STRATEGIES AND PROCEDURES USED BY CHILDREN IN THE LITERACY CYCLE FOR PROBLEM-SOLVING SITUATIONS INVOLVING ADDITIVE STRUCTURES [1]

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ABSTRACT

The Literacy Cycle has been configured as a new space and time for the teaching of mathematics. Several skills and methodologies must be developed in early elementary education to ensure the mathematical literacy of children. In this article we report on a study based on research-action in schools that explored strategies and procedures employed by students to solve problems through addition or subtraction. We note that counting is structured as the main strategy for implementing the arithmetic operations of different types of actions. In some cases, the problems and actions are processed to give priority to the strategy of counting. We understand that the difficulty to quantify and manage sets occurs because of the difficulty of organizing a mental structure or hierarchy, which are fundamental to the notion of numbers.

Keywords: Literacy cycle; Addition; Subtraction; Strategies; Problem situations.

RESUMO

O Ciclo de Alfabetização tem se configurado como um novo espaço-tempo para o ensino de Matemática. Diversas são as habilidades e as metodologias que precisam ser desenvolvidas neste nível de ensino a fim de se garantir a alfabetização matemática das crianças. Neste artigo relatamos um estudo baseado na investigação-ação escolar que explorou as estratégias e procedimentos empregados pelos estudantes para resolver problemas por meio da adição ou subtração. Constatamos que a contagem se estrutura

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como estratégia maior para realização das operações aritméticas em diferentes tipos de ação. Em alguns casos os problemas e ações são transformados a fim de privilegiar a estratégia de contar. Compreendemos que esta dificuldade de quantificar e manejar conjuntos se dá pela dificuldade de organização de uma estrutura mental de classes ou de inclusão hierárquica, que são fundamentais para a própria noção de número.

Palavras-chave: Ciclo de alfabetização; Adição; Subtração; Estratégias; Situações-problema.

1. Introduction and qualification of the problem to be addressed

Math has been one of the most controversial knowledge fields in education, and, as Pires notes (2000), it is a true social filter that determines success and failure both inside and outside of school. This discipline implies approval or retention of students in formal education and influences, in large part, the success in public competition and selection tests.

In addition, math is one of the subjects that show the highest rates of disapproval in the initial years of elementary school. Math also emphasizes learning difficulties, including problems of reasoning, concentration and motivation. Several studies (BECKER, 2012; SILVA, 2010; CARRAHER et al., 2006) have shown that traditional practices dominate the teaching of mathematics and produce problems in learning by not allowing for autonomous and creative thinking. Thus, this is an area of knowledge that is critical for literacy, for understanding the world, and for interpretation in other fields, such as science, history and engineering. In this sense, elementary mathematics is fundamental for any student and citizen, regardless of profession or social position, who live, interpret and interact in the contemporary world.

The resolution CNE number 7/2010 set new national curriculum guidelines for basic education and established the Childhood Cycle or Literacy Cycle, for the first three years of elementary school. In many cases, it was mistakenly thought that the creation of this cycle came to reinforce the teaching of reading and writing in the native language as well as the study of math. However, this is a very limited view, because literacy can be understood in the sense of reading the world and mobilizing knowledge and codes of different fields of knowledge (SILVA & RODRIGUES, 2012; FREIRE, 2002). In particular, in contemporary times, literacy cannot be limited to the native language, because mathematical knowledge has become indispensable in social space-time. Additionally, the new organization of early elementary education requirements caused an estrangement in school communities, which demanded further research regarding their structure and the development of methodologies for the teaching of mathematics consistent with this innovation.

Nevertheless, public policies have mobilized various features to problematize the space-time of the teaching and learning of mathematics with children. These policies include, for example, *Pró-Letramento* (Pro-Literacy), which undertakes a set of actions linked to the training of early elementary school teachers, the *Prova Brasil* and *Provinha Brasil* (Brazil’s Elementary Assessment and Brazil’s Early Elementary Assessment) and, finally, the *Programa Nacional de Alfabetização na Idade Certa* (National Literacy Program at the Right Age), which, in 2014, is currently developing themes focused on mathematics literacy. These different actions and regulations share the fact that their conception of learning is dedicated not to content and information, but to the development of skills and competencies.

What are the skills and competencies deemed to be critical for math literacy in the Childhood Cycle? On the one hand, the first source to consult is curriculum guidelines for this level of education. However, currently, there is not a very clear definition. The *Parâmetros Curriculares Nacionais* (PCN -National Curricular Parameters), which constitute the main guidance document, were published in 1997, when the nine-year primary school was not yet established and no literacy cycle was yet specified. In addition, existing resolutions and alterations of early education feature changes only in terms of structure and function, and do not include more specific curriculum guidance. In 2012 the *Direitos de Aprendizagem do Ciclo de Alfabetização* (Literacy Cycle Learning Rights; BRASIL, 2012) was published, but it has not yet been put into practice in school curricula.

