THE "PRACTICE" OF MATHEMATICAL MODELING UNDER A WITTGENSTEINIAN PERSPECTIVE[1]

Lourdes Maria Werle de Almeida
lourdes@uel.br

State University of Londrina – UEL

ABSTRACT

The objective of this article is looking at the students' *pratice* in mathematical modeling activities under a Wittgensteinian perspective. Starting from the analysis of a modeling activity in two distinct contexts, we present some reflections on specific meanings that may be constituted within the different contexts. What one intends to highlight with this text is the fact that modeling activities are not intended to provide meaning in the classroom to objects or concepts that are in the students' life outside the classroom. Rather than that, it aims to indicate that different meanings are associated to different language-games, which may arise from modeling activities.

Keywords: Mathematical Modeling; Language; Wittgenstein.

RESUMO

O objetivo deste artigo é olhar para o *fazer* dos alunos em atividades de modelagem matemática sob uma perspectiva wittgensteiniana. A partir da análise de uma atividade de modelagem em dois contextos distintos, apresentamos algumas reflexões sobre significados específicos que podem se constituir nos diferentes contextos. O que o texto pretende evidenciar é que atividades de modelagem não têm a intenção de prover de significado na sala de aula conceitos ou objetos da vida do aluno fora dela. Em vez disso, indicar que diferentes significados estão associados a diferentes jogos de linguagem que em atividades de modelagem podem emergir.

Palavras-chave: Modelagem Matemática; Linguagem; Wittgenstein.

The conceptualization and characterization of mathematical modeling in Mathematics Education have been approached in different ways and have been performed under different assumptions in relation to the pedagogical concepts that lead the educational practices and theoretical structuring of scientific research.

In recent paper, Perrenet and Zwaneveld (2012) consider that three leastways relevant aspects regarding the use of mathematical modeling could be taken into consideration.

[1] The text is linked to the performance of the author as a senior teacher at UEL, funded by Fundação Araucaria.

> 1) the lack of unanimity about the essence and the vision of the modeling activity; 2) the almost inherent complexity of the modeling process and, consequently, the complexity of teaching; 3) the fact that mathematical modeling is in the first place always about something, a situation and a problem arising from that situation, and that mathematics is 'only' a part of the whole process (p. 3).

In the present article, although we have no pretension to clarify the problematic addressed in such three aspects, we look upon modeling from a perspective that may somehow contribute to its understanding.

The *place* from where we started in this article considers that, in a general way, a mathematical modeling activity can be described in terms of a initial situation (problematic), of a final desired situation (which represents a solution to the initial situation) and of a group of procedures and concepts which are necessary for one to go from the initial situation to the final one. Literature usually refers to this initial problematic situation as *problem-situation*; and, in general, a mathematical representation or mathematical model is associated to the final desired situation.

Much of the discussion regarding mathematical modeling focuses exactly on such *link* between an initial problem and the obtained solution. Accordingly, different approaches may be found in literature. What students and the teacher do to develop an activity on mathematical modeling is described, for instance, in terms of the characterization of steps and of a cycle well defined by Blum (2002, 2011) and by Blum and Ferri (2009). The students' procedures along the course of the modeling activity are punctuated in terms of *cognitive actions* by Borromeo Ferri (2006) and Almeida and Silva (2002), , for example. The structuring of stages and routs during the development of modeling activities have been approached by Borromeo Ferri (2007). Barbosa (2003) is an example of research in which the author deals with the structuring of the mathematical modeling in the classroom by means of *cases* in a configuration that attributes students and teachers specific actions. According to Galbraith (2012), at the same time that the development of mathematical modeling activities enables the student to engage a problem, it also targets developing in this student what the author calls *intellectual infrastructure*. Under a perspective named *modeling as a vehicle* modeling activities are used by the professor to introduce school content.

Looking at what the student and the teacher do, and how they do it during the development of a mathematical modeling activity is what, in this text, we describe as looking at the *practice of* mathematical modeling, and such act *practice* addresses us to the use we do of language, of mathematics, in such development.

We based our reflections on Ludwig Wittgenstein's considerations on language, especially from his second phase, taking his *Philosophical Investigations* as the major theoretical contribution.

In such phase - usually regarded as the *Second Wittgenstein* - Wittgenstein attempts to undo a referential conception of language, claimed in the first work *Tractatus Logico-Philosophicus*, and starts to deal with language as an activity that establishes processes of action and transformation. Therefore, what he admits is that meaning must not be understood as something fixed and determined, as a characteristic arising from the word itself, but as something that depends on the use within a concrete end, in a peculiar context and specific objectives. These uses are associated to one of the fundamental concepts of the Second Wittgenstein: *Language Games.* Use in different contexts addresses to another of Wittgenstein's perspective on language, the *forms of life.*

Thus, in Wittgenstein's pragmatic theory, linguistic expressions may have distinct meanings within each context they are used, making it possible to perform particular analysis under determined circumstances. What one can do is to examine the different uses and infer the meanings according to such uses.

