

Affine Function: An Analysis from the Perspective of the Epistemic and Cognitive Suitability of the Onto-semiotic Approach

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Received for publication on 5 Nov. 2019. Accepted after review on 12 Nov. 2019.

Designed editor: Claudia Lisete Oliveira Groenwald

ABSTRACT

This article presents the analysis of a series of activities involving Affine Function, taking as reference the components and indicators of Epistemic and Cognitive Suitability, dimensions of Didactic Suitability, the Onto-semiotic Approach to Mathematical Knowledge and Instruction. This analysis is part of a research that aims to investigate the development of an educational project for High School, focused on the study of functions and which takes as theoretical and methodological support the Onto-semiotic Approach assumptions. From the analysis carried out in a series of activities related to Affine Function, it was possible to realize the presence, in a more representative way, of components and indicators of cognitive and epistemic suitability related to Problem Situations, Rules (concepts and procedures), Languages, Logical Reasoning, Reading and Interpretation. Regarding the Arguments, Relations and Analysis/Synthesis components, the analysis indicated the need to develop a wider range of activities that would allow students to expand and deepen their ability to produce justified argumentation and establish relations, which could contribute to the development of analyses and syntheses.

Keywords: Onto-semiotic Approach; Didactic Suitability; Cognitive and Epistemic Analysis; Affine Function

Função Afim: uma Análise na Perspectiva da Idoneidade Epistêmica e Cognitiva do Enfoque Ontossemiótico

RESUMO

Esse artigo apresenta uma análise de um conjunto de atividades envolvendo Função Afim, tomando como referência os componentes e indicadores da Idoneidade Epistêmica e Cognitiva, dimensões da Idoneidade Didática, do Enfoque Ontossemiótico do Conhecimento e da Instrução Matemática (EOS). Esta análise é parte de uma pesquisa que tem por objetivo investigar o desenvolvimento de um projeto educativo, para o Ensino Médio, com foco no estudo de Funções e que toma como aporte teórico e metodológico os pressupostos do EOS. A partir da análise realizada em um conjunto de atividades relacionadas a Função Afim, foi possível perceber a presença, de

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modo mais representativo, dos componentes e indicadores da idoneidade cognitiva e epistêmica referentes a Situações-Problema, Regras (conceitos e procedimentos), Linguagens, Raciocínio Lógico, Leitura e Interpretação. Com relação aos componentes Argumentos, Relações e Análise/Síntese, a análise apontou a necessidade do desenvolvimento de um maior número de atividades as quais permitissem aos estudantes ampliarem e aprofundarem a capacidade de produzir argumentação justificada e estabelecer relações, o que poderia contribuir para o desenvolvimento da produção de análises e sínteses.

Palavras-chave: Enfoque Ontossemiótico; Idoneidade Didática; Análise Cognitiva e Epistêmica; Função Afim.

INTRODUCTION

When we observe the High School Mathematics curriculum with regard to the contents to be studied (Brazil, 2002, 2006, 2018), we identify one as fundamental for the development of Mathematics in High School and in Higher Education in science and technology: Functions. The study of Functions is not only important in Mathematics, but also fundamental to solve problem situations that involve other areas of knowledge, as well as situations that arise in everyday life and in the world of work.

Concerning the teaching and learning of Functions, it is pertinent and necessary to elaborate work tools that direct, deepen and strengthen aspects related to knowledge about the theme as content to be taken to Mathematics classes. Kaiber (2002) considers that the introduction of the concept of Function to students, in many cases, is based on the idea of correspondence between sets. She also points out that, combined with the linear organization of the Mathematics curriculum, this approach transformed the study of Functions in High School, and in the first semesters of university courses in the scientific and technological area, into something formal and abstract, making it difficult for students to understand ideas and relevant concepts.

Currently, the author's notes are considered still valid, although it is recognized that there is a search to assign meanings to the study of Functions considering mainly applications from problem solving, which can be perceived, particularly, on curricular recommendations and current textbooks.

In this context, the Onto-semiotic Approach of Mathematical Knowledge and Instruction (OSA) (Godino, 2011, 2012) may contribute for the elaboration of work proposals that may favor students to appropriate ideas, concepts and procedures in this topic.

The Onto-semiotic Approach, according to Godino (2012), conceives Mathematics from a triple view: as a socially shared problem-solving activity, as a symbolic language, and as a logically organized conceptual system. Regarding teaching and learning, it considers elements that make possible the passage from a descriptive or explanatory didactics to a normative didactics, which provides tools that allow for the analysis of epistemological, cognitive, mediational, interactional, normative and ecological aspects of thought, of language and of situations where mathematical activity occurs.

The analysis of the foundations of the OSA makes possible for us to identify elements that are considered essential to guide both the evaluation of teaching and learning processes and their structuring. Thus, there is space in OSA for discussion and reflection to understand what mathematical objects are, the negotiation of meanings attributed to these objects in the school scope and their articulation in teaching and learning projects that can be broad, as when thinking in organizing a curriculum, or specific, when thinking about developing a certain content or concept (Godino, 2012).

In this context, a study that aimed to investigate the organization and development of an educational project for Mathematics in High School was developed from the perspective of the Onto-semiotic Approach to Mathematical Knowledge and Instruction, focusing on the study of Functions.