Brazil's Elementary Assessment of Mathematical Literacy or Brazil's Early Elementary Assessment[2], as it was customarily named, is a census evaluation that seeks to reach all students and Brazilian schools to map the development of children's learning of the Literacy Cycle. It originated from the references of the PCN and has become an important assessment tool, because it has directly effective practices in the classroom. Thus, we chose to study the skills and competencies listed in the reference matrix of Brazil's Early Elementary Assessment literacy process indicators in the Childhood Cycle.

Depending on the scope of the field and the complexity of the situation, the research was conducted in consortium (SILVA et al, 2013) and encompassed all skills and competencies described in the reference matrix of Brazil's Early Elementary Assessment. In this article, we report the C2 competence, which refers to addition and subtraction.

C2 competence refers to "solving problems through addition or subtraction". From it, two skills are as follows: "descriptors D 2.1 solve problems involving the actions of joining, separating, adding and removing quantities" and "(D) 2.2 solve problems that require the actions of comparing and completing quantities". Based on these references, the aim of this research was to investigate the strategies and procedures that Childhood Cycle students build from problem situations involving these two skills provided by Brazil's Early Elementary Assessment. The study is qualitative and was inspired by participant research in the school.

2. Methodology

This article is the result of a wider project that involves an analysis of different aspects of the teaching and learning of mathematics in the early years of elementary school.

[2]This is not to assume that external evaluation instrument is a curricular reference of what should be taught or regarded as a parameter of good teaching. The Came is used as a sampling of major points that should be taught in the Literacy Cycle in the fields of knowledge of mathematics. It is in these terms that it is used: as a competences and skills indicator that enables a vision, albeit partial, of effective math literacy.

Other supplementary character studies were developed concurrently to constitute a joint action, inspired and adapted from consortium research methodologies (SILVA et al., 2013). In general terms, such modality addresses the realization of collective research, with various related themes, and which, in this case, includes the field of teaching mathematics in the early years of elementary school. Complementary research within this consortium is nearing completion, covering different skills and competencies related to math literacy that may present a more general mapping that occurs in the Childhood Cycle. The advantages of this joint mode are: the possibility to utilize multiple perspectives in the same case, approaches under different aspects, collective and cooperative reflection on data collection and analysis, and the interweaving of various similar themes.

2.1. Delineation

The main objectives of this study are linked to the interpretation, understanding and deepening of a school educational context at a specific level of education, which becomes more complex due to the issue of research. Within the framework of qualitative studies (LUDKE & MENGA, 1986; BOGDAN & BIKLEN, 1994), this proposal was inspired by, more specifically, the school research-action approach. To Carr and Kemmis (1988), research-action provides an opportunity to differentiate between theory and practice and to promote the emancipation of the subjects involved using a dialogue in which all are participants. School research-action involves the cycles of planning, action, observation and reflection (CARR and KEMMIS, 1988), in progressive levels of complexity.

Planning, i.e., the early organization of action, is characterized by decision-making related to research directions. In the initial stage of planning, from the teachers and schools that were willing to cooperate with the study, we built understandings of how we could conduct educational activities in the context of the classroom to collect data that would enable us to achieve the proposed objectives. We devised a particular approach, aiming to make it possible to identify and understand how children related to the competencies required to implement the proposed task.

The second component of the school research-action consists of the implementation of educational activities, developing them directly with students to launch challenges and proposals that may highlight the degree of competence and mastery of the skills involved. Thus, planning is carried out in reality to streamline what was built by the collective imagination of researchers.

Observation, the third point, has the function of documenting the ramifications of action, serving as a substrate for the reflections, being a reflection itself redesigning the shares, i.e., "to observe the process of the action, the effects of the action, the circumstances of the action and its limitations, the mode in which the circumstances and limitations cut and channel the planned action and its effects and other things that may arise" (KEMMIS & MACTAGGART, 1988, p. 19). The records were maintained in a notebook adopted by researchers during and after these meetings and were an important data-collection tool and facilitator of reflection. Given the need for action and simultaneous observation, cooperation and the effective involvement of the teacher are essential for the activity, not requiring a very differentiated school context -which could cause estrangement in children - allowing for the conduct of activities while the others may observe them carefully. In this sense, the teacher participates in the entire process of study design, research and analysis.

At the fourth level, *reflection,* we weighed and evaluated both the individual and collective processes of the school research-action. In this procedure, the central focuses of reflections are educational practices, the results obtained and participant understanding. Through dialogue, we can share common scenarios and solve contradictions and problem situations, including the objective and subjective situations that pertain to the processes of learning, making it possible to identify indicators and to create strategies to qualify actions.

