PROSPECTIVE MATHEMATICS TEACHERS INTERACTING IN A CHAT AND EMERGING DIFFERENT SCOPES ABOUT THE DEFINITION OF POLYHEDRON

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ABSTRACT

In mathematics education, research studies that analyze the construction of geometric concepts through interactions on chat are scarce. This study focuses on prospective mathematics teachers (PMTs) discussing about the definition of polyhedrons. The report is part of an ongoing research project[1] that analyses interactions in virtual learning environments (VLE). One case study will be discussed. The chat proved to be a scenario that improved PMTs reflecting about the definition of polyhedrons in three scopes: one in the context of geometric solids, another one focused on its elements (faces, vertices and edges) and still another one centered in the number of dimensions.

Keywords: Virtual learning environments. Chat. Prospective mathematics teachers. Definition of Polyhedron.

RESUMO

Na educação matemática, estudos que analisam a construção de conceitos geométricos por meio de interações vindas de "chat" (bate-papos) são escassos. Este estudo centra-se em futuros professores de matemática (PMT) discutindo sobre a definição de poliedros. O trabalho apresenta resultados parciais de um projeto de pesquisa em andamento que analisa as interações em ambientes virtuais de aprendizagem (AVA). Trata-se da discussão de um estudo de caso. O bate-papo se mostrou um cenário de reflexão sobre a definição de poliedros em três âmbitos: um no contexto de sólidos geométricos, outro focado em seus elementos (faces, vértices e arestas) e, por fim, ainda outro, centrado no número de dimensões.

Palavras-chave: Ambientes virtuais de aprendizagem. *Chat*. Formação de professores. Definição de Poliedro.

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Introduction

In Brazilian mathematical education, the first studies in synchronous interaction were conducted by Borba and Villareal (2005). Analyzing interaction in chat about mathematics education issues, these authors identified multi-dialogues in their analysis. According to them, in chats, multi-dialogues indicate that interlocutors develop different conversational threads simultaneously. However, we need to study more aspects of learning when individuals interact online on specific mathematical subjects. In this paper we are focusing on synchronous interaction. We build on this to track PMTs reflecting about the definition of a polyhedron and the **question** that drives this research is: Which conceptual scopes regarding polyhedron definition do PMT improve when they interact in online chat?

One research of this nature is important in order to confirm the pronounced impregnation human-technology (Borba & Villareal, 2005) in a kind of environment that constitutes a rich and complex discursive context for interaction. Moreover, developing this type of research is relevant because it is still a challenge to elucidate epistemological aspects (Noss, 2002) concerning mathematical learning in virtual scenarios.

Exploring Geometry in Virtual Environment With PMT

Interactions and thinking are implicit with growth of understanding. Mathematics education researchers have theorized close links between communication and thinking (Sfard, 2008) and between mathematical discourse and collaborative work in virtual environments (Çakir et al., 2009; Stahl, 2006).

Virtual learning environment are mediated by different technologies and artifacts. In this scenario, learning is understood as immersing forms of participation and changing in discourse (Sfard, 2008). In VLE individuals can exchange ideas and develop their mathematics concepts, without hierarchy or domination from one participant on another. In our VLE one way to exchange geometrical concepts is through the use of writing.

Writing about mathematical ideas allows individuals to review, at different moments, their understanding concerning some concepts. In this study we focus on written online interaction regarding the definition of polyhedrons. Defining is an important process in the study of concepts or in establishing *ad hoc* theories, but unfortunately that formative task is absent from current teaching in mathematics (Tanguay & Grenier, 2010). Defining and conceptualizing are intertwined processes. We define the concepts and relationships that we establish in a related way (Gattegno, 1987).

Developing geometry at school and with PMT should be much more than practicing algorithms or memorizing properties and theorems. Working with geometry enables the development of skills such as representing and reasoning, and it stirs the imagination and creativity. Teaching of geometry requires ensuring a sustained focus on the twinned aspects of geometry: the spatial aspects and the issues that relate to reasoning with geometrical theory (Jones, 2012). Exploring the definition of polyhedron can be a powerful pedagogical strategy to improve these two aspects in a sense that when PMT are reflecting about the possibilities of definition they can develop their spatial reasoning based on explicited concepts and related argumentation.