The qualitative research was based on the contributions of the Design-Based Research (DBR) (Godino et al., 2013), involving the organization, application and evaluation of an educational project focused on the study of Functions, together with a group of students from the first grade of High School from a private school in the city of Farroupilha, Rio Grande do Sul, Brazil, during 2018. The educational project was organized around seven topics: initial concepts about Functions, Affine, Quadratic, Modular, Exponential, Logarithmic and Trigonometric Functions. It involved organizing a set of problem situations, construction activities with pencil and paper and Geogebra *software*, learning objects, study materials and selected *YouTube* videos for each of the seven topics covered.

Particularly in this article, we highlight part of the research that refers to an analysis of the application of a series of activities around the study of the Affine Function, from the perspective of the Epistemic and Cognitive Suitabilities of the OSA. The following presents theoretical notions of the Onto-semiotic Approach to Mathematical Knowledge and Instruction, methodological aspects, and the analysis and discussion of activities involving the Affine Function.

ONTO-SEMIOTIC APPROACH TO MATHEMATICAL KNOWLEDGE AND INSTRUCTION (OSA)

The Onto-semiotic Approach to Mathematical Knowledge and Instruction originates from the studies of the research group “Theory and Methodology of Research in Mathematics Education¹” at the University of Granada, Spain, in the early 1990s. The OSA is the result of the analysis of fundamentals, questions and methods of different theoretical frameworks of the Didactics of Mathematics and the Fundamental Didactics of Mathematics, besides the application and expansion of theoretical tools that emerged from experimental works developed by Juan D. Godino², Carmen Batanero, Vicenç Font, Àngel Contreras, Miguel Wilhelmi, Núria Planas, among others (Godino, 2011, 2012).

¹ Research group coordinated by Professor Doctor Juan Díaz Godino (Godino, 2011, 2012).

² The work developed around OSA is available at: <http://enfoqueontosemiotico.ugr.es/>

Through the investigations carried out, the researchers developed theoretical tools that could be used to jointly analyze mathematical thinking, mathematical objects, situations and factors that condition their development (Godino, 2012). The approach seeks to qualify the teaching and learning process by presenting as main features the articulation of the institutional and personal facets of mathematical knowledge, the attribution of a key role to problem solving activity and the coherent incorporation of pragmatic and realistic assumptions about meaning of mathematical objects. Thus, the starting point of the OSA is the organization of an ontology of mathematical objects that considers and articulates the three aspects of Mathematics: as a socially shared problem solving activity, as a symbolic language, and as a logically organized conceptual system (Godino, Batanero & Font, 2008).

According to Godino (2012) the set of theoretical notions that make up the OSA are articulated in five groups or levels: Practice Systems, Configurations of Mathematical Objects and Processes, Didactic Configurations and Trajectories, Normative Dimension and Didactic Suitability. The first four levels of analysis serve as tools for a descriptive and explanatory didactics, while the fifth level is based on the previous four levels and is a synthesis oriented to evaluate whether the activities implemented are suitable or adequate, aiming to identify possible improvements in the teaching and learning process (Godino, Batanero & Font, 2008). The authors also consider that these five theoretical notions can be applied to the analysis of a classroom study process, to the planning or development of a didactic unit or, at a global level, to the development of a course or a curriculum proposal.

In this article, we highlight the theoretical elements of didactic suitability and its analytic tools, two of which will be used for the analysis of a series of activities involving the Affine Function. Godino (2012) emphasizes that the Didactic Suitability can be used as a general criterion of adequacy and relevance of the educators' actions, of the knowledge put into play, and of the resources used in the process of mathematical study, serving as a guide for the systematic analysis and reflection that provide criteria for the progressive improvement of the teaching and learning process.

Godino, Batanero and Font (2008), point out that the Didactic Suitability of a mathematical instruction process is defined as the coherent and systemic articulation of six related dimensions, which are now characterized.

- Epistemic - refers to the degree of representativeness of the institutional meanings implemented or intended, in relation to a meaning of reference.
- Cognitive - expresses the degree of proximity of the meanings implemented against the students' initial personal meanings.
- Interactional - the degree to which the interactions between teacher and students, student and student allow for the identification and resolution of conflicts of meanings produced during the teaching and learning process.

- Mediatorial - expresses the degree of availability and adequacy of the material resources necessary for the development of the teaching and learning process.
- Emotional - refers to the students' degree of implication (involvement, interest, motivation, etc.) in the study process.
- Ecological - the degree to which the study process adjusts to the educational project, to the school, society and the environment in which it develops (Godino, Batanero & Font, 2008).

The scheme shown in Figure 1 highlights the dimensions described. The external regular hexagon represents the suitabilities corresponding to an intended or programmed study process, in which a maximum degree of partial adequacy is assumed, and an irregular inscribed hexagon corresponding to the suitabilities effectively achieved in the study process implemented, with the levels of analysis evaluated as high, medium or low.