Nevertheless, according to Wittgenstein, words are used in many different ways and are family-like related one to another in many different fashions by means of some similarities. Regarding them, the philosopher states: 'I can think of no better expression to characterize these similarities than "family resemblances"; for the various resemblances between members of a family: build, features, color of eyes, gait, temperament, etc. etc. overlap and criss-cross in the same way. – And I shall say: "games" form a family (WITTGENSTEIN, 2012, § 67).

In the context of Mathematics, otherwise, it is not about searching for the meaning of the mathematical objects in the interdependent reality of the mathematical language, or in the multiplicity of uses in empirical situations. Yet, understanding or even *accepting* the different meanings is not always an easy task. In such way, Wittgenstein already stated "We are much more inclined to say "all these things, though looking different, are really the same" than we are to say "all these things though looking the same, are really different" (WITTGNESTEIN, 1939 p.2).

Hence Wittgenstein will have to stress the differences between things rather than the similarities. He gives an example like this:

> An instance of these tendency to assimilate different things to each other was seen when imaginary numbers were first introduced into Mathematics for than some mathematicians said that clearly there could not be such things as numbers which are imaginary; and when it was explained to them that 'imaginary' was not being used an its ordinary sense, but that the phrase 'imaginary numbers' was used in order to join up this new calculus with the old calculus of numbers, then the misunderstanding was removed an most of the mathematicians were contended (WITTGNESTEIN,1939, p.2).

Accordingly, we may think that the *existence* of new meanings is something we face as new language games are presented us, according as we exercise determined forms of live and, according to Martins, (2012), such exercise enables us to know and master a language.

We look at the mathematical modeling activities at this perspective. What language games arise from these activities? What sort of meaning comes from these games? Can *resemblances* and *differences* in the *practice* of mathematical modeling be credited to a *form of life*?

Reflections over such questions may possibly clarify something in regards to the lack of unanimity about the modeling activity or the almost inherent complexity of the modeling process, about the place of mathematics in modeling activities, considering that mathematics is *only* a part of the whole process, as stated by Perrenet e Zwaneveld (2012). In this article, we reflected upon the *practice of modeling* considering some groups of students performing the same activity in different educational contexts.

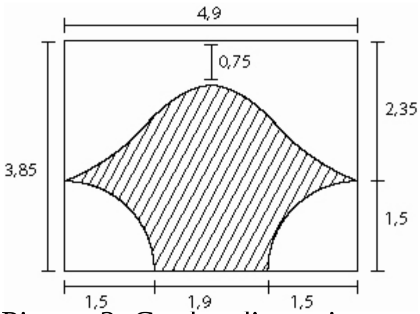
1. Mathematical modeling

In order to subsidize our thoughts on mathematical modeling under a Wittgenstein perspective, we present a modeling activity developed by groups of students in different educational contexts.

The contexts to which we refer consider different physical environments, different students - probably with different interests - in addition to different purposes in performing the activities. Yet, one teacher in these different contexts is the same.

The activity *Planting grass in a garden* has its origin in the image of a garden, actually a photograph of the garden of a teacher's house[2], when she was teaching an extracurricular course of mathematical modeling. The course was developed with college freshmen in Mathematics. The intention in this activity is discussing and obtaining the area of the garden (and, as a consequence, the costs from planting the grass). Picture 1 is the photograph of the garden and picture 2 indicates its measures.

Picture 1: Image of the garden



Picture 2: Garden dimensions

The activity was performed by three different groups of college students from the Degree in Mathematics, during the extracurricular course. Later, in another moment, it was performed by a group of students enrolled in the subject 'Mathematical Modeling in the Perspective of Mathematical Education', taught by the author of the present article during the program of Master Degree in Mathematical Education. In each of these contexts, students are guided by the professor, who is responsible for the systematization of contents. In the end, students are required to explain to the others what they did. In order to look at the *practice* of modeling, we describe how the development of the activity in different contexts came to be.

2. First context: students from the Degree in Mathematics.

The activity had its origin from the students' interest in this context. What the students intended with the activity was to solve the problem. On the other hand, for the professor, the activity would be part of an extracurricular activity linked to the subjects of 'Differential and Integral Calculus' in a way that her intention was to teach mathematics with the activity. We refer here to two student groups using only paper, pencil and a scientific calculator, and a group solving this problem making use of mathematical software.

