

The Quest of the Important in Mathematics Classroom

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ABSTRACT

In this article, some ideas are exposed to take out profit of the properties of the straight line to know and to explore many interesting properties of functions as polynomials or rational functions. Some ideas are also presented to use the graphic calculators in a sequence that allows integrating the arithmetic language with the graphic and algebraic languages. This proposal has been applied in initial mathematic courses of future economists and engineers, obtaining interesting results related to the development of the graphic thought.

Keywords: Graphic Calculators. Graphic Thought. Mathematics Classroom.

A Busca do Importante na Sala de Aula de Matemática

RESUMO

Neste artigo, algumas ideias são expostas para tirar proveito das propriedades da linha reta para conhecer e explorar muitas propriedades interessantes de funções como polinômios ou funções racionais. Algumas ideias também são apresentadas para usar as calculadoras gráficas em uma sequência que permite integrar a linguagem aritmética com as linguagens gráficas e algébricas. Esta proposta foi aplicada em cursos matemáticos iniciais de futuros economistas e engenheiros, obtendo resultados interessantes relacionados ao desenvolvimento do pensamento gráfico.

Palavras-chave: Calculadoras Gráficas. Pensamento Gráfico. Sala de aula de matemática.

INTRODUCTION

In school textbooks, different terms such as open problem arise today open, closed problem, problem, and even investigation, located in epigraphs as “situations and problems”, “activities”, “curiosities”, “investigations”, “Think a little” ... And different meanings of the meaning of “resolution of problems “and” Investigations”.

All these different epigraphs pose an initial question. Why are such diverse terms necessary? It may be to draw attention of the young reader. It could be to make subtle distinctions: for example, on a level basic, it is necessary to distinguish between the

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way of classifying a problem and the various ways to solve it. You can also consider the usual distinction between “Problem Resolution” and “Investigation”. The first of this refers to the fact of trying to get the solution of a certain problem using the strategies and techniques that seem convenient, or through a research work. And it is usually called “investigation” when, in addition, it encourages to be curious, to look for alternative strategies, to consider what would happen if certain conditions are changed or to try to generalize the problem.

However, in open terms it is wise to understand Problem Solving as a convergent activity where students have to get a solution to a certain problem, while Research should be seen as a more divergent activity. In an investigation students are encouraged to think about alternative strategies, to consider what would happen in a certain line of action, or to see if certain changes would change the results. (H.M.I., 1985, p.42)

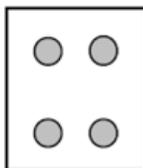
This distinction is interesting, although we sometimes realize that during the resolution of a problem we perform a characteristic task of what is called research in the previous sense. This also happens frequently when we experiment with alternative strategies. In other words, in practice we are really working on a convergent activity. It seems that these expressions are used to refer to the idea that students work creating their own mathematics, as opposed to working on questions or exercises that consist of the automatic application of a previously learned skill or algorithm. Let’s call it research, for example.

All this is important, but we ask ourselves: is that important?

EXAMPLES OF RESEARCH THAT CAN BE PROPOSED IN A CLASSROOM

In various sources and different books there are many examples of activities such as the following:

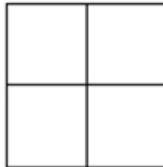
- 1- **Thumbtacks:** How many thumbtacks are needed to fix different sheets of paper on a board? Each sheet of paper is the same as the others, and to be fixed we agreed that you need a thumbtack in each corner.



- 2- **The diagonal of a rectangle:** How many lines does the diagonal of a rectangle cut?

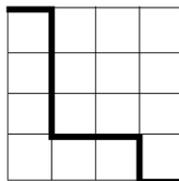


- 3- **The colors of the flag:** A square flag is divided into four equal parts. How many different flags can be represented in this way when it is only possible to use two different colors?



- 4- **The numbers and the four operations:** How many different numbers can be form with four fours and the operations $+$, $-$, \times , \div ? And with any operation?
- 5- **The squares:** How many different shapes can be made with 3 squares united with at least one side?
- 6- **The stamps:** If we have many stamps of 3 pts. and 5 pts., can you do all the values up to 50 pts?
- 7- **House numbers:** Number 2, 5 and 7 are available. How many house numbers can be obtained using these three numbers only once?
- 8- **The routes:** How many different routes are there from A to B?

