

MATHEMATICAL MODELLING IN PEDAGOGIC PRACTICES: HOW TEACHERS DEAL WITH TENSIONS IN DISCOURSES

MODELAGEM MATEMÁTICA NAS PRÁTICAS PEDAGÓGICAS: COMO PROFESSORES LIDAM COM TENSÕES NOS DISCURSOS

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ABSTRACT

How do teachers deal with *tensions in discourses* when they implement mathematical modelling in pedagogic practices? This question is discussed – following Basil Bernstein's theoretical framework – based on a study of three middle school teachers from Brazilian public schools. The nature of the research is qualitative. The procedures used for collecting data were observation (accomplished through recordings of lessons), interviews after each lesson and teachers' documentation of their lessons. In this paper, we will show how teachers dealt with *the tension of unexpected situations*. The results suggest that teachers understand that the modelling task requires producing a legitimate text to develop it in their pedagogic practices. Teachers dealt with the following unexpected situations: *students presented errors in mathematical procedures, students did not bring the requested information to the task, students chose different parameters the teachers had not predicted, students failed to solve the problems of the task and students resisted doing the task*. These unexpected situations were related to the following student actions in the modelling environment: *students' mathematical performance in solving the task problem, noncompliance of tasks in the modelling environment and students' choice of the subject in a modelling environment*.

Key words: Mathematics Education, Mathematical Modelling, Pedagogic Practice, Teachers, Tensions in Discourses.

RESUMO

Como professores lidam com as tensões nos discursos quando implementam a modelagem matemática nas práticas pedagógicas? Esta questão é discutida – tomando como referencial teórico Basil Bernstein – com base em um estudo com três professores do Ensino Fundamental II de escolas públicas brasileiras. A natureza da pesquisa é qualitativa. Os procedimentos utilizados para a produção dos dados foram observação (realizada gravações de aulas), entrevistas após cada lição e documentos dos professores sobre suas aulas. Neste artigo, mostraremos como os professores lidaram com *a tensão de situações inesperadas*. Os resultados sugerem que os professores entendem que a tarefa de modelagem exige produzir um texto legítimo para desenvolvê-lo nas práticas pedagógicas. Os professores lidaram com as seguintes situações inesperadas: *estudantes apresentaram erros nos procedimentos matemáticos, estudantes não trouxeram as informações solicitadas para a tarefa, estudantes escolheram parâmetros diferentes não previstos pelo professor, estudantes não conseguiram resolver os problemas da tarefa e estudantes resistiram em fazer a tarefa*. Essas situações inesperadas foram relacionadas às seguintes ações de estudantes no ambiente de modelagem: *desempenho matemático* dos estudantes na solução do problema da tarefa, *não cumprimento de tarefas* no ambiente de modelagem e *escolha do tema* pelos estudantes em um ambiente de modelagem.

Palavras-chave: Educação Matemática; Modelagem Matemática; Prática Pedagógica; Professores; Tensões nos discursos.

1. Introduction

Mathematical modelling is one way to promote mathematics connections in the classroom (Blum, Galbraith, Henn & Niss, 2007, Lesh, Galbraith, Haines, & Hurford, 2010, Stillman, Kaiser, Blum, & Brown, 2013, Biembengut, 2014, Souza & Barbosa, 2014, Stillman, Blum, & Biembengut, 2015). *Mathematical modelling* is a learning environment in which students are invited to address problems from daily life, professional situations or scientific disciplines through mathematics (Barbosa, 2006). By *learning environment*, we mean the social conditions provided to students for the development of certain activities (Skovsmose, 2001). The problem on which students are invited to work is a *mathematical modelling task*.

In the literature on mathematical modelling and teachers, studies have noted the expectations for teachers in a modelling environment: interventions in the students' modelling performance (Leiß, 2005), interpretations of students' mathematical thinking (Doerr, 2006), and the pedagogical knowledge for teaching modelling (Doerr & English, 2006, Doerr, 2006, 2007). However, the authors also suggest that engaging learners in mathematical modelling tasks is not exactly easy for teachers once they have demanded students work on tasks in a new way via their pedagogic practices. Some studies have clearly suggested teachers' tensions related to mathematical modelling in pedagogic practices (Blomhøj & Kjeldsen, 2006, Doerr & English, 2006, Oliveira & Barbosa, 2010, 2013, 2015). Following Bernstein (2000), we present *pedagogic practice* as the relationships that occur in a certain social context in which someone is in charge of teaching something to others. In a school context, it is understood as the

relationships between teachers and students for the teaching and learning of certain content.