Starting from the image of the garden (picture 1) and its dimensions (picture 2), students began their procedures of going from the initial situation (the image bearing the

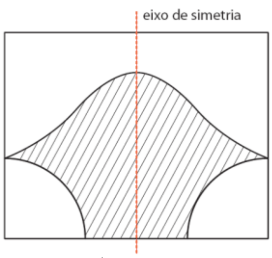
[2] Description of all course activities can be found in Santos (2008).

dimensions of the garden) to the final situation (figuring out the costs with the purchase and plantation of grass).

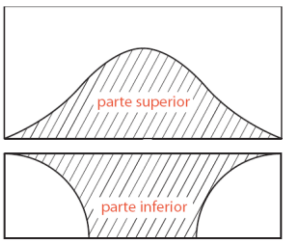
Part of a dialogue between students, recorded in audio and video, reveals how the beginning of discussions was conducted in the group assisted by the professor, aiming to define how their actions to calculate the area would be routed.

> *Prof: How can the area of this region be calculated?*
> *A_B: We're thinking of the Sine Function. Then we should apply the integral.*
> *A_A: OK, but that's only for the upper part of the garden!*
> *A_B: Yep, for the upper part.*
> *A_A: From a certain a to b. Then we could calculate this area... So, then we'd have to calculate counting with this area (pointing at the upper part)*
> *Prof: Yes. Then, what would you do for the lower part?*
> *A_B: Oh, I think we can make a rectangle and then cut off these two side parts, can't we? Do you think this is a parabola? Its area can also be calculated... [A_C interrupts]*
> *A_C: Oh... It looks more like a quarter of circle than a parabola.*
> *A_A: Then we find the area below the curve [commenting on the suggestion by A_B]*
> *Prof: Exactly. One thing I'd like to call your attention to is: this garden is symmetrical (Picture 3). [...] What can this be used for, in the calculation of the area?*

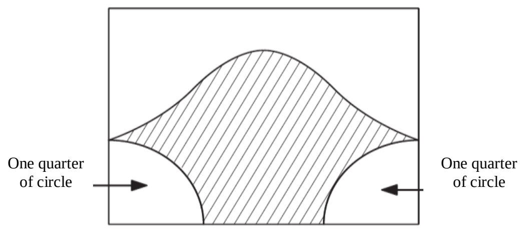
This group of students, starting from the analysis of the picture and the discussions in the group, assisted by the professor, traced the *axis of symmetry*, according to what is shown in picture 3, and considered that it is adequate to divide the region in other two, as indicated in picture 4. From that, one followed what, in Santos[3] (2008), students name definition of *hypothesis.*



Picture 3[4]: Symmetry Axis



Picture 4[5]: Two parts of the region



Picture 5: Identification of the shape of the lower part

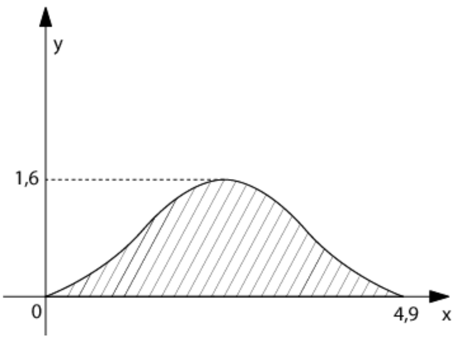
Then, in order to determine the area of the upper part, the challenge would be to adjust a function whose integral, defined in the delimited range, wold fit the value of this area (Picture 6). Students' discussions regarding the shape and what they already knew about trigonometric functions were led them to the definition of *hypothesis* that a sine function

[3] The activity is described in details in Santos (2008).
[4] Axis of symmetry
[5] Upper part Lower part

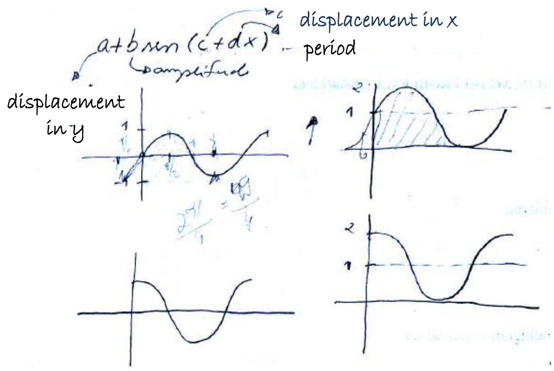
could be used. However, the characteristics of the problem made them establish it should be something like:

$f(x) = A + B \sin (Dx + C)$.

Considering the values in the cartesian plane, as shown in picture 6, students made their *deductions* for the parameters of this function. Picture 7 shows some representations used to aid them in determining these parameters.