Assuming defining and conceptualization as important processes in geometrical thinking, we believe that those processes could be improved even in virtual environments, because in VLE reflections could be interchanged in different discursive ways and moments. Practices that allow PMTs to develop an understanding of mathematics in general and about the nature and function of definitions in particular, contribute to improve their professional knowledge. Furthermore, they enable PMTs to create examples and reflect about which definition should be used in their practices (Zazkiz & Leikin, 2008).

Research Context and Data Source

The Gepeticem environment (http://www.gepeticem.ufrrj.br/cursos.php) is structured around a vision of work that breaks with the axiomatic approach and the memorization of formulae in geometry classes. Below, we illustrate part of the main screen virtual environment, which has been implemented for the study of polyhedron.

Chat: space where interactions will be analyzed in this paper

Although we agree with Tanguay and Grenier (2010) about the importance of the proof in the geometry classroom, we decided, at this moment of our study, to construct our VLE based on a situation related to the activities of defining, exploring and experimenting via different sources (manipulative materials, software, videos etc.). The proposal, which generated the discussion we are analyzing in this paper, was:

Proposal: See below how four prospective teachers characterize regular polyhedron. Analyze and discuss with your partners the definition of polyhedron and regular polyhedron expressed by each one.

PMT	Polyhedron
1	A polyhedron is a three-dimensional geometrical solid the faces of which are polygons.
2	A figure of 3 dimensions formed by polygons.
3	A polyhedron is made of polygonal regions and the space limited by them.
4	It is a solid the surface of which is a finite number of faces (polygons).

Table 1: Chat proposal

The proposal above was elaborated taking into account the answers given previously by PMTs to an earlier task within the unit, regarding their understanding about polyhedrons. The proposal was sent to them, by e-mail, 10 minutes before the chat. In this report the analytical process was focused on interactions in chat. During one semester we implemented five chats. The chat takes about 120 minutes[2]. The transcription of writing chat analyzed here comprised 343 lines, it occurred from 10:05:10 to 11:58:23h. This chat took place one month after the answers for the task were given, and it engaged 12 participants (11 PMTs and the researcher). We used the following procedures for data reduction: chat transcription (a file provided by the platform itself), numbering (in lines) of interactions, removal of lines which contained no ideas related to the concepts we were intent on focusing, re-reading interactions and organizations in turns.

Results

We found that PMTs interacted in three scopes regarding polyhedron definition. PMTs approached aspects associated with geometric solids in general; aspects focused on the elements (faces, vertices and edges), and reflected focusing on the number of dimensions. These approaches were adopted throughout four moments of revisiting the interactions, namely: (1) interactions within the collective (the whole process), (2) interactions from one PMT, (3) interactions of that PMT within the collective, and (4) consideration (in the chat at the ongoing "moment 3") exclusively of words or sentences mathematically related with polyhedrons, and contrast with the proposal of the chat as to elucidate the process of reflecting on the definition.

The first task in the unit about polyhedrons was about the question: *What does a polyhedron mean to you? Give one example.* Some PMTs' answers were: *"A figure with faces", "They are regular polygons", "A solid with four or more faces", "It is a closed solid", "It is a geometrical figure in the space R^3".* As a rule, moments for discussing about some definition and ways to conceptualize some mathematics object do not belong into the usual mathematics classroom lessons in Brazil. As we can see, each PMT's given answer regarding polyhedrons constitutes an interesting field for discussion.

Above, we have made some general remarks about the type of answer gathered in the environment. We are now detailing the moments created to show an in-depth analysis of the process concerning PMT's reflecting about the ways to define a polyhedron. At the first moment, we observed the development of the whole interactive process, as is shown in the following sequence over the first 4 minutes of the debate.

Chart 1. Fragment of the transcription of the chat.

fmagalhaes (10:18:30) : my answer was incomplete
mary (10:18:59): taking into account it's a first contact with the concept of polyhedrons I guess the ideas 3 and 4 were the simplest to understand
fmagalhaes (10:19:13) : … they are too formal for the pupils' understanding
thiago (10:19:33) : I've already got 4
erj (10:21:12) : I found the definitions 3 and 4 are more understandable.
thiago (10:21:21) : I guess 3 was a bit complicated
researcher (10:22:06) : what's making 3 more complicated?

