EMP Educação Matemática Pesquisa
ISSN 1983-3156
A2
http://dx.doi.org/10.23925/1983-3156.2020v22i4p563-578

Research and study paths at the university: a praxeological model of reference related to costs calculation

Parcours de recherche et d'études à l'université: un modèle praxéologique de référence lié au calcul des coûts

Diana Patricia Salgado[1]
Núcleo de Investigación en Educación en Ciencia y Tecnología (NIECyT). Universidad Nacional del Centro de la Provincia de Buenos Aires (UNICEN). Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET). Argentina
https://orcid.org/0000-0001-5130-3624
María Rita Otero[2]
Núcleo de Investigación en Educación en Ciencia y Tecnología (NIECyT). Universidad Nacional del Centro de la Provincia de Buenos Aires (UNICEN). Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET). Argentina
https://orcid.org/0000-0002-1682-9142
Verónica Parra [3]
Núcleo de Investigación en Educación en Ciencia y Tecnología (NIECyT). Universidad Nacional del Centro de la Provincia de Buenos Aires (UNICEN). Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET). Argentina
https://orcid.org/0000-0002-6956-0052

Abstract

This work fits within a wider research whose general objective is teaching mathematics to non-mathematicians at the university, taking central assumptions of the Anthropological Theory of Didactics. This work proposes a praxeological model of reference related to a micro-entrepreneurship costs calculation. The generative question How to calculate micro-entrepreneurship costs? links mathematical and economic praxeologies. This model allows to go through a part of the study programme of a university calculus course.

Keywords: Anthropological theory of the didactic; Course of study and research; Praxeological Reference Model.

[1] - dsalgado@uns.edu.ar
[2] Idem - masamotero@gmail.com
[3] Idem - vparra@exa.unicen.edu.ar

Résumé

Ce travail fait partie d'une recherche plus large dont l'objectif général est l'enseignement des mathématiques pour non-mathématiciens à l'université dans le cadre de la Théorie Anthropologique du Didactique. On propose un modèle praxéologique de référence sur le calcul de coûts d'une micro-entreprise. La question génératrice est Comment calculer les coûts dans une micro-entreprise? Elle associe plusieurs organisations mathématiques et économiques. Ce modèle permet de couvrir une partie du programme d'études d'un cours de Calcul du niveau universitaire.

Mots-clés : Théorie anthropologique de la didactique, Parcours d'étude et de recherche, Modèle de référence praxéologique.

Research and study paths at the university: a praxeological model of reference related to costs calculation

This work is part of a didactic research developed at university level, in a mathematics course corresponding to Enterprise Administration and Public Accountancy. The institution is a public Argentine university, Universidad Nacional del Sur (UNS), with a departmental organization, so that mathematics courses of all majors are lectured exclusively by the Mathematics Department. Also, the courses are developed in a four-month form and particularly the mathematics ones have a theory-practice modality.

Mathematics teaching characteristics at UNS satisfy the conditions that the ATD describes as *visits to works*; i.e., the sense loss of the studied mathematics and the immediate oblivion are confirmed. One of the objectives of the didactic research that is being realised refers to the analysis of the possibility conditions to deal with this phenomenon at least locally, taking the developments of the ATD such as the notions of research and study paths (RSP) and praxeological model of reference (PMR).

The RSP proposed begins with a question related to economy: *How to calculate micro-entrepreneurship costs?* It would be possible through this RSP not only to study different institutional culture works, but also to produce the emergence of other researches. One of our specific objectives is to detect which these "works" are and which of them are linked to the proposed curriculum.

In this PMR the mathematical and economic organizations giving answer to the generative and derived questions are presented. Particularly, the RSP would allow rebuilding mathematical organizations related to two-variable Calculus and an important part of the institutional proposed curriculum. Also, it would be possible to manage the involved organizations integration.

Previous Researches

Different Calculus teaching researches are related to the didactical phenomenon of mathematical content disconnection, rigidity and atomization of mathematical organizations (Trigueros, 2005; Lucas, 2015). At university level and from ATD point of view, some investigations propose an inquiry based calculus teaching (Barquero, 2009; Serrano, Bosch & Gascón, 2007).