Picture 6: The region plotted in the Cartesian plane



Picture 7: Students' auxiliary representations

What this group noticed is that a careful evaluation was necessary, considering the shape of the region and the parameters to the sine function. Thus, together with what they already knew and what they could notice with this activity, the group reached the function:

$f(x) = 0.8 + 0.8 \sin (1.28x - 1.225)$.

The area corresponds to the integral of the function in the indicated range, that is, the area of the upper part is given by:

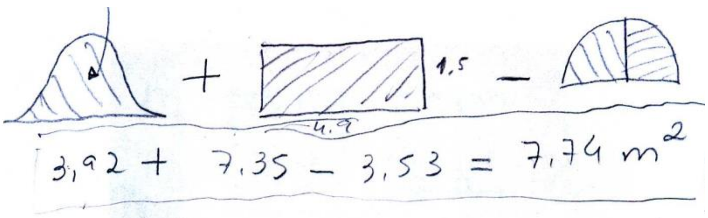
$$A_1 = \int_0^{4,9} (0,8 + 0,8 sin(1,28x - 1,225))dx = 3,92m^2$$

Regarding the area for the lower part, they used the area of the rectangle minus and the area of the two *quarter of circles,* as shown in picture 5. Thus, the area of this region is given by:

-A_2: area of the rectangle: 1.5m× 4.9m = 7.35m^2

-A_3: area of half the circle: $\cong \frac{\pi r^2}{2} = \frac{\pi (1.5)^2}{2} \cong 3,53m^2$

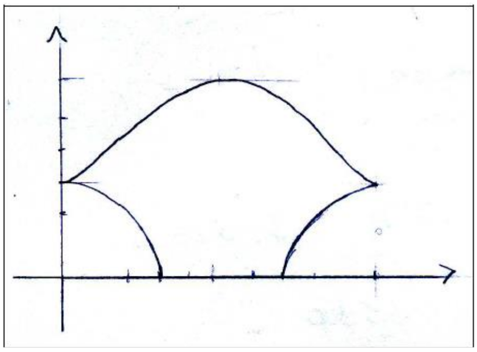
Therefore, the total area of the garden corresponds to $A_1 + A_2 - A_3 = 7.74m^2$, according to the stents' records (picture 8).



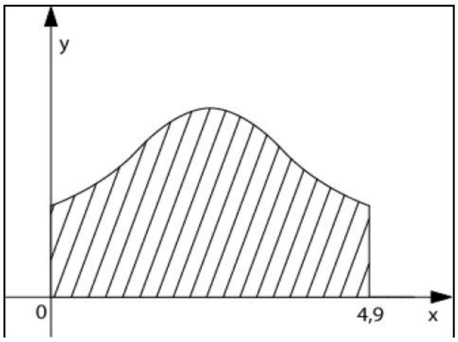
Picture 8: The identified regions and their areas

Considering the price for the square meter of placed grass is about R$4.00 (four Brazilian Reais), it was possible to estimate the costs in R$30.96 (thirty Brazilian Reais and ninety-six cents).

A different approach was made by another pair of students, which chose not to divide the garden region in two parts. These two students started their procedures by making representations, as shown in pictures 9 and 10, considering the *hypothesis* that the region was just one and, in the Cartesian plan, its upper delimitation could be a function such as:
$f(x) = A + B \sin (Dx + C)$.



Picture 9: Garden drawn in the Cartesian plan



Picture 10: Identification of the range

Though in this case, the parameters are other. The function obtained was
$f(x) = 2.3 + 0.8 \sin (1.28x - 1.57)$.

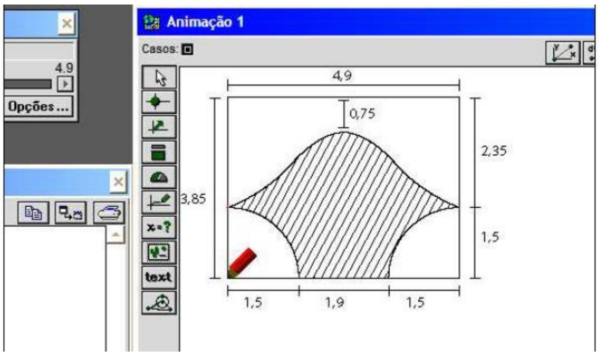
The area of the region in picture 10 is then determined by the defined integral:

$$A_1 = \int_0^{4,9} (2,3 + 0,8 \sin(1,28x - 1,57))dx = 11,27m^2$$

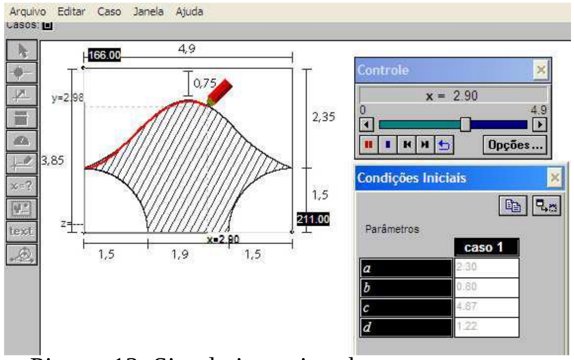
From this result, students subtracted the area of the quarters of circle in the same way as the prior group, doing: $11.27m^2 - 3.5m^2 = 7.77m^2$. In this case, considering the price R$4.00 (four Brazilian Reais) per square meter, expenses would be R$31.08 (thirty one Brazilian Reais and eight cents).