The four levels cited are dynamic, and comprise what are called spiral cycles of school research-action (KEMMIS & MACTAGGART, 1988), which are retrospective and prospective. In the specific case of this research, the steps of the investigation-school action structure are as follows:

Stages	**Description**
Planning	Study of the reality of the proposal. Development of the understanding of the skills and competences to study. Construction of the problem situation. Preparation of materials to be applied.
Action	Action on the third-year classes for gathering information. Proposition of the activities. Preparation of questions during the development of strategies for children.
Observation	Observation of children's conduct, of materials produced and of explanations that have been adopted for some strategies.
Reflection	Analysis of collected data. Reflection on the limits of the problem situation. Development and understanding of how the Literacy Cycle affects children's capabilities, which are reflected in their multiplication and division capabilities.

Table 1 – Detailed investigation-action taken

2.2 Field of study and research participants

From the idea of investigating strategies for resolution, modes of action and procedures of students, it is understood that such level of education aims to develop goals throughout the process, but with the relative assurance of achieving them fully only until the end of the Literacy Cycle. Thus, we think that it is interesting to investigate the subjects who are completing this step, that is, students at the end of the third year.

Two regular education classes from a public inner city school in Rio Grande do Sul participated in this study. One of the classes consisted of 15 students, and the other consisted of 18 students. They were led by two teachers with higher level education, which included the research group in which this study was performed. They led and supported the development of data collection to prevent the estrangement of students, and the researchers conducted a differentiated activity. The criteria for the choice of those students were the availability of the school and the fact that their teachers were on Regent's research team.

In terms of analysis, there was no expectation of an individual performance assessment to observe in which students decided on the proposed problems. We believe that this function is carried out by the fact-finding Stemmed Brazil of mathematics, and their quantitative data are more reliable for the issue. Our focus was directed to the observation of the qualitative data and we did not worry about whether a child would help another child or exchange information during the development of the task.

2.3 The construction of data-collection procedures

Within the perspective of school research-action during the planning stage, there were several structuring movements of the problem-situation to be developed with the students. At this point, researchers and teachers of basic education[3] organized to create didactic situations not much different from the school context, but they focused on demands related to the skills and competences in question.

Within the context of the so-called traditional pedagogies, the contents are understood as a set of information that the teacher should provide students (BECKER, 2012; SILVA, 2010). Learning and teaching modes, from that perspective, involve the memorization of information and the transmission of knowledge by the sensory pathway. The contemporary pedagogical practices and various studies in the field of the foundations of education have questioned such an approach and the retention of the information function. Leaving this questioning aside, the current didactic has been busy creating educational models and curricular references that direct the ideas of skills and competencies as opposed to the perspective of content and information.

Acquiring content and information is an important step of learning processes, but it is not sufficient, given the importance of knowing what to do, interpreting data and mobilizing concepts in situations and problems that we face. Thus, Perrenoud (2000) defines competence as the skill that acts effectively in situations, mobilizing the resources available, i.e., affective or cognitive resources. Similarly, skills set are the set of practical knowledge on the know-how and the development of procedures. They amplify the ideas of the contents, which usually require a more informational background, without taking up learning procedural knowledge and attitudinal knowledge (ZABALA, 2000).

As a didactic strategy and the development of skills and competencies, we have been thinking about the idea of a problem situation. It is a snippet of a complex domain, whose realization involves knowing how to use cognitive and material resources, make decisions and mobilize troubleshooting strategies (PERRENOUD, 2000). Similarly, according Meirieu (1998), problem situations are those that require a student to perform a didactic task he cannot perform without learning something beforehand. In other words, the problem situation is a strategy that aims to develop a skill not only by reviewing the accumulation of prior knowledge. Through it, we can highlight the skills

[3] It is understood that the teachers that make up this study and effectively participated in the design, collection and analysis of research are, in fact, researchers. However, we have maintained this condition to enhance the search experience in conjunction with teachers in the exercise.

and competencies that children possess as well as their learning skills, and reactions to situations with which they were not familiar.

In this sense, action, the second step of school research-action, is directed to a problem situation that requires the mobilization of the skill to develop addition and subtraction ideas, taking into account the different actions that involve these arithmetic operations. To monitor the children's reasoning, they were distributed blank sheets and were told that the tasks would involve calculations, drawings or phrases.