Costa, Arlego and Otero (2013) design and implement a teaching by RSP in the context of an Engineering Faculty in Argentina. Recovering the vectorial calculus sense and raisons d'être was attempted in this path, integrating physic notions. The path permitted the approach to mathematical organizations included in the institutional curriculum.

On the other hand, M.R. Otero et al. (2013) promote and analyse the possibility conditions of a teaching in the sense proposed by the ATD at university level.

The PMR proposed in this work shows different paths in a teaching by RSP, in a regular math course at university level, integrating two-variable Calculus and Economy notions. Some results obtained in the RSP implemented can be found in Salgado, Otero and Parra (2017).

Theoretical framework

The reference knowledge analysis involved in a Research and Study Path (RSP) is an action the didactic researcher must be able to realise. This knowledge integrates the praxeological model of reference (PMR) (Chevallard, 2013) consisting in an always provisory and open analysis of the organizations or praxeologies of one or more disciplines the researcher would meet, or meet again, studying a question Q. What he must know or give himself the liberty to learn about Q is not identified with what a

professor knows or with the way he would answer Q. On the contrary, the researcher must adopt a precognitive posture, “ask the world” in which he is situated.

A PMR elaboration importance lies in its utility as a tool of didactic and praxeological analysis. Following up on the praxeological analysis implies formulating didactics questions as: Where does this “work” come from? Why is it there? How has it been learned by the institution? Which transformations has it suffered? On the other hand, all didactic analysis implies considering how is the praxeology pretended to teach so that it leads to identify the structure and the studied work functioning (Lucas, 2015).

Study programme at the reference institution

The proposed RSP pretends to go through the contents of the first year subject Matemática IIC (MIIC), second four-month period, in the degree of Enterprise Administration and Public Accountancy at the UNS. This subject study programme is divided in four modules:

1. Sequences and series
2. Lineal equations system. Matrix Algebra
3. Functions of several variables
4. Extremes of several-variable functions. Linear Programming.

The main objective of MIIC is to acquire notions related to the more-than-one variable functions analysis, placing an emphasis in its application to administration and economic concrete problems. The programme is developed in two theoretical and two practical weekly classes of two hours each. The evaluation modality includes the approval of partial exams and a final one.

Praxeological model of reference related to costs calculation

Generative question analysis

The PMR described in this work allows us to delimit and analyse the possible research and study paths that arise from the question Q_0: *How to calculate micro-entrepreneurship costs?* We will see that the elaboration of a possible answer to Q_0 results in the study of different mathematical and economic organizations, for example, a mathematical organization (MO) related to differential Calculus and other economic one (EO) linked to costs calculation.

The starting question Q_0 allows the emergence of multiple economic questions as, for example, what is micro-entrepreneurship about? Which are the generated costs? Which is the purpose of doing a cost calculation?, among others.

A first questioning belongs to the notion of micro-entrepreneurship. If its definition and central characteristics are looked up in the Internet, one can find statements such as: micro-entrepreneurship is an earning generating company owned and run by its own entrepreneurs, who work at these companies, in general, without employees. It is an individual or family project requiring very low capital investment.

The notion of micro-entrepreneurship is associated to the idea of micro-credits, born in Bangladesh in the 70's when the economist Muhammad Yunus gave economic aid to a group of poor women to carry out a small business. This initiative led to the foundation of a social bank orientated to the poorest so they could go out of their misery.

A second questioning comes from the notion "costs", understood as all the money needed to produce and commercialise products. Not everything that is an expense represents a cost. An important part corresponds to the necessary capital in order that the micro-enterprise could work. The capital is the value of what is possessed, whereas the cost is the value of what the company uses to generate a product.

Finally, the questioning about "why" a costs calculation must be done is very important for the micro-entrepreneur. In general, a costs study and calculation are needed to fix sale prices. Most enterprises fix their prices, principally and exclusively, taking into account costs, but also costs determination is elementary for decision making because it allows: business results and marginal contribution determination, level of competitiveness evaluation, etc.