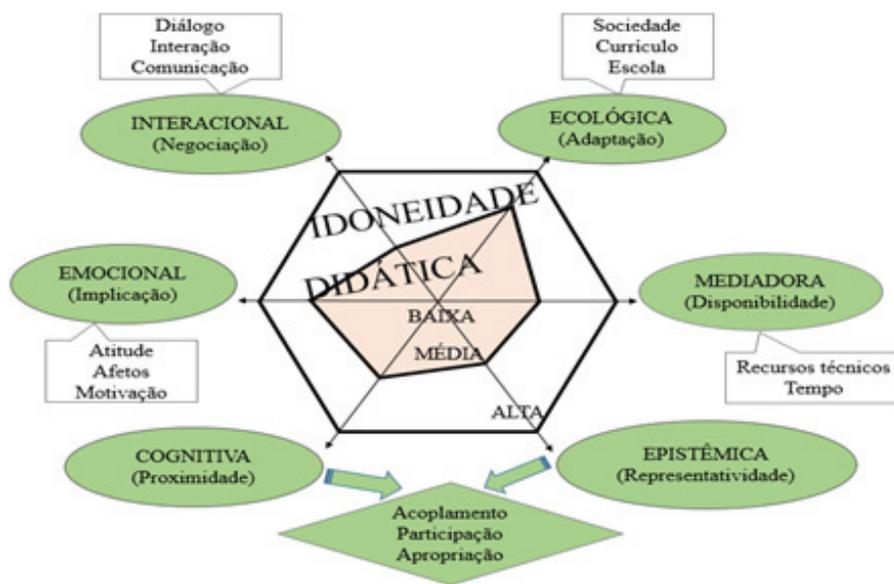


Figure 1. Representation of the Didactic Suitability and its dimensions. (Translated from Godino, Batanero & Font, 2008).

As already highlighted, this article will present analyses performed in the work with the Affine Function, considering the so-called Epistemic and Cognitive Analysis Tools³, which are presented and discussed below. Table 1 presents the components and indicators that constitute the epistemic analysis tool.

³ Components and indicators of the Epistemic and Cognitive Suitabilities taken from Godino (2011) and named by Andrade (2014) "Analysis Tools".

Table 1.
Epistemic Analysis Tool. (Godino, 2011)

Components	Indicators
Problem situations	a) we present a representative and articulated display of situations of contextualization, exercises and applications; b) situations of generalization of problems (problematization) are proposed.
Language	a) use of different modes of mathematical -verbal, graphic, symbolic- expression, translation and conversion between them; b) proper language level for students; c) propose situations of mathematical expression and interpretation.
Rules (definitions, propositions, procedures)	a) the definitions and procedures are clear and correct and are adapted to the educational level to which they are addressed; b) fundamental statements and procedures of the theme for the given educational level are presented; c) situations where students must generalize or negotiate definitions, propositions or procedures are proposed.
Arguments	a) the explanations, evidence and demonstrations are proper to the educational level to which they are addressed; b) situations where students must discuss are encouraged.
Relationships	a) mathematical objects (problems, definitions, propositions) relate to and connect with each other.

Godino (2011) considers that a central and essential point to achieve a high epistemic suitability is the selection and adaptation of problem situations, and the use of different representations, means of expression, definitions, propositions, procedures, as well as justifications allowing for a coherent and thorough analysis of the teaching and learning process to be developed or under development. Thus, the Epistemic Analysis Tool (EAT) permits a look at how the mathematical practices that are enabling students to have access to the institutional meanings implemented or intended are being structured.

The Cognitive Analysis Tool (CAT), according to Godino (2011), makes it possible to identify whether the meanings intended by the teacher are in the students' potential development zone and/or challenge their initial personal meanings. The components and indicators of this tool are presented in Table 2.

Table 2.
Cognitive Analysis Tool. (Godino, 2011).

Components	Indicators
Logical Reasoning	a) situations that make it possible to observe, analyze, reason, justify or prove ideas are proposed; b) situations where students must coordinate previously created relationships between objects (problem, definitions, information) are encouraged.

Components	Indicators
Reading/ Interpretation	a) situations of mathematical expression and interpretation where students can think, analyze and reflect on the information are presented; b) appropriate reading and interpretation of situations at student level are proposed; c) situations that makes possible to analyze or refer to the same mathematical object, considering different representations are presented.
Analysis/ Synthesis	a) situations of particularization and generalization of problems are proposed; b) situations where students must relate mathematical objects (problem, definitions, information) specifically or broadly are encouraged.

Godino (2011) considers that to broaden and deepen the indicators related to cognitive suitability, the primary entities of the epistemic model, the appropriation and understanding of these entities, the individual differences between the subjects involved in the process and whether the intended/implemented contents are appropriate at the student level were taken into consideration.

METHODOLOGICAL ASPECTS

As already highlighted, we present here part of a study that aimed to investigate the organization and development of an educational project for Mathematics in High School, from the perspective of the Onto-semiotic Approach to Mathematical Knowledge and Instruction. The study involved the organization, application and evaluation of an educational project focused on the study of Functions (initial concepts on Function, Affine, Quadratic, Modular, Exponential, Logarithmic and Trigonometric Functions), applied to a group of 26 first-grade students from High School of a private school in the city of Farroupilha, Rio Grande do Sul, Brazil, during 2018. All work was developed with the students organized in pairs (13 pairs) or in small groups.