2.4 Competence and skills involved

To begin the discussion regarding the addition and subtraction skills, we first need to explain the concepts that formally characterize these two arithmetic operations. In conceptual terms we can say that:

> Among the natural numbers are defined two fundamental operations: *adding*, that the numbers $n, p \in \mathbb{N}$ matches the *sum* $n + p$ and *multiplication* that associates the *product* $n \times p$. The sum $n + p$ is the natural number that is obtained from n applying p times in a row to take the operation successor. In particular, $n + 1$ it is the sucessor of n, $n + 2$ is the sucessor of n, and so on. For example, $2 + 2 = 4$, simply because 4 is the successor of the successor of 2 (LIMA et al, 2006, p.38).

We understand then that the sum of all natural numbers is arrived at from the addition of the repeated action of, i.e., 1 + 1 + 1 equals 3, adding one to this forms 4 and so on. Additionally, Kamii (2002) notes that the addition of natural numbers is the mental action (constructive abstraction) to combine two totals to create a total of higher order in which previous totals become two parts, i.e., the sum of 2 and 3 arrives at the result 5. This result is a higher-order total. The parties to which Kamii (2002) refers are commonly called parcels.

During the various stages of the construction of this investigation it was necessary to deeply study the meaning and intentions of the descriptors of Brazil's Early Elementary Assessment to better understand the knowledge and procedures that children use. In particular, the description of the skills related to addition and subtraction involves terms that seemed to be, initially, synonymous.

The reference matrix of Brazil's Early Elementary Assessment indicates that the descriptors that explicitly refer to the skills are "D2.1 - Solve problems that require the actions of joining, separating, adding and removing quantities" and "D2.2 Solve problems that require the actions of comparing and completing quantities". These two descriptors contain several verbs that apparently describe the same actions. However, in cognitive terms, they differ significantly. Still, it is possible to notice that the descriptor D2.1 indicates the same skill actions that involve antagonistic ideas, such as joining and separating or adding and removing, which shows that these different arithmetic operations have interrelated cognitive processes. In this sense, the evidence of Mathematical Literacy Brazil is aligned with the studies of Vergnaud (2009), who departs from the principle that addition and subtraction are part of the same conceptual field.

Before the constructivist studies, the teaching of elementary arithmetic operations was considered to be a natural linear evolution that started with addition, then subtraction, and then multiplication and division, with an emphasis on the teaching of algorithms, which solve numerical problems that involve these four operations. For Vergnaud (2009, p. 197), "Problems of the additive type [...] are those whose solution requires only addition or subtraction", noting that, because the operations of addition and subtraction are closely related, there is no distinction between the conceptual field problems of addition and subtraction problems.

Vergnaud (2009) differentiates between two concepts: the state and the relationship. The first of these terms refers to the numerical quantification; the second term refers to the numerical ratio between two states. From the first differentiation, Vergnaud (2009) classified the problems that involve additive structures in six groups, called "base relations additive structures", namely: a) combination of states; b) transformation of states; c) comparison of states; d) composition of transformations; e) relative states compositions; f) and transformation of relationships. For the analysis of skills and competencies in question, we are interested in the first three groups because they support the description of skills in key words D 2.1 and 2.2 D.

a) Combination of states, also known as "relationship part-part-whole", consists of "the junction" or "separation" of elements of a distinct nature from two different sets in the same set that brings them together. For example: Paul has 2 oranges, and Pedro has 4 apples. How many fruits do they have in total?

b) Transformation of states consists of the "extra" or "withdrawal" of elements in a set that contains only elements of the same nature. It can be "processing of positive states", when there is an extra, or "transformation of negative states", when there is a removal of elements. For example: Ana has 5 dolls, but she gave 2 to Patricia. How many dolls does Ana have left?

c) Comparison of states involves some relation between the quantities involved in the problem. A large part of the problems of comparison of states are characterized by the expressions "more" or "less". For example: Paul is 8 years old, and his brother Pedro is two years older than Paul. How old is Pedro?

In relation to the skills in Brazil’s Early Elementary Assessment, we can say that the actions of joining and separating that exist in the descriptor D2.1 are associated with the so-called problems of (a) combination of states and how the actions of "add" and "remove" relate to the (b) transformation of states. Finally, the actions of the descriptor D2.2, completing and comparing, refer to the (c) comparison of states, in which there is a numerical relationship between two states. In this sense, it is understood that proposing a problem such as "João has five apples and ate two. How many are left?" refers to a transformation of states because they are elements of the same nature. The situation "João has five toys but donated two dolls. How many toys does João have left?" is a combination of states because there is a relationship between toys and dolls. Thus, although in formal terms it is the calculation 5 - 3, it configures itself as different problems, because the cognitive operations needed to solve this problem are different. Taking into account Vergnaud’s conceptual field theory and the indication of skill of descriptors of proficiency C2 of reference matrix, we developed the following problem situation to map out the strategies and procedures employed by the children.