We maintain teachers' tensions as an intuitive notion for now, which will be better defined later. However, our assumption is that tensions are identified in teachers' discourses as part of their actions in pedagogic practices. This assumption delimits the following research question: *How do teachers deal with tensions in discourses when using mathematical modelling in pedagogic practices?* We examined the pedagogic practices of three teachers through an analysis of their discourses. From the theoretical point of view, we mobilize concepts from Basil Bernstein's sociology to frame the research.

This paper is divided into five sections, consisting of this introduction, an outline of the theoretical framework and clear framing of the research question, methodology and context, presentation of the data through some selected extracts, discussion of the findings, and, lastly, an examination of their implications for research and practice in mathematical modelling within mathematics education.

2. Theoretical framework

In this section, Bernstein's theoretical notions (1990, 2000) are employed to highlight the tensions in teachers' discourses and to examine how teachers deal with them (i.e., what they do) when they engage learners in mathematical modelling tasks via their pedagogic practices.

Bernstein (1990, 2000) focused on the movement of discourses from their original site to a pedagogic one, which he called *recontextualizing*. This author employs the word "recontextualizing" to emphasize the transformation of the discourse once it has moved from one place to another. In a few words, any move of discourses means a change in them. The original site, where a new discourse was produced, is named the *field of production*. It refers to original disciplines such as mathematics, physics, chemistry, and so on. Some agents are in charge of appropriating this discourse for pedagogic proposals, for instance, textbook authors, policy makers and teacher educators. They are part of what the author calls the *recontextualizing field*. The author distinguishes between an *official recontextualizing field* that is dominated by the state and a *pedagogic recontextualizing field* that is run from the outside. In Bernstein's theoretical model, teachers are seen as agents who bring discourses from the recontextualizing field into classrooms. These settings are part of the field of reproduction, "where pedagogic practice in schools occurs" (Bernstein, 2000, p. 113).

To understand the process of pedagogic recontextualizing, we will use Jablonka (2007), who presents a description of mathematics as a school subject: "mathematics is a highly specialised activity that consists of a range of practices, some of which employ sophisticated tools and sign systems. The recontextualisation of parts of those practices establish the school subject mathematics as it is defined in curriculum documents" (p. 194). This process involves the selection of those practices and their relocation in school mathematics, and it is operated by a *pedagogic discourse*. Bernstein (1990) defined a *pedagogic discourse* as a principle for the selection of discourses that have been relocated, according to their own order. It is "a principle for appropriating other

discourses and bringing them into a special relation with each other for the purpose of their selective transmission and acquisition” (p. 183-184).

In this sense, once teachers use pedagogic recontextualizing to move everyday life situations into the classroom, the pedagogic discourse selectively relocates and refocuses them in agreement with the rules of the pedagogic practice. In addition, the pedagogic discourse places them in a special relationship with other discourses to constitute its own order. Thus, the movement of everyday life situations for classroom practice is regulated by discourses that had already been socially established and legitimated in the pedagogic context.

Gainsburg (2008) focused on the kinds of everyday life connections that high school mathematics teachers have made in their pedagogic practices. This author pointed out that teachers' main goal when they use everyday life connections is to offer an approach to mathematics content. Thus, the way in which teachers incorporated everyday life situations into their classrooms shows that pedagogic discourse delocated and relocated these discourses, according to its own principle.

Following Bernstein (1990), “in the process of the de- and relocation the original discourse is subject to a transformation which transforms it from an actual practice to a virtual or imaginary practice” (p. 184). As a result, Bernstein (2000) argued that pedagogic discourse is a recontextualizing principle and is not identified with any of the recontextualized discourses. In a study by Gainsburg (2008), teachers were agents of the pedagogic recontextualizing field and had made changes to incorporate everyday life connections into their classrooms. One of the changes was to use everyday life connections to approach mathematical content.