The research was conducted from a qualitative perspective and took as reference, aligned with the constructs of the Onto-semiotic Approach, the Design-Based Research (DBR), which aims to understand the educational interventions employed to promote certain learnings and the underlying processes (Godino et al. 2014). To this end, the four phases of DBR were considered which, according to the authors, concern to:

- Preliminary study of the epistemic-ecological, cognitive-affective and instructional dimensions;
- Design of the didactic trajectory, selection of problems, their rationale and a priori analysis, with indications of the students' expected behaviors and the planning of teacher interventions;
- Implementation of the didactic trajectory, observation of interactions between subjects, resources and learning assessment;

- Retrospective evaluation that puts the predicted and the observed at stake, also reflecting on the norms and on the Didactic Suitability;

Within the adopted methodology the analytic tools taken from the OSA already described (Epistemic Analysis Tool and Cognitive Analysis Tool) both for the development and for the analysis and evaluation of the activities carried out in the educational project were used. The following presents an epistemic and cognitive analysis involving the study of Affine Function.

The article publishes results of the research approved by the Ethics Committee in accordance with the CEP review, nº 1.871.876 and CAAE identification nº 61871316.0.0000.5349.

AFFINE FUNCTION: AN EPISTEMIC AND COGNITIVE ANALYSIS

Here, we present the analysis produced from a set of activities related to the study of the Affine Function, within the scope of the study project on Functions. The project is structured from different strategies and resources, which sought a dynamic, collaborative, but also individualized study, aiming that each student could follow their learning pace, interact and dialogue with the teacher and with their peers.

From the series of activities, two situations that are representative of the work developed will be highlighted: first, the epistemic; and then, the cognitive analysis of the situations will be presented.

The activities proposed for the study of Affine Function aimed to:

- identify the Affine Function in different intra-mathematical and extra-mathematical situations;
- analyze and solve problem situations involving the Affine Function;
- use different (natural, algebraic, tabular and graphical) languages for representations and solutions of the problem situations;
- interpret and determine the domain and image set according to the context;
- identify and analyze the growth and decrease of a Function from contextualized situations;
- interpret the Function signal in the problem situation addressed;
- analyze and interpret variations of a Function (horizontal and vertical displacements);
- use the relevant symbolic language, determining the domain and image set regardless of context.

The first activity (Figure 2) regarding epistemic components and indicators is related to a leaking water reservoir. It is presented in the natural language, and enables the use of

algebraic, graphic and tabular language, as well as retakes, deepens and develops concepts and definitions around the law of formation, zero of the function, domain, image set. Thus, we seek relationships to be established and arguments presented around the rationale for conclusions and observations found regarding the zero of the function, domain and image set. We also emphasize that the use of different forms of representation leads to different meanings of conversion between records and different treatments.

Um reservatório de uma determinada cidade contém 240 m^3 de água. No início de um mês, um duto se rompeu, e está vazando água a uma taxa de 4 m^3 por dia. Com base nessas informações, determine ou construa o que se pede:

- a expressão algébrica de $Q(t)$ (quantidade de água no reservatório em metros cúbicos) em função de t (em dias decorridos desde o início do mês);
- o zero da função e seu significado no contexto do problema;
- uma tabela de valores que represente os dados do vazamento;
- o gráfico de $Q(t)$;
- o domínio da função, justificando sua resposta;
- o conjunto imagem, justifique sua resposta.

Figure 2. Activity leaking water reservoir. (Adapted from Adami et al., 2015).

Another activity suggested (Figure 3) is related to a digital scale of a restaurant, which is used to measure the mass of food each customer consumes in a given meal.

O mostrador digital de uma balança de um restaurante de uma determinada cidade mostra que o preço do quilograma da refeição custa R\$ 29,90. Um cliente ao colocar seu prato sobre a balança, mediu a massa no prato de $0,400 \text{ kg}$ e o valor a ser pago é de R\$ 11,96. Com essas informações podemos construir um quadro com outros valores para a massa (m) a fim de obter os valores a serem pagos ($v(m)$) e depois representar essa situação por meio de um gráfico, conforme figuras a seguir.

Massa (kg)	Valor a ser pago (R\$)
0,200	$29,90 \cdot 0,200 = 5,98$
0,400	$29,90 \cdot 0,200 = 11,96$
0,600	$29,90 \cdot 0,200 = 17,94$
0,800	$29,90 \cdot 0,200 = 23,92$
⋮	⋮
m	$29,90 \cdot m = 29,9 m$

- Com os valores apresentado na tabela e no gráfico estabeleça a lei de formação da função para a situação apresentada.
- Estabeleça o domínio e o conjunto imagem. Explique o significado dos mesmos no contexto do problema.

Figure 3. Activities involving digital scale. (Adapted from Chavante & Prestes, 2016).

In this problem situation we use the natural language, tabular and graphic representations, seeking to provide students with the use of different languages to analyze and solve a presented situation. The institutional meanings intended refer to the relationship of dependence between the quantities involved and the establishment of the law of formation, as well as the establishment of the domain and image set in the context of the problem situation.

The situations presented are just examples of the series of situations used for the study of the Affine Function, which also involved studies related to simple interest, uniform rectilinear motion, arithmetic progression, value of a taxi race, price paid for energy and water consumption, value charged for services, among others. The study project aimed to meet and develop the reference meanings established by the educational institution where the research was implemented.

Table 3 shows the epistemic components and indicators evidenced in the series of activities involving the Affine Function, indicating the degree of suitability. This analysis was carried out from the practices developed with the application of the proposal, that is, although the proposal was conceived considering the assumptions of the Epistemic Suitability, its application and development with the students made possible to establish its degree of suitability in the different components.