Possible answers to the generative question

The first stage in the construction of an answer to Q_0 refers to the different hypotheses analysis that will determine, for example, the variables of the system. De Renolfi and Cardona (2007) affirm that different alternatives require the application of different types of costs, it does not exist an only one cost but different ones for different purposes. These authors consider that different hypotheses related to costs calculation exist; namely, H_1: costs behaviour following an independent variable, H_2: costs relation to the possible product allocation, H_3: calculation extension, H_4: costs considering the relationship to the moment of the calculation and H_5: the relationship to the decision making. Taking into account the above-mentioned hypotheses, different costs considering each one are described below:

H_1: costs behaviour following an independent variable. This hypothesis is considered when the system to model refers to an enterprise whose objective is to fix the article sale price, to cover the minimal cost. Under H_1, the cost depends on one or more independent variables, for instance, production, level of activity, supplies, etc. In this case, costs are classified as fixed (FC) and variable (VC) costs. As a consequence, the total cost is the addition of both: C=FC+VC, which represents a first answer to Q_0. A fixed cost is that whose amount is constant, whatever the value of the independent variable. This doesn't mean that it is invariable in the long-term; for example, rent,

insurances, etc. A variable cost is the one modified in relation to the independent variable value; for example, raw material, direct manpower, etc.

H_2: Considering costs relation to the possible product allocation, costs are classified as direct and indirect. Direct ones are those identified in every produced article, either in its physical aspect, or in its value. They are produced when the activity is carried out; for example, supplies, manpower related to the activity, etc. Indirect costs are those related indirectly to the articles. They are produced independently of the realization or not of a certain activity; for example, taxes, fuel expenses, etc. In this case, the cost (C) is given by the addition of the direct (DC) and indirect (IC) costs: C=DC+IC, which represents a second answer to Q_0.

H_3: considering the calculation extension, costs are classified as total and partial. Total ones are those involved in the totality of a certain activity, while partial ones are those referring to a specific aspect of the activity. In this case, the cost (C) is given by the addition of the total (TC) and partial (PC) costs: C=TC+PC, which represents a third answer to Q_0.

H_4: considering the relationship to the moment of the calculation, costs are classified as real and estimated. Real costs also called historical or retrospective, are those in which the enterprise incurred in the past. They are used to evaluate past actions and to control the management of the enterprise. Estimated costs, also called future or prospective, are those that could happen in a future situation.

H_5: considering the relation to the decision making, costs are classified as marginal, incremental, relevant and opportunity costs. Marginal costs are those required to increase the production in a unit. Incremental costs refer to how much the cost was raised on having increased the activity at a certain level. Relevant costs are those that have a special opportunity for every decision making. Finally, opportunity costs refer to

the value of the rent that might be obtained if the resource was used in its better alternative.

The unitary cost is not included in this classification. It represents the production cost of an article and it is important to fix the sale price. However, searching the Internet how to calculate the costs of a micro-entrepreneurship, principally the fixed, variable, total and unitary costs, are mentioned.

Construction of two models

After realising a costs analysis, the micro-entrepreneur performs a calculation of total production costs; he puts up all data in tables -considering hypotheses H_1 to H_5, or even a combination of them- and performs simple arithmetic operations to answer Q_0, by adding the registered information. This leads to consider both an MO related to arithmetic operations and an EO referred to costs according each hypothesis.

Supposing that one wants to calculate the total costs, it is necessary to determine the fixed and variable costs. Whereas the fixed ones can be calculated monthly, the variable ones, by produced unit. Table 1 shows a list of costs if only one product is manufactured:

Table 1

Costs in the manufacturing of an article

Fixed	per month	Variable	per unit
Gas	F1	Materials	V1
Electricity	F2	Labels	V2
Rent	F3	Package	V3
Taxes	F4		
Fixed Costs per month	FC=F1+F2+F3	Total Variable Costs	VC=V1+V2+V3

The total fixed monthly costs FC emerges from the addition of all fixed costs. The total variable costs per unit VC is the addition of the different manufacturing expenses. If x is the number of manufactured products in a month, the total cost is $C=FC+VC.x$, with FC, VC positive real numbers.

If on the other hand, two articles are manufactured, variable costs are calculated per unit (see Table 2).

Table 2

Costs in the manufacture of two articles

Fixed	Per month	Variables		Per unit	total per unit
Gas	F1	Art. 1	Materials	V11	VC1=V11+V12+V13
Electricity	F2		Labels	V12	
Rent	F3		Package	V13	
Taxes	F4				
		Art. 2	Materials	V21	VC2=V21+V22+V23
			Labels	V22	
			Package	V23	
Fixed Costs	FC=F1+F2+F3+F4				

The fixed monthly cost FC emerges from the addition of all fixed costs. VC1 and VC2 are the variable costs to manufacture articles 1 and 2, respectively. If x and y are the number of articles 1 and 2 manufactured in a month, respectively, the total cost is given by $C=FC+VC1.x+VC2.y$, with FC, $VC1$, $VC2$ positive real numbers.