Empirical studies have suggested that teachers play a crucial role in ensuring the implementation of modelling in pedagogic practice (Doerr & English, 2006, Doerr, 2006, 2007, Oliveira & Barbosa, 2010, 2013, 2015). Following Bernstein, they are key recontextualizing agents. Doerr (2006) examined the ways in which teachers identified, interpreted and responded to students' work with modelling tasks. The results suggested that teachers develop sophisticated schema to understand the diversity of students' ways of thinking. Teachers' actions supported students' engagement in the task and led them to review and refine their own mathematical thinking. Similarly, the results in Leiß's study (2005) pointed out that teachers' interventions were important in terms of making students understand the problem, finding an appropriate model for the situation and reflecting on the model. These results mean that promoting modelling in the classroom has provoked some changes in the pedagogical relationship between teachers and students.

Teachers have had contact with mathematical modelling through teacher education programs (in-service or pre-service) in Brazil and in many other countries, as shown in results from research in the community of mathematics education conferences, which are pedagogic recontextualizing fields, as named by Bernstein (2000). The discourse on modelling, which teachers move from a pedagogic recontextualizing field to the classroom – defined as a field of reproduction (Bernstein, 2000) – is a specialized discourse that Barbosa (2006) names *school mathematical modelling*.

Bernstein (1990, 2000) uses two concepts to stress relations of power (classification) and control (framing) in pedagogic practice. On the one hand, classification embodies power relations between different categories, such as kinds of discourses. It is defined by the space between categories that is maintained by power relations. Bernstein (2000)

defines this space as *insulation*. On the other hand, framing regulates relations within a context. It refers to different forms of legitimating communication in any pedagogic practice. Framing refers to control of communication (selection, sequencing, pacing and criteria) in pedagogic relations, such as between teachers and students. Following Bernstein (2000), classification establishes that regulate what meanings are relevant in a context, and framing establishes that regulate how the meanings are to be put together to create the legitimate text according to this context. In short, “classification provides us with the limits of any discourse, whereas framing provides us with the form of the realisation of that discourse” (Bernstein, 2000, p. 12).

Regarding these discussions, what happens when teachers move mathematical modelling to the classroom? How have teachers moved it to the school context? We used the aforementioned concepts to understand how teachers have engaged learners in mathematical modelling tasks via their pedagogic practices. In particular, how have they dealt with tensions when promoting modelling in the classroom?

Empirical studies have discussed teachers’ dilemmas and uncertainties when teachers engage learners in mathematical modelling tasks via their pedagogic practices (Blomhøj & Kjeldsen, 2006, Doerr & English, 2006, Oliveira & Barbosa, 2010, 2013, 2015). Blomhøj and Kjeldsen (2006) discussed three teachers’ dilemmas. The first dilemma was to understand the phases in the modelling process from a holistic point of view or as an inner part of the modelling process to work with the mathematical content. The second dilemma was the goal of modelling as educational or as a means for motivating and supporting the students’ learning of mathematics. The third dilemma was how to develop students’ autonomy when working with projects.

Doerr and English (2006) identified teachers’ uncertainty on how students can develop mathematically viable solutions. This uncertainty relates to the teachers’ legitimate actions in promoting modelling via their pedagogic practice. It constitutes a kind of uncertainty regarding how to develop the task and which solutions the students might develop to solve the problem. These dilemmas and uncertainties are seen as results of trying to place a new discourse in pedagogic practice.

Following Bernstein’s theory, mathematical modelling in the classroom might mean the weakening of classification and framing because other discourses are moved to the classroom to solve the problems investigated by the students. As a result, mathematical modelling requires different forms of legitimate communication to deal with the problems from everyday life, modifying the control over communication. The teacher must consider the following questions: How to select the contents? How to sequence the actions? What is the pacing? How to evaluate? Thus, these dilemmas and tensions represent the insulation between a new discourse addressed in modelling tasks and discourses historically present in pedagogic practice. Consequently, it can modify the values of classification (what can be said) and framing (how it can be said) in mathematics lessons.

Based on Bernstein’s theory, we use the expression *tensions in discourses* to understand how teachers engage learners in mathematical modelling tasks via their pedagogic practices. Teachers manifest *tensions in discourses* through contradictions, cleavages and dilemmas that occur because of the space between categories (discourses historically present in the pedagogic practice and discourse on mathematical modelling). “The classificatory principle creates order, and the contradictions, cleavages and

dilemmas which necessarily inhere in the principle of a classification are suppressed by the insulation” (Bernstein, 2000, p. 7).