[2] Two regular classes of 60 minute each.

rschiaro (10:22:10) : It's a solid, the surface of which is a finite number of faces

At this first moment, we tried to have an overview of interactions, the subjects' motivations, their curiosity and the elements that appeared to have caused them some cognitive unbalance. For example, the perceptions about the answers that had initially been presented (fmagalhaes 10:18:30: *my answer was incomplete*), familiarity (or lack of it) with the subject (mary – 10:18:59: *taking into account it's a first contact with the concept of polyhedrons*) doubts or questioning addressed to the group (diegolima – 10:24:50: *A polyhedron is a solid, right?*), and agreements (thiago 10:25:20: *yeah… I was thinking about that*; mary 10:27:08: *I also think it should start by defining what a solid is*). That is why we consider important to get to know the cognitive group as it constitutes itself in the first place (Stahl, 2006), and, from there, later develop an analysis oriented toward one of the participants. So, at a second moment, we randomly chose one PMT. As a matter of fact, we picked the first one who entered the chat to interact online. We withdrew the other participants' interactions and we analysed only this one student's. In this case, we have *erj*.

Chart 2: Focus on a single student.

erj (10:21:12): I found the definitions 3 and 4 are more understandable

erj (10:23:13): I guess when the definition talks about a number of dimensions it gets difficult for the pupils to learn. That's why I like 3 and 4 better

erj (10:24:52): The problem in 1 is the "three-dimensional"

erj (10:28:50): I think a solid, kids already understand better what it is **without a definition that's too formal**

erj (10:36:06): One more thing, ... I think we should define a polyhedron **without prior citing** its elements (faces, vertices and edges) and then **identify** them later

erj (10:45:23): How about: "A polyhedron is a geometric solid, formed by a finite number of polygonal regions (polygons)

erj (10:55:05): But, wait a minute, just **any polyhedron**, **it doesn't have to be regular polyhedrons.** That would be for a regular polyhedron!

erj (11:07:56) : Let's discuss the definitions presented for a regular polyhedron!

erj (11:08:53): In definitions 1 and 3 for a regular polyhedron, the word "Angle" is not mentioned anywhere. Definitions 2 and 4 depend on that, though!

The analysis focused on the ideas and interactions of just one PMT *erj* helps us in obtaining further information about the learning process of this individual within the constituted collective group. At this analytical moment, what interests us mainly is to identify the movement from individual ideas to collective thinking (*but, wait a minute, hey you guys/ if you'd think best*) and backwards, as all the participants have a possibility to think and talk, without asking for permission. It is also important to highlight that even selecting just one interlocutor, it is visible that his/her thinking process is constituted and takes into consideration the contributions from his/her pairs. The contribution from *erj* (10:45:23) shows that the PMT considered a definition for a polyhedron. Nevertheless, as the chat encourages the collective reflection and a belonging to the group, then he/she has the possibility to analyze the idea together with the group. The discourse markers (*Hi you guys! I guess, I found it, how about, but, hey!, could we, if you think fit, let's*) are all examples of how the interactions are shared with the collective group. This process, according to Stahl (2006) widens the individual skills of the people involved. At a third moment, we analysed PMT *erj* in the constituted collective group. What follows is an illustration of PMT *erj* interacting with pairs.

Chart 3: Focus on the students as a collective unit.

fmagalhaes (10:18:30) : **my answer was incomplete**.

mary (10:18:59): taking **into account it's a first contact with the concept of polyhedrons** I guess the ideas 3 and 4 were the simplest to understand

fmagalhaes (10:19:13) : … they are too formal for the pupils' understanding

thiago (10:19:33) : I got 4

erj (10:21:12) : I found the definitions 3 and 4 are more understandable

thiago (10:21:21) : I found 3 a bit complicated

erj (10:28:50): I think a solid, kids already understand better what it is without a definition that's too formal

rschiaro (10:29:11) : I guess so

paulocesar (10:29:23) : I agree with Estevao

fmagalhaes (10:29:33) : Sorry, I found 1 and 2 easier to understand.

mary (10:30:59) : I guess so too

The above interactive process shows us how the answers can be revised (fmagalhaes– 10:18:30), challenged (researcher–10:31:05) and caused (alinets–10:36:41) within the collective group. Besides, examples are requested (thiago–10:37:04) and relations with the PMT's practice (erj–10:28:50). This individual/collective back and forth movement, with contributions which differ in nature, has to be valued in the processes of initial training of teachers. With yellow markers we illustrated how the interactive dynamic in the chat fosters a moment to deepen the PMT's knowledge about what a definition should involve (Zazkiz & Leikin, 2008). Furthermore, in accordance with Borba and colleagues (2010), the way in which PMTs learn can contribute to the way in which they perceive and develop mathematics in their lessons. Finally, at a fourth moment, we highlighted words related to the definition of a polyhedron. We marked (in bold), in the chat above, only the words related mathematically to some definition of a polyhedron and we contrasted them against the chat's proposal. This analysis allowed us to concentrate on the analytical process focusing mainly on the definition, as we show below.