Derived questions from Q_0: answers according to the models

The search of an answer to Q_0 originates more questions, such as:

Q_1: How many articles can one make with a certain amount?

Q_2: Which is the marginal cost?

Q_3: How does the total cost change considering a modification in the variable costs?

Q_4: Which are the maximum and the minimum cost?

In order to answer Q_1 to Q_4, it is necessary to answer Q_5: How do we reply to each of Q_1 to Q_4?

Considering a numerical model, where the micro-entrepreneur performs a regular register of the costs, one can deduce conclusions from these data in tables; such as, estimating short term costs, calculating marginal costs, analysing the cost variation,

determining how many articles can one manufacture with a certain budget, etc. From specific data, estimations or exact calculation will possibly be done in order to answer Q_1 to Q_4, among other questions.

Considering a functional algebraic model, if the micro-entrepreneurship, for example, is engaged in the production of two articles, the model *C=FC+VC1.x+VC2.y*, with *FC*, *VC1*, *VC2* positive real numbers, can be written as:

$$C(x, y) = c_1 x + c_2 y + FC,$$

with *FC* representing fixed costs, c_1, c_2 variable ones, and *C(x,y)* cost function with variables *x* and *y*. It will be showed that this model also allows us to answer Q_1 to Q_4.

Up to here, two possible paths are observed. They allow to find answers not only to the generative question, but also to the derived ones.

Considering the functional algebraic model, one can answer Q_1. Supposing a certain amount of money for expenses, the answer will indicate which the production with this budget is.

If one has an amount of *K* monetary units to carry out the project and to use it completely, one models this situation by the equation *C(x,y)=K*, which represents a level curve of the surface whose equation is *z=C(x,y)*. The curve corresponds to a set of points in the XY plane for which *z=K*, with *K* a positive constant. From an economic point of view, this is an isocost curve (Mochón and Beker, 2003). Every point (*x*,*y*) belonging to it brings a production level for which the cost is constant. In this way, this analysis that answer Q_1 allows to get into an EO related to Costs and an MO referred to two-variable Calculus and Analytic Geometry in the plane.

With regards to Q_2: *which is the marginal cost?* this means: how much the cost varies given an increase in the production in one unit? In the case of the functional

algebraic model in two variables considered here, the question can be reformulated as: *Given a fixed number of manufactured articles of one type, how does the cost change if an increase in the number of manufactured units of the other type happens?*

The rate of change of a variable in relation to another, here, how does *C* change for an increment of *x* or of *y*, requires to get into the "work" of derivative. In this way, given the function *z=C(x,y)*, one can answer Q_2 by taking partial derivatives: $\frac{\partial C}{\partial x}$ and $\frac{\partial C}{\partial y}$, which calculated in a point (x_0, y_0), represent the approximated change of *C* for each unit increase in *x (y)* keeping *y (x)* fixed, which are called marginal costs.

For example, $\frac{\partial C}{\partial x}(x_0, y_0)$ represents the approximated change of *C* for each unit increase in *x*, keeping *y* fixed, at the moment in which the production level is $x = x_0$ and $y = y_0$.

The question Q_3*: How does the total cost change considering a modification in the variable costs?* states that c_1 and c_2 change. If c_1 and c_2 depend on the production, for example, $c_1 = f(x)$ and $c_2 = g(y)$, *C(x,y)=f(x).x+g(y).y+FC*, then the approximated changes of *C* are given by: $\frac{\partial C}{\partial x} = f'(x)x + f(x) = c_1'(x)x + c_1$and $\partial\frac{\partial C}{\partial y} = g'(y)y + g(y) = c_2'(y)y + c_2$.

In this way, answering Q_2 and Q_3, one gets into an EO related to Costs, specifically Marginal costs, and an MO referred to two-variable Differential Calculus.

Finally, considering the functional algebraic model, an answer to Q_4: *Which are the maximum and the minimum cost?* can be found studying an MO referred to two-variable Differential Calculus.