In this sense, discourse on modelling has moved to the classroom through a pedagogic recontextualizing process. As a result, it might change the values of classification (what can be said) and framing (how it can be said) in a mathematics lesson. It means that *tensions in discourses* are interpreted in terms of a recontextualizing process because taking a new discourse to classroom involves crossing the insulation between its original site and a pedagogic site. This new discourse has been positioned by the pedagogic discourse, presenting a discontinuity in relation to the discourses historically present in the pedagogic practice. This discontinuity is justified by the insulation among discourses that are positioned in the pedagogic practice.

The expression *tensions in discourses* has its origin in the discontinuity between historically legitimate discourses and a new discourse positioned by the pedagogic discourse, when teachers decide what can be said and how it can be said when they engage learners in mathematical modelling tasks via their pedagogic practices. *Tensions in discourses* are constituted through the contact between discourses that have moved, through a pedagogic recontextualizing process, to the school context. In our study, we discuss the notion of “*tensions in discourses*” as a starting point to capture the discontinuity between historically legitimate discourses and a new discourse positioned in the pedagogic practice. This discontinuity is manifested in the teachers’ discourses through contradictions, cleavages and dilemmas that occur in specific moments in pedagogic practice and shows the tensions in discourses that we call *situations of tension*.

3. Method

In this section, the mathematical modelling tasks developed by the teachers are described, as well as the rationale for data collection and analysis.

3.1 Setting and participants

This study is based on data from research that investigated tensions in teachers’ discourses when using mathematical modelling in their pedagogic practices (see Oliveira & Barbosa, 2010, 2013, 2015). The context is the first modelling experience of three middle school-level teachers from public schools in the northeast of Brazil. During data collection, those teachers were finishing an in-service training program for non-certified mathematics teachers at a public university in the state of Bahia, Brazil. It is common for non-certified teachers to teach at schools because of the lack of teachers in the Brazilian educational system. We selected these teachers for this study because of their willingness to participate in the research. We requested an authorization (Term of Free and Known Consent) from the teachers to collect the data in their classrooms.

The teachers that developed their modelling-based lessons were Boli, Maria and Vitoria (chosen pseudonyms for them). Each of them has been teaching for more than 20 years in public schools with classes of disadvantaged students. Two of the five mathematics lessons per week were set aside to develop modelling tasks. Each lesson lasted fifty

minutes. The teachers organized the modelling environment according to what Barbosa (2003) calls Case 2; in other words, teachers present a problem and students should collect data and investigate it. However, they also elaborated some tasks framed in Case 1 (Barbosa, 2003) during the development of the modelling environment. Then, they presented some problems with quantitative and qualitative data and students worked on them. Each teacher's class was organized in small groups of students who worked on the tasks assigned by these teachers.

Boli developed a modelling task entitled "Basic Basket of Goods" in two middle school classes (9th grade). He justified the use of modelling in order to make students "understand the presence of mathematics in everyday situations". He reserved two lessons per week for eight weeks for this task. He organized the modelling in several phases, such as discussing the theme, introducing a problem, defining the products and quantities of a basic basket, students working in groups, students getting information, defining families' expenses, making calculations and comparisons and drawing graphs.

Maria developed a modelling task entitled "Analyzing Water Bills" in a middle school class (6th grade). She said, "Teachers introduce mathematical modelling in their pedagogic practices to facilitate the learning of mathematics by students". For that purpose, she reserved two lessons per week for ten weeks. She organized the task in several phases, such as discussing the theme, introducing a problem, students working in groups, students getting information, analyzing tables, making calculations and comparisons.

Vitoria developed a modelling task entitled "The Minimum Wage and a Family's Cost of Living" in a middle school class (8th grade). She said, "Students understand how mathematical contents are applied in everyday life". She reserved two lessons per week for eight weeks. She organized this task in several phases, such as discussing the theme, introducing a problem, defining families' expenses, students getting information, establishing the products and quantities of a basic basket of goods, collecting data, making calculations and comparisons, and elaborating tables.