Table 3.
Epistemic Analysis: Affine Function.

PROBLEM-SITUATIONS COMPONENT		
Indicators	Indicators highlighted	Suitability
a) we present a representative and articulated display of situations of contextualization, exercises and applications; b) situations of generalization of problems (problematization) are proposed.	- The activities proposed refer to a series of problem situations pertinent to the study of the Affine Function, presented, predominantly, in the form of problems focused on situations of reality, daily life or other areas of knowledge, in which students needed to relate the concepts, definitions, laws, procedures and properties already studied or under study. - Regarding generalization, situations that enable students to conjecture, justify, deduce, establish relationships and laws of formation, identifying characteristics and properties of the Affine Function are proposed.	High
LANGUAGE COMPONENT		
Indicators	Indicators highlighted	Suitability
a) use of different modes of mathematical -verbal, graphic, symbolic- expression, translation and conversion between them; b) appropriate language level for students; c) situations of mathematical expression and interpretation are proposed.	- The language used is proper to the level of the students, being presented in the form of the natural language and algebraic, symbolic, tabular and graphic representations. - It was possible to identify the presence of the different forms of representation and the conversion between them, making it clear when they referred to the same object under study.	High

RULES COMPONENT		
Indicators	Indicators highlighted	Suitability
<p>a) the definitions and procedures are clear and correct and are adapted to the educational level to which they are addressed;</p> <p>b) fundamental statements and procedures of the theme for the given educational level are presented;</p> <p>c) situations where students must generalize or negotiate definitions, propositions or procedures are proposed.</p>	<p>- The definition of Affine Function was developed, and the domain, contra-domain, image set, zero of the function, Function signal, intersection with axes, increase and decrease were established.</p> <p>- In the activities the definitions, propositions and procedures are worked out clearly and according to the educational level of the students. The study of properties and procedures were developed through situations in which students had to observe, analyze, conjecture, conclude and evaluate from manipulations or constructions, seeking regularities, particularities and generalizations.</p>	High
ARGUMENTS COMPONENT		
Indicators	Indicators highlighted	Suitability
<p>a) the explanations, evidence and demonstrations are appropriate to the educational level to which they are addressed;</p> <p>b) situations where students must discuss are encouraged.</p>	<p>- In general, the activities proposed moments when students needed to discuss considering a given problem situation, either through the discussion of a property, a definition or a law of formation, or even from a series of actions taken by them to represent, construct and analyze the object under study. However, it is considered that a larger series of construction activities should have been presented in Geogebra, as well as new activities that would allow students to develop autonomy regarding the procedures and solution strategies, as well as the appropriate rationale for each situation presented.</p>	Medium
RELATIONSHIP COMPONENT		
Indicators	Indicators highlighted	Suitability
<p>a) mathematical objects (problems, definitions, propositions) relate to and connect with each other.</p>	<p>- It is considered that it was possible to establish relationships between the properties of the mathematical objects under study, especially the relationships between the law of formation, the tabular and graphic representation of situations. However, their relationship with the domain and the image set was not evident in the context of some situations under study. It is noteworthy that the relationship between the domain and the image with the values of the intercept x and y were not properly explored.</p>	Medium

From the aspects highlighted, it is possible to observe that the epistemic components and indicators are strongly present in the activities proposed, and the analysis sought to identify the extent to which they could be implemented in the work developed with the students. Although every study project was conceived seeking the maximum suitability in each of the partial aptitudes, the work dynamics did not always enabled the adequate development of activities that related all epistemic indicators.

Regarding the Problem situations (examples highlighted in Figures 4 and 5), their high representativeness was considered, because the concepts around the Affine Function

were developed through situations of contextualization and application related to everyday situations, from the world of work and other areas of knowledge. This also led to consider the Rules component with high representativeness, because, for the resolution of each problem situation, it was necessary to use different concepts, definitions and procedures of resolution. As for the Arguments component, it was ranked medium suitability, since not all activities required from students a solution that needed justification or proof. In this context there are activities of a more procedural nature, which, although requiring knowledge on the subject, lead the justifications to be implicit.

Regarding Languages, we considered high suitability, considering that different forms of representations were used in the series of activities, as in the examples presented in Figures 4 and 5, both in natural language and algebraic, symbolic, graphic and tabular representations. These different forms of representation were explored by considering conversions and changes in meaning between them, wherever possible, as already noted.

On the other hand, the Relationship component was considered with medium representativeness, since relationships between the law of formation, the analysis of graphs and tables through exercises and problem situations were evidenced. However, to achieve a high degree of suitability, it is considered that it would have been necessary to establish the relationships between mathematical objects in a larger number of problem situations.

Aiming to observe students' learning, as well as the conflicts and difficulties met and presented during the study of the Affine Function, a cognitive analysis was carried out through the components and indicators of the Cognitive Analysis Tool (logical reasoning, reading/interpretation and analysis/synthesis). It was also considered pertinent to perform an analysis around the institutional meanings and the meanings declared by the students. Thus, we used the components of both the Epistemic Analysis Tool (problem situations, languages, rules, arguments and relationships) and the CAT, having established meanings of reference intended with the study material for the components of both tools. According to Lemos (2017), in this way, we seek evidence of what students were able to do/ understand/mean, based on a series of activities and problem situations proposed.