A



B

- 9- **Odd and even numbers:** Choose any number: if it's even, divide it by two; if it's odd, add one. Repeat as many times as you can. What happen?
- 10- **The calculator:** Suppose you have a calculator whose only buttons that work are 4, 3, -, \times , =. What numbers can be obtained?

These, and many others, can be found in different texts, and each teacher has his own opinion and his preferences about the effectiveness of each of them. All are important but, nevertheless, we ask ourselves: is that important?

DIVERSE RESEARCH CLASSIFICATIONS

You can consider the previous examples and work them as investigations. These can be classified in different ways. One of them could be the one Burghes (1984), which distinguishes four types:

- a) *Eureka investigations*: they are characterized by needing an adequate idea (“happy” idea “ \hookrightarrow ”) to solve the problem, without which few useful progress can be made. Example: The magic square.
- b) *Escalator-type investigations*: those that do not depend on an idea but allow to achieve certain progress throughout the investigation. Example: The diagonal of a rectangle.
- c) *Decision problems*: originally obtained from real situations in which decisions must be made without clear evidence of what they would be the most appropriate. Example: The routes.
- d) *Real problems*: those that have a direct relationship with the citizen, but that the context in which they find themselves has little to do with Mathematics. Example: Seals.

Another possibility would be to classify them according to their objective (Rawson & Chamoso, 1999):

- a) to contact with daily life,
- b) to introduce a new concept,
- c) to develop and strengthen known concepts,
- d) to develop reasoning and explanation.

It could also be done in a similar way to Corbalán (1994) when it refers to games that, according to him, can be:

- a) *Of knowledge*, those that refer to one or several of the usual topics of the Mathematics programs (cite numerical, geometry and probability examples).
- b) *Of strategy*, in which one would try to start up one or several typical procedures of resolution of problems or the habitual ways of mathematical thought (like the Nim or the Sun and Shade).

There are many other possibilities of organization. All of them are important, but we ask ourselves: is that important?

WHAT IS THE IMPORTANT?

Gairín Sallán (1996), in his article “From the saying to the fact ...” and based on his experience, makes a great effort to explain two different arguments of the demonstration of the resolution strategies of a concrete example: initially in a “said” way, with the need

to use high mathematical knowledge, in the same way that any textbook would, and later in a “done” way, with demonstrations that can be expected from a student working in a similar way to how mathematicians do. But perhaps it would be closer if we could discover how the mind of each student works when it works, instead of how the teacher believes that the mind works when it develops a certain activity.

We refer as research to wander with ideas as far as possible, in a free way, with dialogue and discussion among the students. It is important to choose them in an appropriate way to use them at the specific moment. Although inserting several of them previously has been only with the pretension of their inclusion in the article as a complementary and decorative element, this has not been an obstacle so that the final list is compiled after arduous discussions. For example, there were divergent thoughts about the relevance of the problem of the calculator, in spite of considering it very interesting and of using it frequently as a teaching element in class. But he reminded himself that he had once interesting results with a student: It was a student which usually does not do his homeworks, but faced with the challenge proposed by the teacher if all those numbers could be built with those buttons up to 100, not only arrived the next day with the problem solved, but presented in a delicious double paper, written on both sides, in which it was proved that it was possible to do it until number 132. Normally he did not like to repeat similar activities, but on this occasion he did it gladly because he had control of the work he was doing. That is to say, he not only arrived with the homework done, but he carried out more than what had been requested. Perhaps at other times it had been used as a routine problem and its advantages had not been discovered. Perhaps the problem itself was not so important, but rather the form of its use.

Another investigation that presented conflicting opinions worth mentioning in relation to its inclusion was that of the stamps. Its simplicity seemed like it did not more than itself being practically an exercise, although it would allow, for example, to generalize changing the information of the statement. Our surprise was great when falling by chance in those moments in our hands Gardiner’s book *The Art of Investigation* (1989), written for readers with a certain level of knowledge in Mathematics and in which, after a serious introduction in which he tries to explain what it means by research, focuses on two specific cases to exemplify in a practical way. One of them is the mentioned of the stamps, to which he dedicates 50 dense pages trying to exploit the possibilities of it (tables, multiples and dividers, Diophantine equations, geometric constructions, areas ...). Maybe not he had sufficiently thought about the problem and its possibilities.