3.2 Data collection and analysis

The research was framed according to the qualitative perspective (Denzin & Lincoln, 2005) because its purpose was to analyze how teachers dealt with tensions in discourses when engaging learners in mathematical modelling tasks. Each teacher was videotaped during the modelling-based lessons. The videotaping focused on the teachers and their interactions with students, and these videotapes were transcribed. After each lesson, interviews were conducted with each teacher that described how the modelling task was developed. The interviews were also recorded and transcribed. The teachers wrote narratives about each lesson, which were analyzed. In this paper, we used data from interviews to identify how teachers dealt with the tasks, and we have used data from class videotapes, teachers' narratives and interviews.

Bernstein (2000) argues that empirical data and theory should form a dynamic relationship to describe the objective of the research. With the purpose of producing theoretical understanding, based on the collected data and guided by a research question, we analyzed the teachers' pedagogic practices in terms of the relationships between agents (teachers and students) and discourses that were present in the

pedagogic practice, as well as discourses on mathematical modelling. The data analysis found some inspiration in the analytical procedures of grounded theory (Charmaz, 2006) that refers to the elaboration of codes and categories for the transcribed data. The data analysis happened in three phases, as follows:

First phase: reading of the transcripts of the videotapes and interviews, as well as teachers' narratives on their lessons;

Second phase: identification of extracts that were read, line by line. We used an open-ended coding of the tensions in discourses – *the tension of unexpected situations* – that we identified in teachers' pedagogic practices in order to verify how the teachers understood and dealt with this tension;

Third phase: classification of the codes into more general categories. We examined the tensions in discourses – *the tension of unexpected situations* – to interpret how each teacher dealt with these tensions. After that, we classified codes into general categories and attained an understanding of the research problem by integrating the results in the literature and Bernstein's theory.

4. Tensions in teachers' modelling-based lessons

In this section, initially, we present the situations in which tension of unexpected situations occurred when teachers implemented modelling in their pedagogical practices. They planned several tasks to implement the modelling environment but faced unexpected situations. This tension was identified when teachers decided what to do and how to carry out planned tasks. We refer to unexpected situations as those situations that happen to interrupt the sequence and pace of lessons, and, consequently, the tasks planned. Then, we discuss how teachers deal with *the tension of unexpected situations*. To discuss the aim of the paper, we selected excerpts referring to each teacher's lessons. These excerpts refer to the transcripts of the remarks made from video and audio recordings of teachers in classes and interviews.

Situation 1: Students presented errors in mathematical procedures

This situation of tension happened in Boli's class when he asked the students, organized in groups, to present calculations of a family's costs, with the minimum wage as a parameter. He asked each group to put the table with the expenses and their percentages on the whiteboard. "What have you noticed here for each calculation that you did as a percentage of the family's expenses?" Then, Boli discussed the values found by the groups and asked the students to explain the values they found. Boli noted that some calculations were incorrect. "But there are some calculations that may be wrong because you hurried and such. Therefore, we will try to review some calculations to see if they are correct. I would like us to do some calculations again". In the interview, Boli commented on the students' incorrect calculations:

"I noticed that the calculations were wrong and they did it again. I was worried about such high values that they found. I was worried about the very big difference they found. When the students presented the errors, I had to stop everything and ask them to do the

calculations again. I directed the groups to do the calculations again. [...] I asked them to calculate the percentage of each item on the minimum wage. Thus, I found that almost everyone had no idea of percentage and rule of three”.

This situation indicated that the students did not know how to perform the task because they did not know how to use the problem's mathematical content contained in the task. To deal with this situation, Boli explained how to find the percentage of expenses. After the students did the calculations, Boli asked the following questions: “What did you check? What happened? What did you realize about the calculations you did? Does the minimum wage really pay all the expenses?”

The students' incorrect calculations were a situation that constituted a tension in the development of the task on a family's expenses using the minimum wage as a parameter. Boli did not expect this to happen. The situation indicated that the students did not know how to solve the problem proposed in the task. Boli had to consider how to deal with this situation and what to do to get the students to perform the task. Boli realized that the students performed the wrong calculations because they did not know how to mobilize the mathematical content to solve the problem. He guided the students on how to do the calculations, asked them to do the calculations again and explained the content used in solving the problem.