In Table 4, we present each component used in cognitive analysis, identifying the meanings established and evidenced in the series of activities, together with the indicative of the degree of suitability deemed appropriate.

Table 4
Synthesis of the cognitive analysis.

PROBLEM-SITUATIONS COMPONENT		
Meanings intended	Meanings declared	Suitability
<p>a) Identify the Affine Function in the problem situations proposed in the different intramathematic and extramathematic contexts.</p> <p>b) Resolve situations proposed using the concepts, definitions and procedures related to the Affine Function.</p>	<p>- The students did not present difficulties related to the identification of an Affine Function, as well as its characteristics in the different problem situations proposed.</p> <p>- Most students were able to solve the situations proposed correctly by using appropriate procedures, definitions and concepts.</p>	High
LANGUAGE COMPONENT		
Meanings intended	Meanings declared	Suitability
<p>a) Identify and recognize the use of different modes of mathematical -verbal, graphic, symbolic- expression, translation and conversion between them.</p> <p>b) Recognize and use the mathematical language appropriate to each situation.</p>	<p>- Students identified and used the different forms of mathematical representation or expression and the conversion between them properly, but found it difficult to use and understand the symbolic form.</p> <p>- They used different ways to represent the domain and the image set of the Affine Function, even though in some cases they were not correctly established in the context of the problem situation.</p> <p>- They used the language appropriate to each situation proposed.</p>	High
RULES COMPONENT (DEFINITIONS, PROPOSITIONS, PROCEDURES)		
Meanings intended	Meanings declared	Suitability
<p>a) Recognize what characterizes an Affine Function.</p> <p>b) Identify situations involving the increase and decrease of an Affine Function.</p> <p>c) To represent graphically an Affine Function and establish the algebraic representation from its graphical representation.</p> <p>d) Identify the domain and the image in situations involving an Affine Function;</p> <p>e) Calculate the root of the Function.</p> <p>f) Determine the law of formation of an Affine Function, given by a problem situation.</p>	<p>- Students are considered to recognize the elements of a Function.</p> <p>- There were no mistakes in the classification regarding the increase or decrease of the Function.</p> <p>- There were no difficulties in the graphical representation, but difficulties in understanding the domain set and image of problem situations.</p> <p>- They used properly the mathematical procedures to calculate the root of a Function as well as its coefficients to determine the law of formation.</p>	Medium

ARGUMENTS COMPONENT		
Meanings intended	Meanings declared	Suitability
<p>a) Identify and justify the domain and image sets in the problem situation.</p> <p>b) Identify and justify the increase and decrease of a Function by either an algebraic, tabular or graphical representation, considering the context of the problem situation.</p>	<p>- Students identified and represented mathematically the domain and image set but they were sometimes unable to justify what such sets or intervals represented in each situation.</p> <p>- It was not difficult for them to identify the increase or decrease in a graphical representation, but, in some cases, they did not use adequate rationale for the increase or decrease in a context or in a requested interval.</p> <p>- Mostly, activities where a rationale was requested were left blank.</p>	Low
RELATIONSHIP COMPONENT		
Meanings intended	Meanings declared	Suitability
<p>a) Understand the relationships between different representations of an Affine Function.</p>	<p>-We consider that the students partially established the relationships between the different forms of representation of the same object in the situations presented, for which we may think that they had not yet appropriated the ideas or concepts of the Affine Function properly.</p>	Medium
LOGICAL REASONING COMPONENT		
Meanings intended	Meanings declared	Suitability
<p>a) Observe and analyze the graphical representation of the Affine Function in the Cartesian plane, seeking to justify its increase or decrease.</p> <p>b) Analyze the domain and image set.</p> <p>c) Establish strategies to determine the law of formation of an Affine Function.</p>	<p>- From the observation and analysis of the graph of the function, the students justified its increase or decrease and represented the domain and image set of the different situations.</p> <p>- They used the points found to determine the coefficients of the function, seeking to show the respective law of its formation.</p>	High
READING/INTERPRETATION COMPONENT		
Meanings intended	Meanings declared	Suitability
<p>a) Read and interpret properly the information and situations proposed.</p> <p>b) Identify, understand and apply the concepts and definitions.</p>	<p>- Appropriate readings of what was required in each problem situation addressed were carried out.</p> <p>- We realized that students were able to understand and apply the relevant concepts and definitions properly.</p>	High
ANALYSIS/SYNTHESIS COMPONENT		
Meanings intended	Meanings declared	Suitability
<p>a) Synthesize to express the concepts constructed or revisited.</p>	<p>- The students found it very difficult to carry out the syntheses requested. We perceived that they were not used to summarizing the situations and concepts worked, which made the focus of the work turn to that construction.</p>	Low

As shown, in the activities involving problem situations, the students participating in the research did not find it difficult to identify the characteristics of an Affine Function in the different problem situations and other activities using either Geogebra or pencil and paper. In general, they used definitions and procedures that were suitable for the situations, as can be observed in the solution presented by pair B, highlighted in Figure 4.