Interesting is the diagonal of the rectangle. It has been experienced in the classroom and it has served to demonstrate that the capacity of reasoning should not be underestimated of students due to their young age. It has been done in the following way: observed with video a class of 20 people of 10-11 years working in several groups. The same research has been presented to teaching students and to active teachers, so that they can work with them and discover their difficulties. Subsequently the video has been posted. It has been used as a stimulus to use the same topic in different environments. Many teachers think that doing this research is often too difficult for the students of those

years, and never will. But they can see in the video that this is not the case, because if they are allowed, you can never guess what the boys are capable of doing. What is certain is that the capacity that a person can reach will not be known if they are not given the opportunity to discuss alternatives, if we only ask for content, if we restrict ourselves to following the indications of a textbook.

It had been agreed that in the initial explanation of the research approach the teacher spoke little, asked very brief questions and left the work for the students. Subsequently they were allowed to act, in a group, with two teachers pending only to check whether an appropriate question fit of suggestion was necessary in some case. Conversations were observed in the groups of students like the following:

Girl A: That's because it's exactly the same.

Girl B: If you cut this piece in two parts and fold it in half, then ...

Girl C: Do not worry about that: if it's wider, exactly the same thing happens. If it is narrower too.

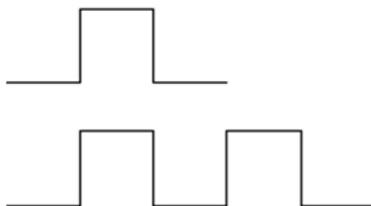
And they guess:

Girl D: The sides of the boxes minus two are added, that is equal to the number of cuts.

After this hypothesis someone observes that there is a special case that does not fit it, the $1 \times n$. Therefore we must continue, until partial solutions are reached.

That is, it is observed that many conditional sentences appear, the method of research to formulate conjectures. And they are able to separate what they need of what they do not need. This, done by 10-11 year old students, makes it seem that it is really working in the form of investigation in the same way a researcher would do it: they observe first, they explain what they observe and they conjecture a general formula that they check later. Isn't that what we want?

Let's see another case. It's about building doors with chopsticks or blocks. One is possible to do it with 5, two with 9, three with 13 and so on.



It asks: how many bars are needed to build 10 doors? And in general? Two students have been experienced in class and videotaped. In the transcription of the conversation it has been possible to discover what the students do: they build, they experiment, they support, they diverge, they ask, they deduce, they correct, they verify, they explain, they

direct, they answer, they suppose, they generalize. That is to say, it is observed that the students solve:

- *Building*, in what has to do with the development of the structure, and be manipulative or numerically: "... If we multiply by 5, it will be equal 10 times", "Put on top the pen cap", "Check the 4 again. We remove 1, we remove 2, 4 the same ... but you only remove 1, right? (while building) "...
- *Experiencing*, trying to see what happens in some particular cases: "Let's try for 17", "Let's try it with another" ...
- *Supporting* affirmations of what has been said, even if it is not fully aware of the procedure: "Yes, therefore we have to go well now", "Okay, right. And that for the number you want "...
- *Diverging*, disagreeing, showing new ways that can open new possibilities of the problem: "Will it work also for odd numbers?" ...
- *Asking*, as a fundamental part of the problem solving process, in which the students examine how the structure is composed: "Was 6 the next?", "Yes ... and if we apply this for 10?" ...
- *Deducing* (from a particular experience students use the language to express relationships that they recognize): "10 has to be 41, because if we add four each time it would not be 42 "...
- *Correcting*, which demonstrates some reasoning ability and understanding a certain part of the problem: "45 cannot be right", "It has to be confused then. Maybe it's 8 "...
- *Checking* to verify the correct development of the reasoning: "We are going to check it ", "Let's check 8 again. 4 equal to 17, 5 equal to 21 "...
- *Explaining*, linking past and present experiences with reasoning that are being carried out: "We have just solved it for 4. If we consider that previous for 2, 18 ago, we remove 1, it is 17. Therefore it is counted by 2 and we remove 1 ... ", "It will be 61 because it looks, adding 41 equal to 62, but remember, this is when we are removing 2 "...
- *Directing* the action of other people in order to instruct, demonstrate and formalize the strategy that is being used: "... so if you remove 2 there and 3 there, what would you remove for 6? Let's try the 6 ", "No, do 9 "...
- *Answering* critically and justifiably the affirmations made, adding other points of view: "It cannot be 33, it has to be 32. You have to remove another, remember?", "No, do 9. It should be 37. If 8 is 33 and we add 4, it will be 37 "...
- *Assuming*, formulating hypotheses and conjectures: "10 could be equal to 42 then", "73 should work" ...