Another group of students in this same course developed the activity in the computer lab, making use of two software programs, *Modellus* and *Mapple*,, which they had already worked with and knew some of the software tools and how to use them.

With the evaluation of the region and its representation in the software (Picture 11) this group, also assisted by the professor, did not consider dividing the region in other two smaller ones. Then, in the same way it was done in classroom, initially it consisted in building a mathematical model whose graph would adjust, the best possible way, to the line delimiting upperly the region whose area it was intended to be calculated. Using the software's *animation* window of *Modellus*, the image of the garden was inserted with its dimensions, being the origin to the Cartesian system defined as pointed by the arrow in picture 11.



Picture 11: Drawing of the garden in the software



Picture 12: Simulation using the parameters

In this case, the group defined the *hypothesis* that a sine-type trigonometric function would be appropriate and started from the general expression:

$$f(x) = A + B \sin (Dx + C).$$

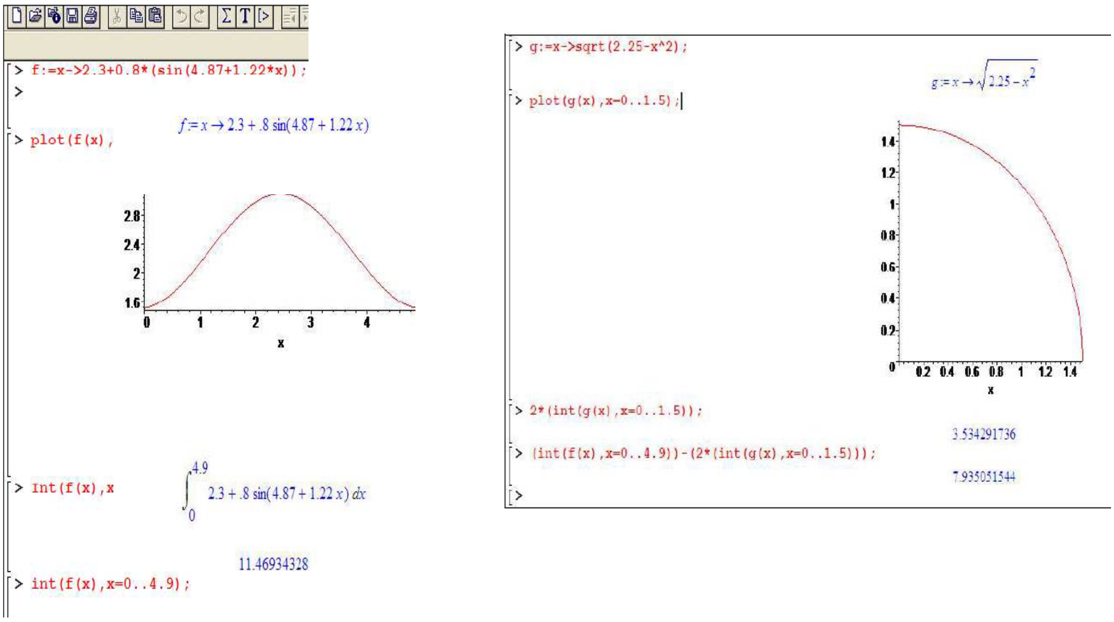
Nevertheless, the determination of parameters would be based upon the use of the software tools, focusing on the analysis of characteristics such as period and amplitude (picture 12), reaching the function:

$$f(x) = 2.3 + 0.8 \sin (1.22x + 4.87).$$

In such case, the area of the region is given by the integral:

$$A_1 = \int_0^{4,9} (2,3 + 0,8 sin(1,22x + 4,87))dx = 11,46 m^2$$

This group also preferred to use another software program, *Mapple*, in order to calculate the integral, as well as to define the area of the circular region, as shown in picture 13.



Picture 13: Students' records using the software *Mapple*

In this case, the total area corresponds to 11.46-3.53=7.93m^2. Also taking into consideration the cost of R$4.00 (four Brazilian Reais), per square meter of grass, the cost of gardening ads up to R$31.72 (thirty-one Brazilian Reais and seventy-two cents).