2.5 Delineation of the problem situation

The problem situation proposal was designed to build a context familiar to students and in which they could handle transactions across its different actions. We opted for a situation in which it would be possible to provide figures for students that they could lean on during the resolution of problems and find that the vast majority of the items came from some type of visual support. The problems were offered for groups of three children so that they could share views and we could see more clearly the strategies and procedures negotiated in the collective. The problem situation involved the management of challenges with different engravings of animals, with which they held together the actions of joining, separating, adding, removing, comparing and completing quantities.

For the purpose of the elements that comprise and characterize a problem situation, we have worked mainly with those indicated by Perrenoud (2000), Macedo (2002), Meirieu (1998) and Zabala (2000), including a significant context of the situation, the obstacles to be faced, the character of the challenge of a situation, prior knowledge that students needed to possess, the learning that can develop, the resistance that tends to be found in solving the tasks and the possibilities for validating the strategies used.

The table below shows the characterization of the problem-situation:

Problem situation	Students should organize engravings so that the skills of joining, separating, adding, removing, comparing and completing were achieved.
Obstacle	Organize the distribution through the notions of set.
Challenge	Separate all animals according to specific features.
Knowledge	Notions of addition, subtraction, comparison and separation.
Learnings	Establish relations of addition and subtraction through sets.
Resistance	Understand that all items are part of a larger set of animals. Manage the stock within that set in conjunction with its subsets.
Validation	Perform the actions of joining, splitting, adding, removing, comparing and completing quantities by counting with the images of support.

Table 2- Detailing the problem situation proposal

2.6 Description of the problem situation

Six problem situations were developed, one for each field in the additive skill, which students reported as they solved the problems presented. Because the research was exploratory, we established some criteria to be reviewed over the course of the activity; however, we believe that throughout the analysis other factors were also relevant, especially regarding types of students, while students tried to solve the problems.

Initially, students were divided into two large groups and were presented various figures with animals to paint. The objective was to become acquainted with the material, and the vocabulary was adjusted as the nomenclature given to each image. There were four parrots, five chickens, five cows, three pigs, three horses, eight ducks and a rooster. Over the course of the activity, however, these quantities were subjected to minor changes to surprise the groups. The figures presented were as follows:

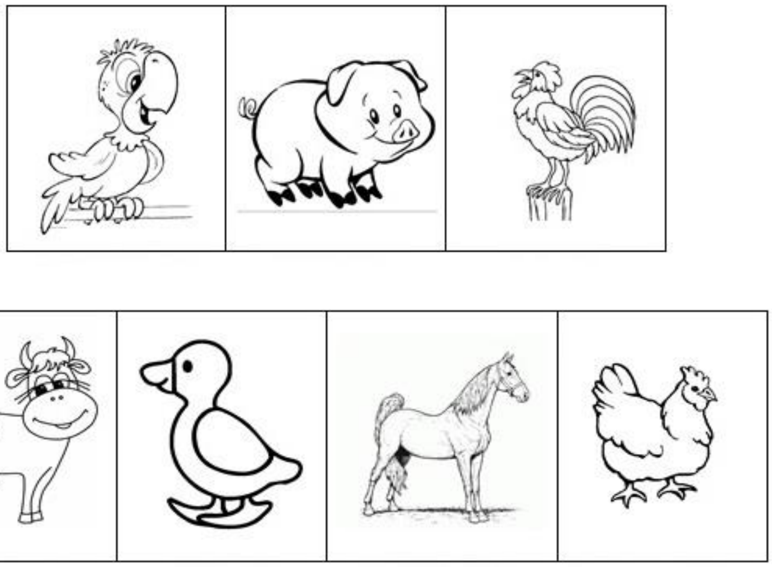

Figure 1 - Figures available for the students to perform the activity

Then, the researchers were arranged in a corner of the room with a certain amount of figures that presented several animals of each species. Three students from the two major groups that were painting were chosen at random to sit at the table of the researchers and respond to problem situations that were previously drawn up. Before the students responded, the researchers asked whether they knew the species of the animals and explained that they lived at "Cocoricó Farm". Based on this information, the children answered questions, which are detailed in the following sections. After this group ended, another group of three students sat at the table with the researchers, and the same procedure was performed.