The search of an answer to Q_4 can generate more questions: How to calculate maximum (minimum) cost? What techniques to employ?, What constraints do the variables have, if any? These questions are solved using techniques to find relative or

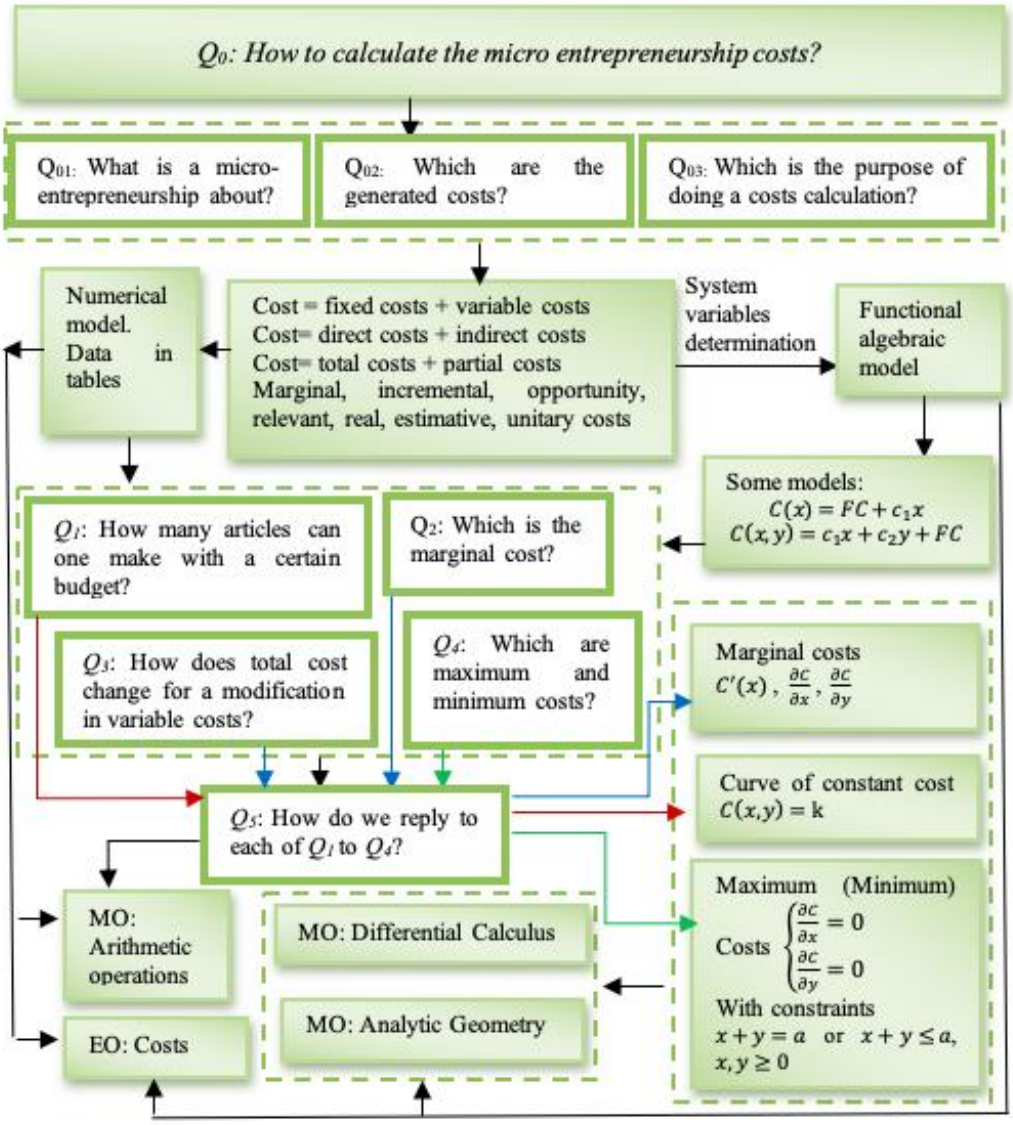
absolute extremes of cost function $C(x,y)$. If there are constrains, one employ techniques of constrained optimization. In this case, the variables take positive values, then, one has at least the constraints: $x \geq 0$, $y \geq 0$.