Situation 2: Students did not bring the requested information to the task

This situation of tension happened in Maria's class when she asked the students if they had brought the water bill requested in the previous class. “Let's look at the water bill. Did you bring the water bill? Do you have more than one water bill here? Please, lend me a bill here”. In the interview, Maria explained what happened:

“I thought about working with them on how to measure and the amount to be paid. Only when we started the first task and I asked them to bring a water bill, many did not bring the water bill. What happened? The first problem arose: one student had a group where all members lived in the countryside. They had no water bills because they had no running water at home. Other students said they had lost information about the water bill. My concern: everyone needed to participate in the task. At that moment, one group had not brought the information and another group had lost the water bill information. I was afraid. I thought the task invitation might have failed at something”.

She noted that some had lost the information and others could not bring the information about the water bill. Maria did not expect this to happen. She proceeded as follows, as reported in the interview:

“I was worried about how to lead the class. What to do? I had the idea of putting the students who brought the information in contact with those who had not brought it. [...] There was a group where all members were from the countryside. No one had a water bill. They had no running water at home. Others had water pumps. One student asked, 'Teacher, what are we going to do?' The groups that brought more than one water bill, I asked them to lend to colleagues who had not brought the water bill. I lent the school's water bill. They did the task with the information from the school's water bill”.

Students did not bring the requested information to the task, which is a situation that constituted tension in the development of the water-bill task. Maria had to consider how

to deal with this situation and how to develop the task without the information on the water bill. Not all students had running water at home. Maria got the water bills from some groups that had brought more than one. She also worked with the school's water bill. With these water bills, all students had the information to do the task, and students from both urban areas and rural areas were able to analyze the urban water bills.

Situation 3: Students chose different parameters not predicted by the teacher

This situation of tension happened in Maria's class when she asked the students, organized in groups, to choose three items from the water bill. One student said, "Professor, only three items to choose from the water bill?" In the interview, Maria said the students did not understand the task.

"[...] when I asked the students to choose three items in the water bill that would draw their attention, some students said, 'Teacher, what do we choose? What don't we understand? What don't we know?' [...] I want you to highlight what most calls your attention. Now, you will choose three items that have caught your attention. [...] Some students found it difficult to choose the items in the water bill because they thought the task was to choose what they had not understood in the water bill. I guided them and they made the choices".

The groups chose the following items: Hydrometer, payment and water clarity (group 1); interest, consumption and hydrometer (group 2); consumption, expiration date of the water bill and payment (group 3); parameter, consumption and registration (group 4); parameter, consumption and payment (group 5); hydrometer, parameter and location (group 6); hydrometer, parameter and consumption (group 7). After the groups chose their items, Maria asked the groups to indicate the most commonly chosen items and they said the following: parameter, consumption and hydrometer. Maria said that these items would be worked on in the next lessons, but other items would be studied as well.

"At the moment, I thought about consumption, but they chose parameters, treatment. The students chose several things and I was not planning on that. [...] They surprised me because I thought the students would choose a single item in the water bill. They chose items in the water bill that I had not expected. When I did the planning, I did not think about that. For example, one group surprised me when they chose the parameters. [...] Another group chose the notice about the expiration date of the water bill."

The different responses of students to the choice of items created a situation that constituted a tension in the development of the water-bill task. Maria thought that they would choose a single item, not several items, when analyzing the water bill. The situation indicated that she would have to work with all the items chosen by the students. Maria had to consider how to deal with this situation because she had not planned the development of the task this way. What could she do to work on the different choices? After asking each group to put their choices of water bill items on the whiteboard, she proposed to the students that the items most frequently chosen by the groups would be discussed in class. This reduced the number of items for the students to work with.

Situation 4: Students failed to solve the problems of the task

This situation of tension happened in Maria's class when she accompanied the students in solving problems about water wastage in domestic activities: washing clothes, watering the garden, washing dishes, washing the sidewalk, washing the car. She noted that the students were not solving the problems. "Why are you not solving the problems?" "Teacher, we have no idea how to do it". Maria explained what happened: "Some students said they did not know how to do the calculations. One student asked me, 'How do I do it?'" In the interview, Maria commented on the difficulties students experienced in solving problems:

"I accompanied the groups in solving the problems and realized the difficulties they had in resolving the calculations. [...] I am teaching in 6th grade and I taught them in 5th grade, so more or less, we know what is going to happen. At that moment, I was worried that most of the students had stopped doing the task. What am I going to do to get them to solve the problems? [...] I decided to go to the whiteboard and show how to solve one of the problems. After I showed them how I was doing the problems, I realized that they started doing the task. It had already made other problems easier."