The problem situations were developed initially for specific quantities of the species. For example, in the joining activity, we asked: "*How many four-legged animals live on Cocoricó Farm?*" However, when one group dissolved and returned to painting, we believe that a possible communication with other students who had not participated in the activity could have elicited answers from other students when they participated. Thus, throughout the activity, we slightly changed the wording of each problem situation. For example, when assessing the ability to gather, we asked, "*How many two-legged animals live on the Cocoricó Farm?*" (rather than four-legged animals) to eliminate the possibility of adjustment by prior knowledge of the response. We could not see discussion occurring among the students. The questions used to assess each of the actions were:

Action	Question
Join	How many four-legged animals live on Cocoricó Farm?
Separate	All Cocoricó Farm animals were invited to a party, but horses cannot enter because they are very brave. How many animals will be able to get into the party?
Add	Two more chickens have arrived to live at the Farm. How many chickens live on the Cocoricó Farm now?
Remove	Three ducks left to live on a neighboring farm. How many ducks remain on Cocoricó Farm?
Compare	The neighboring farm has five more parrots than Cocoricó Farm does. How many parrots

	live on the neighboring farm?
Complete	The owner of the Cocoricó Farm is a heavy sleeper and would need five roosters to be awake. How many cocks would they need so that the owner could wake up?

Table 3- Proposed issues on situation-problem

3. Analysis and Discussion of data

3.1 Joining skill

The adding skill was tapped by all figures, illustrated in Figure 1, and the following question was asked: "How many four-legged animals live on Cocoricó Farm?”

After receiving the figures, all groups left them grouped in a single lot. Most groups used the withdraw skill from the four-legged animals, thus forming a subset of the larger set, which we call the animal set.
When asked for the first time, some groups named only one representative from each species and disregarded the others. The excerpt of the following protocol illustrates this strategy:

> The student took a picture with a pig, another with a cow and another with a horse and said there were three animals with four legs. They did not consider all the figures of the same species, taking only one representative of each species in the count.

So, in these situations the researchers were asked questions such as "What about those there?" or they were asked "All?" The researchers then retold the students to consider the other animals. In one of the groups, for example, the children got all the figures and spread them on the table. A student said to another member of the group:

> — We have to take those with two legs here.
> Then, they begin counting the figures all mixed and they say:
> —1, 2, 3,...? This cannot because it has two legs, 4, 5. This also does not ...
> Another member of the group says:
> — Wait, we have a chicken without wanting to.
> And start counting from the beginning until you reach the correct answer.

Thus, all students used the unit count as a strategy. They counted, pointing directly at the figures. Only one of the groups organized the pictures of animals in accordance with the species forming subsets. Other figures were disorganized. Nonetheless, this group that organized the figures used the animals - joint count - without establishing a relationship between animals and species.

In summary, we highlight two different resolution strategies:

1) Organization according to the species, regardless of the number of individuals and counting only one representative of each species with four legs,

2) Random organization of the figures on the table and one-by-one counting of individuals, without distinguishing the species, instead of analyzing whether each individual had four legs.

3.2 Separating skill

In the separating skill, the students were asked: "All animals of Cocoricó Farm were invited to a party, but the horses cannot enter because they are very brave. How many animals will be able to get into the party?"

Most of the groups again blended figures, though some made a partial classification, leaving figures with animals of the same species. One of the groups again left the mixed figures on the table and started counting from the figures given:

> —1, 2, 3. This is a horse so you can't. 4, 5, 6. This also cannot ... As for not separating the horses from other animals, there was some confusion at the time of the count, and one student said to another:
> —We had a horse too, look - and pointed to the horse that was together with other animals that had already been counted. Then, they started to count again, in the same way as they were doing before, but being more careful not to include the horses.

In another group, a student suggested the idea of separating the horses from the other animals:

> —Let's get all the horses you have there, so it's easier to count. Then, another student began to separate the horses and put them in a corner of the table so they weren't counted. From there, they began to tell what was left.
> — 1, 2, 3, 4...
> They counted the unitary shape, figure by figure, until they reached a final response.

In general, most groups separated the animals and then made a unit count of other animals without organizing them according to species. One of the groups, however, began counting and had to start over once they found an animal that should be separated. This situation happened twice, until one of the group members noted that a species should be separated so that the counting could be performed properly. We note that there was no attempt to estimate the result. The count was purely unitary.

Two strategies were observed:

1) Separation of the horses with the larger body, noting each of the other figures that should be included therein. This was the strategy of all groups, except for one.

2) The same group that in the skill to join the count from the species number and not the quantity of individuals returns to perform the same strategy.

3.3 Adding skill

The adding skill was tapped through two figures with chickens and the following question: "Two more chickens came to live on the farm. How many chickens live on Cocoricó Farm now?"

Some groups chose not to use the chickens and counted on their fingers or mentally. Here, the strategies outweigh the direct counting of figures. As an example, we present the dialogue below:

> — Just add two more chickens.
> And started counting on their fingers.
> — 1, 2, 3, 4, 5, ... until you reach the answer.

Another group preferred to separate chickens from other animals and only then began to count them, along with the two chickens that were added. The group was asked:

> — How did you get this response?
> — It's simple, we just put two chickens. As we had 5, we got 7.

