body. Here again, the linear system is for small amplitude oscillations and the damped and driven cases can be also considered, corresponding to the same mathematical model, but in which the parameters have a different physical interpretation.

However, as it refers to a suspended oscillating body, this is not a suitable physical model for the Movediza stone system. Since that, the base of the Stone was not flat, it is necessary to consider more precise models of the real situation. This leads to the mechanics of supported (and not hanging) oscillating rigid solids. In this case, we consider a rocker-like model in which the Movediza stone base is curved and it lies on a flat surface, where the oscillation is related to a combined translational and rotational motion (OTERO et al. 2016).

The application of Newton laws to the rocker model of the stone leads to a differential equation where the parameters are specific of the Movediza system: mass, geometry, inertia moments, friction at the base, external torque, etc., which is given by the following effective Harmonic oscillator mathematical model of the Movediza physical system:

$$\ddot{\varphi} + \gamma\dot{\varphi} + w_0^2\varphi = (M_0 / I)\cos(wt) \qquad (1)$$

The stationary solution to equation (1) is

$$\varphi(t) = \varphi_M \cos(wt - \psi)$$

being the amplitude φM and the phase ψ

$$\varphi_M = \frac{M_0/I}{\sqrt{(w_0^2 - w^2)^2 + w^2\gamma^2}} \qquad \psi = tg^{-1}\left(\frac{\gamma w}{w_0^2 - w^2}\right) \qquad (2)$$

The maximum of φ_M is for $w_m = \sqrt{w_0^2 - \frac{\gamma^2}{2}}$. The parameters: M_0 (external torque), I (inertia moment), w_0 (natural oscillation system frequency) and γ (damping coefficient), must be estimated. Detailed data about the shape, dimensions and center of mass position of the Movediza stone are available (PERALTA et al. 2008) after a replica construction and its relocation in 2007 on the original place (although fixed to the surface and without possibility to oscillate). These data bring us the possibility to estimate some parameters in our model, as e.g. mass, inertia moment, and the distance of 7.1 m, from which the external torque could be exerted efficiently by up to five people (according to historical chronicles) to start the small oscillation. By using these values, it is possible to study the behaviour of the $\varphi_M(w)$ function for w_0 in a range of frequencies between 0,7 Hz and 1 Hz, historically recognized (ROJAS, 1912) as the natural oscillation frequencies in the Movediza stone system and calculate for each case the maximum amplitude $\varphi_M(w_m)$..

The Stone would fall if $\varphi_c \leq \varphi_M(w_m)$, being $\varphi_M(w_m) = M_0 / w_0 I\gamma$. Note that if γ is very small (as is expected to be in this case) we can neglect it from om $w_m = \sqrt{w_0^2 - \frac{\gamma^2}{2}}$, leading to $w_m \approx w_0$, which is the approximation that we mentioned in the previous section and we will use hereafter. By using this approximation in equation (2) (left) the falling condition becomes $\varphi_c \leq (M_0 / w_0 I\gamma)$.

The value of φ_c can be determined by an elementary stability analysis, which per the dimensions of the base of the stone and the center of mass position is estimated to be approximately of 6.

Note that in the present model γ is a free parameter, for which we set "ad doc" a magnitude order $\gamma \geq 10^{-2}$. This is justified in the frame of a more sophisticated model that we will comment briefly below. With this constraint, we find several situations, comprising different torques within the mentioned frequencies interval, supporting the overcoming of the critical angle, i.e., predicting the fall.

Finally, in search of a more appropriate approximation of the physics model for the damping that is clearly not due to air, we consider a more sophisticated model of the stone as a deformable solid, where the contact in the support is not a point but a finite extension, along which the normal force is distributed, being larger in the motion direction and generating a rolling resistance, manifested through a torque contrary to the motion. The rolling resistance depends on the speed stone, giving a physical interpretation to the damping term. Therefore, the physics behind the damping is the same that makes a tire wheel rolling horizontally on the road come to a stop, but in the case of the stone, the deformation is much smaller. Although the deformable rocker model has extra free parameters, tabulated values of rolling resistance coefficient for stone on stone, which are available in the specialized literature, allowed us to estimate and justify the damping values that we incorporate otherwise ad-hoc in the rigid rocket Movediza model.