- *Generalizing*, studying what would happen in any case: “Therefore we have to find out what would be for any sequence of any number “...

In those situations, in addition to the above, interesting circumstances are observed worth mentioning:

- Students take the initiative:** It's observed that the students are who are in control. For example, a bad notation comes up but was decided by them. In the problem of the diagonals it does not seem correct to put $2 \times 3 = 3$, with the sense of length \times width = number of cuts of the rectangle diagonal. However, it was their choice. In other circumstances, similar representations have been observed: that of the doors wrote $2 = 9$. What would the parents say if they saw it! The sense is well, but it is only understandable if we place ourselves in the context of its resolution. We accept it as a short, convenient and simple way to express yourself on paper. The advantage of allowing students to experience their representations gives opportunity the teacher to talk to them about its meaning. You have to explain to them that they have to think about why they write and who is going to read what they write.
- Resolution of doubts:** In the same way, inaccuracies arose when the diagonal passed through a vertex common to four squares, since you could understand that should be counted once or twice (that is, it was a common point of a vertical line and another horizontal). It was a good time that the professor took advantage to encourage a dialogue between the students to reach a kind of agreement, in which they spoke what is a line, what is meant by cutting point, concept of tangency ... Finally they decided how to consider it, and they did it as they wanted (in fact a course considered it as a single cut-off point and another as two).
- The importance of the teacher:** On one occasion when they were working similarly, after the teacher raised the research to two students, by letting them solve it, he observed how, after five minutes, they were convinced that they had solved the problem. But he remained silent, patient, he endured without saying anything with great courage and got his reward: the boys, together but alone, without the teacher adding a single word, resumed the problem until they got the correct answer and a generalization of the that both were convinced (after thirty minutes). Subsequently, he saw them much more animated when doing a second problem. This also brings us to raise and meditate that, in many occasions, this way of working takes a lot weather. But even if more time is spent than is usually available in other cases, the students have worked until the last moment and, what is more important, they have solved the problem.
- Student errors in the process:** The teacher's correction is the immediate reaction that arises in the teacher's mind, but for the students it may be more positive to abstain and check if they are able to resolve the difficulties

by themselves, so that they discover the path that can be followed and the appropriate solution.

- e) **Use of material:** Students use manipulatives elements to form an idea of the solution and formalize it, although everything depends on the difficulty to find this one. For this reason it seems important to provide the students' opportunities to use concrete tools that can help you become familiar with the problem and to find a solution, in a way that helps them establish a relationship between thought and action. This can also be used to check partial results obtained or the final solution, which helps to get small successes.
- f) **The importance of the distribution of resources:** It was observed in the case mentioned in section c how the boy who had pen and paper, who in the principle seemed to be the most timid, it was he who was taking the initiative becoming the real leader. This has been proven on other occasions where we had the opportunity to perform experiments in a classroom, distributed in different groups of 4 or 5 people working on the same investigation in parallel: each member of some groups were given a paper with the problem, in others a couple of sheets were left for all the components and in other cases only one for the whole team. It was precisely this form of distribution that marked the subsequent way of working of each group, which subsequently affects the quality of the dialogue and discussion of the answers: those that had one for each member of the team worked almost exclusively on an individual basis; when they had been given two, two subgroups were made, and when there was a single one for the whole group, the work was really from a single joint team.
- g) **Group work:** A great motivation was seen during the resolution of a problem when explaining the process to a partner. It is also interesting observe how each group worked in an environment of respect, with which each one of its members played a significant role. In the previous examples we can verify the high level of participation and contribution of ideas. Mutual respect helps express these ideas in a comforting environment and in a democratic society. This is essential because each component has different peculiarities: one prefers to inform, another to ask, some are very skilled at specifying the main points, others prefer to joke...
- h) **The importance of overcoming stages:** Achieving small successes provides the student with joy and confidence in the experience. In addition, overcoming those phases must serve the teacher the opportunity to consider rewarding the achievement obtained. Also, to celebrate before the class the overcoming of stages can be used to motivate the students: the thought is stimulated when it is necessary to explain the process of reaching a solution.
- i) **Little written expression:** Students usually do not write much, and what they write does not usually express what they have thought. It is not usually