One of the groups also used mental calculation:

> — Two more chickens came to live on the farm. How many live on Cocoricó Farm now?
> —7- Performing the mental calculation.
> — You think it's seven? Another group that had said here that it is three. What do you guys think?
> — Don't. The answer is seven. Because five plus two is seven, said another student group member while counting with fingers.
> — 5, plus two more chickens, gives seven, and just add it all together.

In short, for addition two groups organized the figures before the procedure removing only the figures of the species of animals in which there would be growth and three groups added with disorganized figures. The strategy used was again the unit count, not estimation. Some students used their fingers to count, although most performed the counting directly using the figures, adding the figures of animals that were already in the farm animal figures. One group counted mentally.

3.4 Removing skill

In the removing skill, students were asked, "Three ducks went live in a neighboring farm. How many ducks remain in Cocoricó Farm?". Because most groups had not organized the figures to resolve the issues, they had trouble removing the ducks. In general, after finding the ducks, students counted on their fingers. This approach is illustrated by the following dialogue:

> — The answer is three.
> — How did you arrive at that answer?
> — I don't know. You said it in the question, so I think it's three.
> — If you think that's the answer, then use the figures to show me what you've done.
> The student thought and then began counting the ducks of unitary form using the figures.
> — 1,2, 3, 4, 5, 6, 7, 8 – featured figure by figure.
> Then, he thought a bit and then said:
> — If I had 8 (shown in the left hand opened and the right hand with the thumb, index and middle fingers open) and 3 are gone (bent his pinkie, ring

and middle of the left hand), then 5 (counting on his fingers from the left hand indicator until the middle finger of his right hand).

However, instead of removing the ducks, this student counted on his fingers from three up to eight. This procedure turned the action into removal, which is usually understood as a subtraction, in a count. Instead of 8 - 3, the student turned the problem into starting with 3 and asking, “How far is 8?” He, then, proceeded to perform an action to complete the problem that is closer to addition. In essence, the action is modified to be capable of being performed by counting.

One of the students initially separated the ducks from other animals and then replied that there were eleven ducks. A student of another group, which was no longer performing the activity, turned back and said:

— It's not 11. Is the answer 5? – addressing the classmate.
The student who was participating in the activity looked at the set of ducks organized ahead and asked why the answer should be 5.
— The answer is three.
The researcher then asks the student:
— How did you get this response? (referring to 11).
The student, without answering, went back reading the question silently, and then spoke:
— Oh, no ... it's too little they left.
Then, the student went back to the group of ducks, organized and removed three figures, counted again the set and got the correct answer.

Thus, to remove, all groups used the unit count strategy. All groups reported directly in the figures, although two groups also counted on fingers after removing the animals to answer how many remained. Of the groups that organized the figures before the procedure to add in the previous problem, two continued with the disorganized figures to remove, but one group concluded that it would be better to organize the pictures for the new problem. The groups that organized the animals in the activity continued organizing the activity of removing and were successful. We note that this task was more difficult, especially in removing, because the unit count involves a more sophisticated structure: the general set and two subsets, one of which remained and one of which left. The manipulative material, in this case the figures, helps to reach the answer when counting; however, this strategy evolved to more sophisticated forms by count subsets or the sum of instalments, because students are restricted in the manner of counting the figures to arrive at an answer.

3.5 Comparing skill

In the comparing skill, we asked the following question: "*The neighboring farm has five more parrots than Cocoricó Farm does. How many parrots live in the neighboring farm?*" The groups again left the mixed figures on the table. Soon afterwards, one of the researchers posed a question to one of the groups, and one member replied:

— But there's no way to solve it because they're missing parrots.
— Why do you think they are missing?

— Because we only have 4 parrots and the answer is 5.
— And you know it's 5?
— You saying it there in the question.

In this matter, there was much doubt among the children because they did not understand what we wanted to say "most", although the question was asked several times. Two students separated only parrots, and we began to observe the same thought process. One student said to another:

— Parrots are missing here.
— Of course not. We have to add. You're not seeing it?
— But you don't have anything to add here We only have 4 parrots.
— Of course it has. Look, if the neighboring farm got 5 more parrots than ours, you mean we have to add 5 with 4 (he began counting on their fingers) which will give 9, which is the answer.
— Ah, now I understand. I just thought I had to use the figures.

One of the groups did not find it difficult to solve the problem:

— It's 9!
— Why is this is the answer?
— Because we already have 4, and there is 5 more which makes 9!

After the students were given five more pictures with parrots, they struggled to formulate a response, because they did not have the figures to manipulate.