Chart 4: Focus on “words” related with polyhedron definition

Mary (10:18:59): considering it’s a first contact with the **concept of polyhedron** I found the ideas 3 and 4 simpler to understand researcher (10:31:05): So, how would this <u>new definition</u> be, using **geometric solids**

erj (10:36:06): One more thing .. I think <u>we should</u> define polyhedron without prior citing its elements (**faces, vertices and edges**) and then identify them later.

Alinets (10:36:41): but hey, we live in **3D** why do you guys think that kids will not be able to understand the **three dimensions** ... I think it's **easier** than **flat**! * kkk

By highlighting the words in the definitions contained in the proposal we observed: a solid three-dimensional geometric figure of three dimensions, polygons, polygonal regions, and the space limited by them, solid surface, finite number of faces (polygons). In this reflective process we observe how learning aspects that relate reasoning and geometrical theory (Jones, 2012) can be improved through the interactions regarding the considered definition.

Concluding Remarks

In this study, we have analysed online PMT interactions that focused on ways to define polyhedrons. Participants showed they deepened conceptual aspects in three scopes: one associated with geometric solids in general; another one with aspects focused on the elements (faces, vertices and edges), and a third one, focusing on 3D. These approaches are not sequential, hierarchical, nor individual. They arose from the discussion and they developed and deepened with the constituted online group.

In order to identify the three scopes above, our analysis switched from a global look on the interactions (on the motivations of participants, the expression of their curiosity, their doubts, etc., to a focus where we tried to highlight the mathematical ideas, (individual and collective) that were most explicit in their interactions.

We verified that the chat can be an educational space where the ideas of the PMTs can be challenged and reviewed by the individual within the collective group. Perceptions of previously given answers, familiarity with the subject under discussion, doubts or questionings addressed to the collective group, among others, can be observed in the interactive dynamics. Examples are continuously required and relations with the teacher-to-be practices may emerge.

We believe that, besides specific conceptual thinking, when we materialize the opportunity for the PMTs to experience this kind of environment, we are contributing to their possibilities to innovate and implement their professional practices in this type of virtual scenario (Borba et al., 2010). The reflexive back and forth movement between individual/collective, with contributions different in nature (asking, answering, questioning, asking for an example, explaining, etc.) should be valued in the processes of teachers’ early training in virtual environments. In our analysis, for instance, we observed that there was changing in thinking resulting in learning (Sfard, 2008) in these aspects:

PMT Answer (task 1)	PMT reflection at chat After the given answer	Learning aspects observed Moment 1+2+3+4
"**Figure** with faces"	"It's a solid, its surface has a **finite number** of faces" / Chart 1 - rschiaro (10:22:10)	Moving from figure (in general) to specific (solid). Attention on the finite number of faces
"They are regular polygons"	"A polyhedron is a **geometric solid** formed by a **finite number** of **polygonal surfaces** (polygons)" / Chart 2 - erj (10:45:23)	The emergence of the idea of finite and making clear relations between polygon and surface
"It is a geometrical figure in the space $\mathbf{R}^3$"	"but hey!, we live in **3D** why do you guys think that kids will not be able to understand the **three dimensions** ... I think it's **easier** than flat!" / Chart 4 - Alinets (10:36:41)	The importance of exploring 3D then paying attention to flat aspects (then observing flat aspects)

Table 2: Exemplifying learning aspects observed throughout the interactions

Epistemologically, our analysis didn't reveal anything different from what may happen in a conventional classroom. Nevertheless, taking the interaction into account, we can see differences in the ways the PMTs reflected on the possibilities in defining a polyhedron. All the participants had opportunities to share their previous ideas and to go through them and revise them with their pairs within the group. Furthermore, *cognitively,* we can say that the online discussion gave the PMTs an opportunity to reflect upon the aspects involved in accordance to an adopted definition. The discussion was not channelled only to one idea. As we have shown in the table above, different rearrangements of meanings of previous ideas were made possible. Maybe this diversity in forms of interaction and mathematical reflection is one way to perceive changes in the realm of *epistemologies at work* (Noss, 2002) in virtual environment.

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