There was another interruption of the lesson: "My students failed to turn liters into cubic meters. At this point, I stopped the class to resume the problems in the next class". A student in one of the groups asked, "Teacher, how do I know when it is in liters or cubic meters? How do I know I have done it right?" Some students asked, "What are we to do?" The students did not solve the problems because they did not know how to transform the units of measures. They would have to put the consumption in cubic meters and consult the table to tell the amount to be paid. Maria had to explain the volume content and how to convert measures so students could solve the problems.

That students could not make the calculations was a situation that constituted a tension in the development of the water-bill task. The students could not do the task because they did not know what mathematical content to use to solve the problems. Maria did not think this could happen because the content required to solve the problems had been studied in the previous series. Maria dealt with this situation by explaining the content of volume and capacity to the students so they could use it in solving problems.

Situation 5: Students resisted doing the task

This situation of tension happened in Vitoria's class when she asked for information about the monthly expenses of the students' families. She said that the students did not bring the information about the task, they did not want to organize in groups and they did not participate in the completion of the task.

"I almost panicked because most of the students did not bring the information. I asked them to organize in groups and they resisted. They did not want to organize in groups. [...] At the time of the task, some students did not participate. They resisted a bit to organize in groups, but ended up organizing in groups".

Vitoria commented, "Some students said that they forgot the information and others missed the class. My first difficulties arose in the development of the task. I observed

that class activities were being fragmented". Although she had difficulties in carrying out the task, she continued to ask the students to organize themselves in groups with the objective of accomplishing the task planned for the classes. She explained the reason for the students' resistance.

"I think the resistance of the students is not because they do not want to do the task, it is that they have difficulties. Sometimes they are discouraged because they have difficulties. I do not consider that they are resistant. They like it and work well, especially when I do a different task".

The resistance of students to doing the task was a situation that constituted a tension in the development of the task on the expenses of a family making the minimum wage. The students' difficulties with the mathematical contents and their discouragement about accomplishing the task were the justifications presented by Vitoria for the students' resistance to the task. Vitoria had to consider how to deal with this situation and how to deal with students' resistance. Vitoria continued with the guidelines for them to accomplish the task: She asked the students to organize in groups; she guided the groups on the information needed to solve the problems; and she clarified doubts about the content used in the task problems.

5. Discussion and Conclusions

In the previous section, we present situations of tension related to the tension of unexpected situations. This tension refers to the discontinuity in the teachers' discourses regarding what to do and how to do it when unexpected situations happened in the modelling environment. This discontinuity was identified when teachers interrupted the sequence and pace of pedagogical practices.

Teachers dealt with the following unexpected situations: *students presented errors in mathematical procedures, students did not bring the requested information to the task, students chose different parameters the teacher had not predicted, students failed to solve the problems of the task and students resisted doing the task*. These unexpected situations related to the following students' actions in the modelling environment: *students' mathematical performance in solving the task problem, noncompliance of tasks in the modelling environment and students' choice of the subject* in a modelling environment.

The unexpected situations *students presented errors in mathematical procedures* and *students failed to solve the problems of the task* refer to students' *mathematical performance* in solving the task problem. These situations occurred in the pedagogical practices of the teachers and interrupted the sequence of the task when the teachers accompanied the students in solving the problems. Boli's students used incorrect procedures to solve the problem, and he had to explain the content required to solve the problem, even though the students had already studied it in a previous grade. Maria's students did not know how to mobilize some mathematical content already studied in previous years to solve the problems. Vitoria's students did not know how to use mathematical content during the modelling task and were unable to use this content in solving the task problem. Thus, students' mathematical performance was related to mathematical procedures and the use of mathematical content to solve task problems.

The unexpected situations *students did not bring the requested information to the task* and *students resisted doing the task* refer to *noncompliance of tasks* in the modelling environment. These situations occurred in the teachers' pedagogical practices. For Maria and Vitoria, these situations interrupted the sequence of the tasks when the teachers presented the tasks, requested information to solve the problems and accompanied the students in solving them. In Maria's classroom, the students did not bring the information requested in the task, in this case the water bill, because some students forgot to bring it and other students had no running water at home. The subject of the task was not part of these students' reality. Vitoria's students also did not bring the information requested in the task, and they resisted doing the tasks she proposed. The interruption of the development of the task sequence was related to the students not having information to bring because the task subject was not part of their reality, and they had difficulties in doing tasks proposed in the mathematics classes. In addition, Professor Vitoria's students did not participate in investigating the proposed problem in an autonomous way. Victoria intervened and guided them to find the solution to the task problem.