For the comparing strategy, students used the unit count. Only one group used fingers and figures to count. Counting on fingers without figures was also done by another group. The other groups failed to produce an efficient strategy to solve the problem. One of the groups presented random responses, with expressions of laughter. Apparently, the term "most" in the statement of the problem, which had to be repeated an average of 3 to 4 times for each group, simply made no sense to that group, who used trial and error to obtain the answer. A frequent response to the reply was the number of farm animals at Cocoricó or just 5, ignoring the "most" stated in the problem. The other two groups simply did not respond, expressing difficulty in understanding the question.

3.6 Completing skill

In the completing skill, students were asked the following: "The owner of Cocoricó Farm is a heavy sleeper and needs five roosters to wake up. How many cocks are needed for the owner to be able to wake up from his heavy sleep?" Once again, the students counted on their fingers to resolve the issue, as shown in the dialogue:

— If he needs 5 and there's only 1, then 2, 3, and 4 is missing to get to 5 – and began counting on their fingers from the number 1. So he needs 4.

Another group, however, did not understand the question and began giving random answers. Another group organized their response by subtraction calculation. Let's see their approach:

— There are 4!
— How do you know that the answer is 4?
— Because he needs five cocks and only has one. He is missing four!
— What did you do to find this answer?
— He needs 5 cocks, right?
— Yes!
— Then, he only has one - lowering a finger - so he needs four cocks!

Thus, for situations that involve the completing skill, three groups used the strategy of unit counting on their fingers to reach the answer. The figures were not available, which led them to use this counting mechanism. The other two groups did not understand the statement that was repeated several times and responded by trial and error until they reached the answer. Some students tried the strategy of adding, which was misguided, and responded that the answer was actually the sum of the number of farm animals with the number of missing animals.

4. Final Considerations

From the problem situation proposed we understand that addition and subtraction ideas form operation systems that are used by the children, without a concomitant worry about the fact that the problem is more suitable for this or that operation. The strategy for a resolution almost always turns to the counting that involves pointing procedures about figures or manipulating fingers. Even if all students in a classroom already learned resolution by algorithms and had paper and pencils available, none of them mentioned using these tools in the same way that did not demonstrate interest in the records through drawings or symbols.

This predominance of counting as a strategy may reveal a problem regarding the cognitive processes of adding. In general, the procedure involves counting with fingers and tapping the image being recorded. We noticed that the students carried out operations taking into account quantity sets, such as four ducks and two more chickens. They used a strategy of identifying the items that they would like to record and then creating one large set whose amount was uncovered through a unit count. Thus, the students were unlikely to quantify the animals in terms of sets and were likely to quantify only the handle. To perform the scores, they were always individual and did not use strategies such as counting by twos or threes. This difficulty of working with sets in different development unfolds actions that compose adding.

When elements arise and break from the whole, as in the action of adding (when they get two more chickens at the farm), students used fingers as intermediaries, tainting the initial set without adding the figures provided. In the same sense, in the action of removing, a similar difficulty appears. In this case, the totality is a set that must be broken into two others: one of which remains and one of which is left. This type of action makes counting a strategy; thus, the procedures change. Some students turn the situation into an action to complete to be able to count; however, doing so does not lead to a withdraw action. In other words, to ask, “If of 8 ducks, 3 are gone, then how many are left?” the student does not perform 8 - 3 = 5, but if the question is ”From what I can tell you, number 3 will arrive at 8”, they reverse the situation to be able to perform a count as a strategy to find the answer.

This behavior that we identified in children is also quite common in adults, such as in the case of money. We pay, for example, a bill of $37.00 with a $-50.00 bill. The collector, who does not usually perform a 50 - 37 calculation, performs a count: He puts a bill of $10.00, and after 47 more says $2.00 and says 49, ends with $1,00 and says 50. In fact, what he did was count from 37 to 50, without anticipating the outcome, so much so that it is usual to check the resulting value while recounting the change and then finding out the amount to give. This example demonstrates that counting persists as strategy and is not restricted to children.

The comparing skill, in which students wondered how many more animals were in the neighboring farm, became especially difficult for the same reason: necessarily comparing actions demanded the identification of two sets. We believe that it is necessary to gradually increase the level of difficult. As in this case we did not provide the additional figures, students could not use the feature count, and, without this option, it seemed that the vast majority of the children formulated a strategy for how to proceed.

Thus, counting is a larger strategy to perform arithmetic operations with different types of skills. In some cases, the problems and actions were transformed to prioritize the strategy of counting. Supported by Kamii (2002), we understand that this difficulty to quantify and manage sets occurs because of the difficulty of organizing a mental structure and hierarchical inclusion, which are fundamental to the notion of numbers. In this sense, we understand that teaching is transmissive and relies on orality, favors learning based on the announcement of the numerals in order, and facilitates learning of the notions of seriation, but the idea of classification is challenging. In the same sense, counting is structured as a strategy in that it allows one to achieve a result without developing more sophisticated cognitive processes.

5. References

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