Discussions considering the area values, obtained in the three ways of solving the problem, were sent by the professor, so each group of students could expose what they had done and how they had reached a certain measure, validating thus their practice in the activity.

3. The second context: a group of students from a post-graduation program

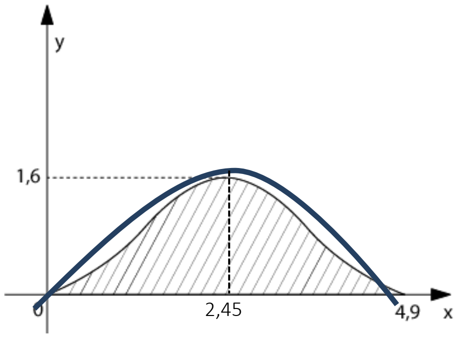
Another context in which the same activity was undertaken is a subject called 'Mathematical Modeling', taken by Master's and Doctor's Degree students in a Post-graduation Program in Teaching Sciences and Mathematics. Here, I refer to the approach by one the groups, which was formed by three doctoral students - one of them a Higher-Education professor and two Elementary Education teachers - and a master's degree student and Elementary Education teacher. This group's intention with the activity was different from those in the previous context. These ones, as teachers, were more interested in the activity as something they could later use in their own lessons.

This group also considered two regions for the area to be delimited, as indicated in picture 4. For the lower part, they proceeded exactly in the same way of the first group students of the Degree in Mathematics, using the area of the rectangle of sides 1.45m and 4.9m minus the two quarters of circle of radius 1.5, having the area for this region 7.35-3.53=3.82m^2.

What this group would change in their procedures concerns the area of the region's upper part, especially regarding the determination of the function that delimits it. The students, making a representation as displayed in picture 14, defined as *hypothesis* that such delimitation could be given by a parabola, a graph which is associated to a function of the second degree.

These Students, considering that the quadratic function is an expression of the type $f(x) = ax^2+bx + c$ and starting from the analysis of the graph shown in picture 14, obtained the necessary information for determining the parameters *a, b, c*, having the function

f(x)=-0.2665x2+1.306x



Picture 14: The approximate region given by a parabola

In this case, the area for the region corresponds to the value of the integral:

$$A_1 = \int_0^{4,9} (-0,2665x^2 + 1,306x)dx = 5,22\text{m}^2$$

So, the total area of the garden is 3.82 + 5.22= 9.04m^2

Considering the same price of R$4.00 (four Brazilian Reais) per square meter of placed grass, in this case expenses would amount in R$36.12 (thirty-six Brazilian Reais and twelve cents). After the performance of their activities, students made some comparisons between their own results and the ones obtained by groups in another context, once the costs of gardening arising from their approach would be greater.

4. A view on the modeling activity based upon Wittgenstein's philosophy

The itinerary of Wittgenstein's thought on language in the writings of his *Philosophical Investigations* expresses the dynamic aspect associated to the process of action and transformation by means of language. What becomes evident, particularly from this stage of Wittgenstein's, is that our condition of relationship with the world is mediated by language.

Wittgenstein's conviction that there is no universality in language, yet, it is bearer of ambiguity and expressions do not have a definitive meaning, was confirmed in his writings by the term *language games:*

> Here the term "language-game" is meant to bring into prominence the fact that the speaking of language is part of an activity, or of a form of life. Review the multiplicity of language-games in the following examples, and in others: Giving orders, and obeying them — Describing the appearance of an object... Reporting an event — Singing catches — Asking, thanking... Play-acting... (WITTGENSTEIN, 2012 §23).

Therefore, the different practices in which language is used, or contexts in which it includes itself are called language-games, in a way that the philosopher states 'I shall also call the whole, consisting of language and the actions into which it is woven, the "language-game"' (WITTGENSTEIN, 2012, § 07) and, in another moment, he points 'the meaning of a word is its use in the language' Wittgenstein (2012, § 43) and Wittgenstein (2012, § 7) named this use *language-games.*

In this sense, the use does not increase the instrumental character of language, but language's intertwining as an interactive practice in which a life-form reflects itself and, at the same time, reproduces it. Meaning is not in the rigid definition of a concept, because it is possible to use such concept in a way that it is not totally closed by a limit, and that does not stop us from understanding it. Therefore, the meaning can only be considered within a context and practice. It seems to be what Wittgenstein intends to say:

> And this is just how one might explain to someone what a game is. One gives examples and intends them to be taken in a particular way. — I do not, however, mean by this that he is supposed to see in those examples that common thing which I—for some reason—was unable to express; (WITTGENSTEIN. 2012. §71).