significant. Neither do it orderly. This is an aspect of the teaching-learning of Mathematics on which much information is needed. Rawson (1997), Mitchell and Rawson (1998, 2000) have studied different possibilities that Primary students use when writing about Mathematics. From these studies stand out the few occasions of normal classroom development in which teachers help and direct their students in the ability to write the process of solving problems in different ways.

All this arises from the opportunities that spring from the dialogue, from the interaction of the students when they communicate. For example, in the case of doors, students did a very good generalization job, but it was because they were in a position to do so. And the students were not particularly bright. They were of medium level. In some moments of the difficult daily path of teaching we ask ourselves if students are capable of carrying out a certain activity, and organize and think to achieve an adequate response. And it seems very clear: they do it whenever they are given the opportunity to do so.

BUT, WHAT IS REALLY IMPORTANT?

There are different ways of understanding the meaning of Mathematics, as well as different points of view on how learning processes can be performed in relation to them. For example, some consider that there has to be a solution for each problem. For them Mathematics consist in finding answers to problems, and their teaching is usually aimed at achieving results, that is, to get the correct answer.

In contrast to the consideration that learning is nothing more than instruction, others have a point of view of Mathematics as something of a creative nature that requires initiative, imagination, reflection and flexibility. Actually, they consider solving problems as how to identify the problem category, select the appropriate strategy and apply it. Undoubtedly, they must be taught and know strategies, but doing it this way would turn Mathematics in something similar to a mere application of rules mechanically.

Another possibility is to understand Mathematics as a process that emphasizes the need to understand, following the procedures and skills of a researcher. In that case, problems should be presented as something that stimulates work of students, as well as an opportunity for resolvers to develop an effective method of work. In that way, the activities would serve as experiences that develop strategies, allow to formulate conjectures and tabulate results, identify the objectives, encourage discussion and reach agreements sets... Thus, Problem Solving emphasizes the interactive nature of Mathematics and encourages students to make decisions to achieve the objectives that are intended. With this they pass from being passive recipients of knowledge to active students. This makes them present a wide variety of answers. They also intend to present situations that allow them to generalize according to with their levels of development. This ability to generalize is an important step, since being able to acquire a skill in certain conditions is something very different than showing different skills in the same situation, or the same skill in different situations.

All this does not eliminate the value of mastering the skills of the calculation, but it must be for the purpose of considering them as tools basic for problem solving.

Bruno D'Amore (1997) relates, in his article "The mental images of texts of problem situations influence their resolution?", the revealing phrase that a ten-year-old student gave to a problem that he did not understand exactly the statement: "The important thing is not to understand, but to solve the problem". But, really, what do we want with education? Remember, understand and be able to use those memories, but not located on three different levels, but present in every situation and in any human activity.

Education should not be confused with memorization because, in reality, knowing with accuracy, that Felipe II was King of Spain, the number of years that he lived, the extension of his dominions under his mandate and the name of his children does not seem too relevant. This can be found in any book that deals with the subject, but an encyclopedia with legs is not usually understood as a well-prepared person. Perhaps it is more important to know what these facts had to do with the development of humanity, in its time and in relation to the current situation. Although one cannot fall into the opposite pole by identifying education as mere reasoning. According to Bruner (1962, p.34): "Teaching is not creating small living libraries, but rather that students think for themselves, mathematically, to consider issues as a historian does, to take part in the process of constructing the knowledge. Knowledge is a process, not a result".