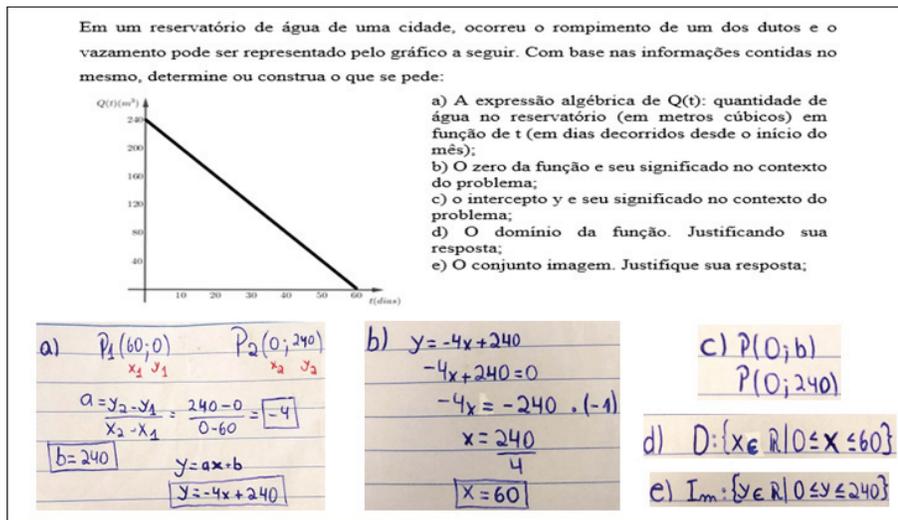


Figure 4. Solution by pair B (Activities adapted from Adami et al., 2015).

To establish the law of formation, the students used two points belonging to the function graph limited by the coordinate axes, determined the values for the angular and linear coefficients, substituting them in the algebraic expression " $f(x) = ax + b$ " (Figure 8, item a).

It is thus conjectured that this pair, while not using the quantities presented in the statement of the question, read satisfactory the situation proposed, using proper concepts, definitions or resolution procedures related to the Affine Function. Similar situations were presented by other pairs of students in this situation and in others, which allowed us to identify the suitability in the problem situations component as high. The same solution disclosed presented elements that allowed us to consider that the Languages component reached a high degree of suitability, since the students used different mathematical languages and representations in the situations under study and made conversions between them.

On the other hand, the Rules component reached an average degree of suitability, since the students found it difficult to understand and represent the domain and image set, as can be observed in the resolution submitted by the students of the pair F, who used, to solve the same problem, two other points on the graph (Figure 5), which were not defined or visually highlighted on the graph. With the points selected, they made a

construction that misrepresented the initial situation, as it did not consider the intervals $[0, 10]$ on the x axis and $(200, 240]$ on the y axis. Using the formulas “ $a = \frac{\Delta y}{\Delta x}$ ” and “ $y = ax + b$ ”, they determined the law of formation, giving a correct outcome.

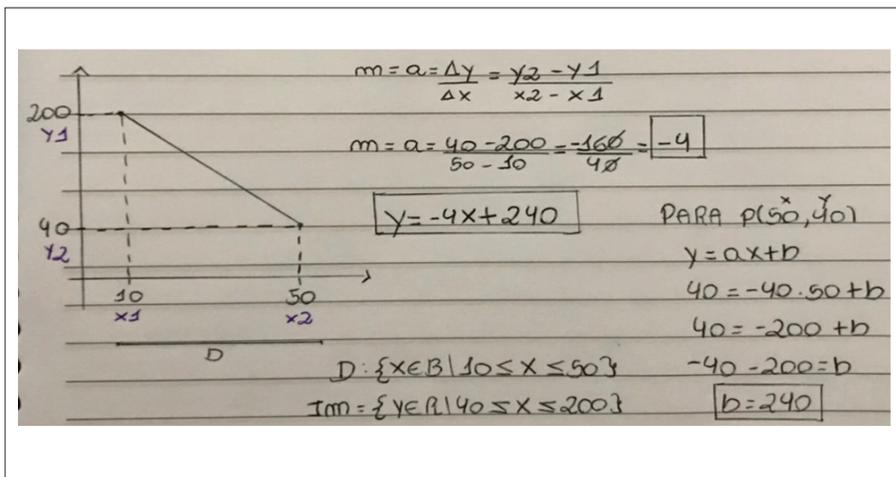


Figure 5. Resolution of pair F.

Even though they reached the expression that represents the situation being studied, the resolution was incorrect. As the points used belonged to the domain of the function, they reached the correct law of formation, but the misrepresentation carried out led to a misunderstanding and misrepresentation of the domain and image set of the situation addressed, which can also be observed in Figure 9.

This type of solution was used for the analysis and discussions with the group of students, retaking the concepts involved, making room for students to reflect, present arguments, and have the opportunity to assign new meanings to the domain and image sets involved in the issue.

Regarding the Arguments component, we understand that it reached low suitability, as the students were given situations in which they needed to debate from a given problem situation, either through discussing a property, a definition or a law of formation, or from a series of actions carried out by them to represent, construct, or discuss the object under study. However, even if the solution presented pointed to the understanding of what was being requested, they could not use the proper rationale when expressing themselves in writing.

In the answers of student pair B (presented in Figure 8), it is possible to notice that they determined the zero of the function and the y intercept, which, in principle, leads us to understand that they appropriated the concepts involved, however, they could not justify or discuss their meaning in the problem situation addressed. This fact was also noticed in the solutions presented by other pairs, and in large part the students did

not answer what had been requested in relation to justifications, only highlighting the points where the graph intercepts the abscissa and the ordinate axis, or the intervals of the domain and image set, without explaining their meanings.