With such understanding coming from Wittgenstein, we may consider that there is not exactly a problem in the assertive by Perrenet e Zwaneveld (2012, p. 3) in relation to "the lack of unanimity about the essence and the vision of the modeling activity". On the contrary, we can ponder that the possibility to consider different configurations for the development of a mathematical modeling activity may be useful, especially if we can associate to them specific characteristics or a particular context.

In this sense, even if the issue in this article was only one single activity, the contexts in which this activity was undertaken are distinct, and the students' use of language are not the same either.

Though, it is necessary to consider that what regulates the different uses of a word are the rules, turning the use of language not only into an arbitrary activity, but also one that is configured according to our experience, by our understanding the rules in each specific context. According to GOTTSCHALK, (2004), then

> *Learning* the meaning of a word can consist of the acquisition of a rule, or a set of rules, which govern its use in one or more language-games. One of the consequences of such idea in education is that there is no meaning in teaching the specific meaning of an independent word and its several uses. A word only acquires its meaning when one works with it; that is, following a rule in a determined linguistic context. (p.321)

Nevertheless, one must ponder that not all activities on the resolution of a problem may be characterized as mathematical modeling. Accordingly, it seems more adequate to use a characteristic already presented by Bean (2001): a modeling activity requires the performance of simplifying approximations, and the definition of hypotheses by the one who undertakes the activity. That is, *modeling* in association to a path going from an initial to a final situation which we referred to involves these two actions. However, these simplifications might be attached to a form of life.

Although the expression *form of life* is a relatively rare occurrence in Wittgenstein's writings if compared to others as language-games, for instance, it seems to be of great importance to his thinking. "To imagine a language means to imagine a form of life" (WITTGENSTEIN, 2012, § 19), that seems to me as a good example of such importance.

Yet, several of Wittgenstein's interpreters have risked presenting a more detailed explanation for such term. Glock (1998, pp174), for example, states that a form of life consists of: "a cultural or social background; the totality of community activities in which our language-games are immersed". Moreno (2003, p 129), in turn, considers that "forms of life are integrated systems of conventional actions, immersed in the effective practice of our life with language".

These authors' definitions encourage us to consider that the different contexts in which the activity of modeling *Planting grass in a garden* here developed may be associated to a form of life.

Indeed, the language in which students are immersed in at a Degree in Mathematics course might have some specificity regarding its employment in calculating the area of the garden. The relation between the calculation of the area and the calculation of the defined integral is part of a ruled system in which these students are found. What would be left for them to do - which would be specifically of the activity - is to calculate the function whose integral is the approximate value of the desired area.

It's precisely in this aspect that the definition of hypotheses is used bye the students in the different contexts in which the modeling activity was developed. Still, what is the meaning of the word *hypothesis* under the aspect of the development of a mathematical modeling activity?

According to Abbagnano (2007), *hypothesis* concerns a rubric that can only be proved, examined and verified indirectly, considering its consequences. Japiassú and Marcondes (2008), on the other hand, consider that a *hypothesis* is a provisory explanation of a phenomenon, and it must be attested by experimentation.

Eventually, would these be the meanings to the word *hypothesis* in the modeling activity performed by those students? Then, what would such *experimentation* be? Would we be able to verify the *hypothesis* by means of its *consequences*?

In fact, such discussion indicates that maybe, in linguistic terms, it is not exactly what the word hypothesis means in such this context. In the language game to the modeling activity, it seems that the term is closer to what one could call a *well-based supposition*, a sort of *guide* for a search. In other words, starting from the reality (the image of the garden) and the students' knowledge about functions, regarding the aspect of the function graph at least (being these the fundamental elements for the supposition), students start their search. So, there was an expectation in relation to what they could (would) find.

Accordingly, Wittgenstein (2005) states: "the manner you search for something somehow expresses what you expect to find. Expectation prepares a pattern to measure the fact. If there was no connection between the expectation and reality, you could expect nonsense" (§33). Therefore, in a certain extend, the hypothesis expresses a possibility of connection between the expectation and reality.

Referring to what the students produce from the hypotheses, mathematical models, we can start our reflections with Wittgenstein's affirmation: "We shall remember that, in mathematics, the own signs *do* mathematics, they do not describe it" (WIITGENSTEIN, 2005, §155).

It is possible to say that these models may be used to organize this experience of calculating the areas of non-regular regions (the garden). Probably this is precisely one of the most relevant aspects of a modeling activity. The activity we described points to the fact that, within the context of a Degree in Mathematics course, students reached different models, different mathematical suppositions from hypothesis (well-based suppositions). They could see how each parameter of the built function interferes in the graphic aspect of the function. And they were able to do it in different ways. Furthermore, the group which used the laboratory can enter the language-game supplied by technology. Understanding the role software programs play in this case, being able to see how it collaborates in the construction of models and in their graphic representations, also favors the organization of the experience we refer to. In accordance to such thought, Wittgenstein states: "Every time we are able to use a representation other than this or that one instead, we take another step towards our objective, which is to understand the nature of what is being represented" (WITTGENSTEIN, 2009, p. 37).