Mechanization education should also be distinguished, because to emphasize working on a long list of sums can be a trivial mathematical activity. These activities do not necessarily make students especially competent when they use numbers. Perhaps it would also be interesting to have in tells how each person interprets their meaning. Let's see an example that can show that distinction between education and mechanization. It takes imagination to turn a page of sums into an activity that stimulates the action of thinking. For that, you have to look at that page with a new vision. For example, a restless teacher may suggest, in the first place, doing the sums that cannot easily be done mentally. Later it is about inventing the story of a problem where the solution:

- is among the answers of the first and the last sum of each row of sums,
- be among the largest and smallest solution in each row,
- is greater / smaller than the plus / minus solution of each row of sums,
- is greater / less than half of the largest solutions in each row...

Many students and professors comment that what they normally do students is to prepare an exam, and that is usually done "the day before". There is also talk of attending to diversity, as each student must maintain their own pace and achieve their results. Fighting against that is easier if you try to make students think for themselves. With this we would like to comment and criticize the well-known Chinese proverb "I listen and forget, I see and remember, I do and I understand". In our opinion we should include a word between do and I understand: I reflect.

In the textbooks of years ago the names *mental*, *algorithm* and *problem* were used. Now things have changed, and it seems that today's society demands to teach thinking in an increasingly competitive context. But in many facets and situations of our life we act in the first way that occurs to us, without understanding and without even trying to understand. That is, what is done is to solve practical problems immediately, and then observe the success or failure of these actions. However, when we act as teachers we do it reflexively, so that the understanding of a problem precedes the performance before it. We would like students to follow a similar path.

There is talk about reflective mathematics, that students have an attitude criticism, but what does that mean?

AND HOW DO YOU DO THAT?

Research (or any other denomination) seems adequate, understood as a way of working that tries to establish an environment where it awakens the thought. The difficult thing is to measure "how much is good". The important thing is to use them as a means and not as an end. Until they are done, you do not know what they can be or what they can get to. The students are looking for confidence when facing any problem. And young people have to be encouraged to investigate when they study teaching, when they learn to teach. According to Halmos, (1991, p.33): "A teacher must show his students how to solve problems – and solve problems is to do research, which can and must be done on many levels different – and a teacher must be able to recognize in his students the ability to investigate, to be able to encourage them and give them adequate advice".

In this situation, students should be presented with the possibility of apply what they know in changing and unfamiliar possibilities, which allows to follow their own mathematical paths and develop their own ideas. For this presentation plays an important role, especially in favor of interest and motivation. And even a period of preparation can be interesting. It would also be favorable if they were easily understood and approachable. Situate students in groups can encourage cooperative work, but that does not ensure that the discussion takes place.

Also keep in mind that: "on the methodology based on the resolution of problems we would like to add that as important as the resolution of problems we consider to be the 'proposal' of new problems, that is, that students bring real-life or imagined problems to the classroom, for which there are that look for the mathematical approach. The fact of extracting the mathematical scheme of problematic situations, giving them mathematically treatable form according to known models, is very important and must be stimulated. It is the better way to stimulate creativity "(Santaló, 1992, pp.86-87).

Polya used to say that if you cannot solve a problem you should choose a part of it or look for another problem that seems simpler, that is known how to solve, and then solve it. Then solve problems is to look for problems or, better, find the correct approach to them. The same principle applies in education. It comments on Socrates, considered by

many to be one of the best teachers of all times, who said: “Do not tell them things, ask them! Do not solve the problems, plant them. The best way to teach is not to dictate, but ask, identify new goals and justify the facts”.

Our opinion has already been exposed, that there is no point in seeking method par excellence, but it is more convenient to choose at any time the adequate, depending on the activity to be carried out and the characteristics of the specific group of students. There are no magic wands. Any strategy of work may be important, but research is still ongoing why certain methodologies get better results than others, with the intention to continue improving.

Yes, yes. But that, how is it done?

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