Conclusions

Which was the role of the students and the teacher during the SRP? Independently of the difficulties presented, as mentioned before, the TTs experienced a genuine SRP within its means. However, at the beginning there was a visible initial reluctant attitude on the part of the TTs: Why physics should be studied if we are teachers of mathematics? Later, it was gradually understood that the idea was to experience a genuinely co-disciplinary SRP, analyse it and comprehend the teaching model supporting an SRP. In an SRP, the students and the teacher integrate the study community facing together situations of study and research. In both implementations, the TT's studied physics and mathematics thoroughly and showed a good disposition to deal with questions they had never considered before. It is important to highlight the role of the teacher in the SRP. For the teacher, the question Q_0 was also an open question, for which, especially in the first implementation, did not have any a priori closed answer. In this sense both, the students and the teacher took a genuinely active part in the SRP.

Which mathematical and physical contents were studied along the SRP?

In both groups, interdisciplinary education is alien to the students, due in part to the imperative of traditional pedagogy. In this sense, the SRP device is very appropriate to foster interdisciplinary study, because it allows studying only the necessary mathematics or physics to answer a question, returning to the original problem. However, it is not only important to decide what content to study, but how to use them, and so the physical and mathematical models and their rationale emerge.

In this work, the question Q_0 triggers, on the side of the physics, the study of oscillating systems, which leads to the study of resonance, motivated by the most plausible conjecture about the fall of the stone. In turn, this physics calls for the study of the equations of motion of these systems, which through Newton's laws give rise to second order differential equations.

Which mathematical and physical models were developed by the students during the SRP?

The construction of a possible answer to the question Q_0, driven the study and the analysis of several physical models related to oscillating systems like springs, single pendulum and physical pendulum, including damped and driven oscillators. However, none of these physical models are adequate to the stone. By reanalysing the real system in more detail, students realized that previous models do not describe some essential aspects of the stone, the most important being the fact that the real system is an object supported on a surface and that is not hanging, like the previous physical models. Then, in the search for a reason to make a supported physical stone model oscillate, the hypothesis that the contact surface between the stone and the base is not flat, but some of the two or both have a certain curvature, emerge, something that in fact had some historical evidence. In this way, the physical model of the rocker arises as the most appropriate to describe the oscillations of a supported object.

Although the students understood that the physical model of the rocker is able to oscillate and could somehow describe the oscillations of the stone, the dynamic analysis of its motion is not within reach of the students, so the mathematical model of the rocker was introduced by the teacher. At this point the students recognized the mathematical similarity with the equations of motion of the pendulum, although the physics is completely different.

Here, it is important to notice that the most relevant obstacles there were not only in the physics knowledge, insofar as the physical model was sophisticated and the physical knowledge necessary to treat it was expanded, but in the difficulties of the TT's to use functional modelling involved in the solution of the differential equations. We can say that some aspects of the modelling processes in the sense of the ATD (Barquero, Bosch and Gascón, 2011), were accomplished, and because the SRP evidence the inadequate of the available already made answers to treat the motion of the stone, and the increasing complexity proposed by the SRP.

Which are the most relevant constraints to develop the SRP in this level?

Even though the TTs had studied the ATD and other didactic theories, they did it in a traditional way comparable to the traditional training they got. This is reflected in the difficulties they had to understand and to use both physical and mathematical models. It was not expected that the TTs developed the models by themselves, but it was expected that they used the mathematical results presented in the physics textbooks in a pertinent and exoteric manner. This fact did not occur in the first group and improved in the second one from the didactic decision to make a previous incursion into mono-disciplinary SRP, particularly suitable for evidencing the role of the functional modelling. Moreover, this allowed teachers to discuss the relationship between the mathematical model and the physical model and the meaning and role of the parameters.

The TT's behaviour is interpreted from the fact that although they have experienced four years of "hard" university studies, the utility of the science they aim at teaching had never been visible. The epistemological conception about the mathematics produced by the traditional paradigm is so ingrained, that it is complex to reverse it. This would be, in our view, the most relevant drawback to permit the TT's at least understand what an SRP is and how the modelling activity works. However, it is important to notice that the sporadic incursions in the modelling activity do not seem enough to allow the TTs to develop such school practices. Although the predominant teaching is mainly traditional, the TTs will face increasing demands for a change to a mathematics teaching based on research, questioning and modeling. It is unlikely that a teacher whose training has been answers-based teaching can teach by means of questions. Therefore, our final message is that the training of teachers must change profoundly.

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