Considering the students within the context of the doctor's and master's degree course, we may conjecture that they have other side-experiences. Maybe, because of that, we are able to say they constitute another form of life and what they did reflects such form of life and, at the same time, consolidates it. Certainly, in these cases, the students who already were teachers/professors for different levels of education were concerned about solving the problem in a way they could later explain to the others. Such concern seems be based greatly on the mathematical aspects in the construction of the model, rather than on its relation to the calculation of the non-rectangular area. Even if it was grounded in a hypotheses (well-based supposition), this relation would only become relevant afterward, when the quadratic function was already built and its integral calculated according to what we presented in section 2.3. It is possible that such students' strategy reflects what Wittgenstein expresses in:

> Naturally, I may write the solution to an equation of the 2nd degree by chance, but I'm not able to understand it by chance. The manner I reached the solution disappears in the thing I understand. I therefore understand what I understand. That is, chance can only refer to something external, as when I say 'I found it after drinking coffee'. The coffee is not contained within what I discovered. (WITTGNESTEIN, 2005, §157).

In general terms, what this activity allows us to say is that the very language-games constitute meaning. Nevertheless, games constituted in the students' *practice* may also seem to be grounded in rules. Yet, Wittgenstein understood that 'obeying a rule' is a practice' (WITTGENSTEIN, 2012, § 202). According to Gottschalk (2004), such idea of Wittgenstein's drives one's attention to the public character of the rule, in a sense that our form of life organizes our actions *a priori*. That is, what these students already knew (perhaps because they were students applying for a Degree in Mathematics, or for having already concluded this course) about functions of the type $f(x) = A + B \sin (Dx + C)$ and the type $f(x)= ax^2+bx + c$, as well as what they knew or would learn about defined integrals, enabled them to configure what rules of this language-game could be associated to the calculation of the garden area.

For Wittgenstein, mathematical propositions or mathematical sentences are normative. He understood that mathematics was not descriptive, it does not refer to reality. Yet, according to Gottschalk (2004), it provides us conditions for the understanding in determined contexts.

Thus, when students used the *well-based supposition* that the delimitation can be given by a parabola as shown in picture 14, they used mathematics as normative. However when they used points from the curve that outlines the region in order to determine the parameters to the function $f(x)=ax^2+bx + c$ they could use descriptive affirmations like as: *The parabola passes through (0,0)*; *The abscissa of the vertex of the parabola is 2,45; the value of a is 0,2665; the quadratic function will be* $f(x)=-0,2665x^2+1,306x$.In this sense, the rule *The quadratic function is given by* $f(x)=ax^2+bx + c$ *and its graph is a parabola,* has no meaning for itself, but it furnishes the elements for the meaning of the parabola in this language-game.

What one must highlight is that, with the practice of mathematical modeling activities, one does not search for the meaning by school activities for questions or problems students' have in their lives outside the classroom, as some may suggest. Contrary to that, what activities of this kind are designed to do is to contribute so the student understands the meaning of mathematical concepts in the different language-games.

Considering the apparent intrinsicvagueness that Wittgenstein considers concepts may have, its application in determined contexts attributes them meaning in these different contexts. Neither should one argue that, in modeling activities, the student *discovers* the meaning to mathematical propositions on his own. Rather than that, these activities may provide students with different manners of seeing.

Accordingly, *the practice* in modeling activities is aligned with the Wittgensteinian perspective which, in the words of Gottschalk (2008, p.90), 'suggests a teaching and learning concept in which the teacher's role is to teach meanings from the *use* that is made of them in their respective linguistic contexts'. Therefore, the different manners of seeing, when dealing with a problem to which many different forms of solutions may be associated or different concepts could be used, even if suggested by the teacher, must be validated within this context. In the classroom who that validates these different manners of seeing is the teacher.

On the one hand, the *place* from where we started this article considers that, in general, a mathematical modeling activity can be described in terms of an initial situation, an end situation and a set of necessary concepts and procedures. On the other hand, a view upon the *practice* in such procedures, under a Wittgensteinian perspective, makes us conclude that language-games in these activities add specific meaning to mathematics, to the problem, and even mathematical modeling itself, as an activity which demands the employment of simplifications and the formulation of well-based suppositions. In this sense, it may contribute to the development of an ability to describe this or that which, in a certain way, shall be "determining" not with regards to the meaning of a word, because this issue is simply rejected, but in relation to the manner a word may be understood by a form of life.

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