Lastly, the unexpected situation *students chose different parameters not predicted by the teacher* refers to the *choice of the subject* by students in a modelling environment. This situation occurred in Maria's pedagogical practice and interrupted the sequence of the tasks when she discussed the items chosen by the students to analyze the water bill. The students chose various items from the water bill, leaving Maria apprehensive about how to work with different items at the same time.

The teachers planned several activities to develop modelling in their pedagogical practices. These activities were planned and structured for two of the four weekly classes in mathematics. All the activities contributed to solving problems with reference to day-to-day situations in the modelling tasks. However, unexpected situations occurred, altering the sequence and pace of the planned activities, constituting tension in discourses called *tension of unexpected situations*.

The tension of unexpected situations refers to the discontinuity between what to do and how to do it in conducting modelling tasks when unexpected situations happen in classroom practice. This discontinuity was identified when the teachers recontextualized mathematical modelling in their pedagogic practice, and some unexpected situations interrupted the sequence and pace of the pedagogic practices. The tension of unexpected situations was related to the discontinuity between discourses present in the pedagogical practices of the teachers: a discourse related to the purpose of following the sequence and pace of planned lesson activities, and another discourse related to how to deal with unexpected situations in the modelling environment (Oliveira & Barbosa, 2010, 2013, 2015).

Modelling is an open-natured activity in which there are not *a priori* strategies to follow in solving problems arising from day-to-day situations. The teachers were concerned about how to conduct the modelling task when unexpected situations interrupted the sequence and pace of the classes. The teachers dealt with the unexpected situations by performing some action already carried out in their pedagogical practices. One possible explanation is that the teachers employed the present predictability in their pedagogical practices within the tradition of school mathematics to structure the activities developed in the modelling environment (Oliveira & Barbosa, 2010, 2013, 2015). As mentioned by Penteado (2001), in traditional education, the comfort zone, in which the actions of the teacher and students are practically predictable, prevails. However, teachers also had

to consider what to do and how to deal with the unexpected situations that challenged prior planning.

Studies indicate that teachers mobilize pedagogical knowledge to deal with unexpected situations in solving students' problems in the modelling environment (Doerr, 2006, 2007; Doerr; English, 2006; Oliveira & Barbosa, 2013, 2015). In terms of Bernstein's theory, teachers established a sequence and pace for the tasks, structuring possibilities for "how to talk" and organizing the interactions between teacher and students in the classroom, but unexpected situations occurred in class, interrupting the planned activities. The teachers recognized that they needed to produce a legitimate text (what to say) to deal with these situations because they did not know what to do and how to do it in the modelling environment. Thus, there is the recognition of the specificity of the context (classification) for the production of the legitimate text (framing). In addition, they continued developing the planned sequence for the modelling task. However, the teachers performed some actions already used in pedagogical practices to deal with these situations and to follow the sequence and pace of the activities planned in the modelling environment. The teachers acted the following way: they organized the students so that they had the information to solve the problem; they listened to the students' unexpected interests, but they guided the students to the interests planned by them; and they guided the students in how to solve the problem.

In our study, we noticed that teachers had carried out actions in order to deal with the tensions that occurred during the modelling tasks in their pedagogic practices. They looked for guidance to understand and to develop modelling in their lessons. This tension can indicate that teachers need support when they are promoting modelling in their classroom practices for the first time (Oliveira & Barbosa, 2010, 2013, 2015).

In this way, *tensions in discourses* can contribute to teachers' professional development, because in order to deal with them, teachers carry out actions, use strategies and produce pedagogical knowledge in the accomplishment of new practices in their lessons (Oliveira & Barbosa, 2010, 2013, 2015). Doerr (2007) indicated that modelling tasks demanded teachers' pedagogical knowledge to hear unexpected approaches, to support students in making connections to other representations, and to listen to students interpret and explain their answers. Tensions in discourses can provide opportunities for teachers to act and form strategies to produce legitimate texts about learning environments to teach math.

6. References

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