Another possible constraint is that of fixing the production in a units which leads to search for the extremes of $C(x,y)$ restrained to the constraint $x + y = a$, with a positive integer, or the constraint could be an inequality $x + y \leq a$, if the production is up to a units. One can use techniques of extremes calculation to solve this kind of problems, for example, the method of the Lagrange multipliers -constraints given by one or more equalities- or techniques of linear programming -linear constraints, given by inequalities and linear cost function-.

It is easily seen that considering the functional algebraic model the search of answers to questions Q_2 to Q_5, leads to the study of an MO referred to two-variable Differential Calculus, which includes dealing with functions of two variables, the calculation of partial derivatives and extremes. The generative and derived questions are shown in Figure 1, as well as the possible organizations to rebuild in the RSP.

Figure 1

Generative and derived questions. Possible MO and EO to rebuild in the SRP



Discussion

This PMR shows that Q_0 is open and generates multiple questions that can result in the study and research of mathematical and economic organizations.

Firstly, two possible paths are observed in order to search an answer to Q_0, one path using a numerical model and another, a functional algebraic one. In the first one, adequate costs are considered according to the micro-entrepreneur objectives, which leads to meet an EO related to Costs and an MO referred to arithmetical operations. In

the second one, the variable number determination will imply the one or more variable functions study. Therefore, to answer the derived questions, for instance, one will get into the MO study related to two-variable differential Calculus. Another possible path is to consider firstly the numerical model and then to decide to take the functional model.

In the mathematics university level course where the RSP is performed, Q_0 would allow, at least, rejoining an MO relative to two-variable Calculus and an EO referred to Costs. The answers to questions Q_0 to Q_5, using the functional algebraic model, could lead to the study of an MO related to two-variable differential Calculus, which are part of modules 3 and 4 of MIIC programme study. In this way, it would be possible to go over the institutional proposed curriculum using a different pedagogy.

Conclusions

In this work we have presented a PMR belonging to an RSP related to a micro-entrepreneurship costs calculation. Possible paths are detected; namely the one using a numerical model and functional algebraic one. Besides, it is shown that these paths would allow going through a part of the reference institutional proposed curriculum.

References

Barquero, B. *Ecología de la Modelización Matemática en la enseñanza universitaria de las Matemáticas.* Tesis Doctoral, Universidad Autónoma de Barcelona, 2009.

Chevallard, Y. (2013). *Analyses praxéologiques: esquisse d'un exemple*. IUFM Toulouse, France, 2013. http://yves.chevallard.free.fr

Costa, V., Arlego, M. & Otero, M. R. Enseñanza del Cálculo Vectorial en la Universidad: propuesta de Recorridos de Estudio e Investigación. *Revista de Formación e Innovación Educativa Universitaria,* 7(14), p. 20-40, 2013.

De Renolfi, M. C. & Cardona, G. Costos forestales. *Serie didáctica* Nº 30, Universidad Nacional de Santiago del Estero. Argentina, 2007.

Lucas, C. *Una posible razón de ser del cálculo diferencial elemental en el ámbito de la modelización functional*. Tesis Doctoral. Universidad de Vigo, 2015.

Mochón, F. & Beker, V. *Economía: principios y aplicaciones*. Buenos Aires: McGraw Hill Interamericana, 2003.

Otero, M. R., Fanaro, M.A., Corica, A.R., Llanos, V.C., Sureda P. & Parra, V. *Teoría antropológica de lo Didáctico en el aula de matemática*. Tandil, Buenos Aires: Dunken, 2013.

Salgado, D., Otero, M.R. & Parra, V. Gestos didácticos en el desarrollo de un recorrido de estudio e investigación en el nivel universitario relativo al cálculo: el funcionamiento de las dialécticas. *Perspectiva Educacional*, 56(1), p. 84-108, 2017.

Serrano, L., Bosch, M. & Gascón, J. Cómo hacer una previsión de ventas. Propuesta de un recorrido de estudio e investigación en un primer curso universitario de administración y dirección de empresas. *Actas del II Congreso Internacional sobre la TAD*. France, 2007.

http://www4.ujaen.es/~aestepa/TAD_II/Comunicaciones_TAD_II/25-Serrano_Bosch_Gascon_congres_TAD_2.pdf

Trigueros, M. La noción de esquema en la investigación en matemática educativa a nivel superior. *Educación Matemática*, 17(1), p. 5-31, 2005.