When asked why they were not justifying their answers, one of the students reported that: "I don't know what to write, I know the graph cuts the x and y axis. Isn't it already a justification?" After this discussion, when asked again about the meaning of the zero of the function in the context of the problem, or about what happened on the 60th day after the leaking began, the students answered that on the 60th day the reservoir would be empty. Then, pair E asked, "Is that what we must answer?" These students' demonstrations suggest that in many situations students have an argument to justify their procedures, they just cannot trigger them and present them in writing properly.

It is possible that this way students employ to solve the situations proposed may be related to the work that, usually, is performed in Mathematics classes, when the solutions are based on procedures, such as using algorithms and rules.

The Relationship component was considered in the proposal, but we think that an medium suitability was achieved, because the students could not adequately relate certain concepts and definitions involving the Affine Function, mainly regarding the domain and image set in the context of problem situations. However, they established pertinent relations between the law of formation and the tabular and graphic representation of the situations presented.

The cognitive component Logical Reasoning reached a high degree of suitability, since the students could observe, analyze, infer, conjecture and prove their understandings and procedures regarding the solution of the different problem situations they faced. In addition to the individual and twosome workspace, discussions in the large group on issues that generated conflict constituted a space where students could debate, resignifying many concepts.

Regarding the Reading/Interpretation component, we understood that a high degree of suitability was achieved, since the language used was adequate at the student level and that the different forms of representation of a function referred to the same mathematical object under study. Thus, the students did not present relevant difficulties regarding the situations of mathematical expression and interpretation, in which it was necessary to think, analyze, reflect and infer on the information contained in the problem to arrive at a solution.

Regarding the Analysis/Synthesis component, the degree of suitability was low. Even though the activities provided solutions both regarding the particularization as generalization, and the relationships between mathematical objects, in a broad or specific way, the students found it difficult to synthesize the concepts and definitions approached in relation to the Affine Function, as can be observed in Figure 6.

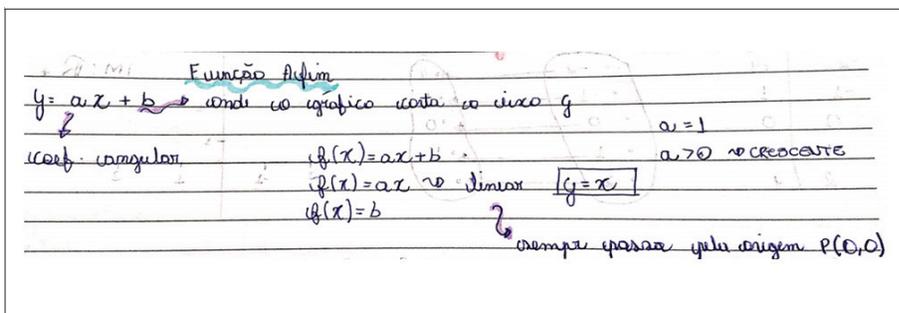


Figure 6. Synthesis performed by group C.

This type of synthesis was used for analysis and discussion in the large group, returning to the concepts involved and possible ways of presenting a synthesis. At the end of the discussions and reflections, the pairs resumed their activities of synthesis. In the new schemes, students listed the concepts approached satisfactorily, presenting the generic form of the Affine Function, particularizing the constant, linear and identity functions, presenting the conditions to determine whether a function is increasing or decreasing, the mathematical rules to determine the root or zero of the Function, the coefficients of the function, as well as the study of the signal and the study of inequations.

Again, we believe that the students found it difficult to carry out this type of task due to the lack of habit summarizing what had been addressed. It is also noteworthy that, after the discussions, two pairs of students (G and K) based their synthesis on a mental map presented at the end of the material adopted by the school, adapting it to the study in question.

Those moments of discussion brought us to realize that students were able to broaden and deepen ideas, concepts and procedures related to the mathematical object Affine Function, consolidating meanings already attributed as well as assigning new meanings. Forms of representation were also expanded, and the development of argumentation was justified.

FINAL CONSIDERATIONS

The analysis of the Affine Function made possible for us to focus, from the perspective of the Onto-semiotic Approach of Mathematical Knowledge and Instruction, on the epistemic and cognitive suitabilities, aiming to contribute to the selection, development, application, analysis and evaluation of activities for an educational project for Mathematics in High School .

In this sense, the proposal of activities around the topic sought to enable students to resume, deepen and develop notions, concepts, definitions and procedures articulated from confronting problem situations; using digital technological resources (GeoGebra

software, videos, learning objects); carrying out activities of construction with pencil and paper, which also made possible establishing relationships and producing arguments. We also concluded that the use of different resources and activities allowed students to deepen their knowledge, as well as overcome difficulties presented in previous years and during the development of the study of the Affine Function.

In the analysis, the problem-situation components, languages and rules concerning epistemic suitability reached a high degree of suitability. We emphasize that the activities developed and analyzed presented problem situations related to everyday issues and other areas of knowledge; the use of different forms of representation of a same object; and the presence of appropriate procedures in both the explanations, and the resolution of situations proposed, which were often approached from different perspectives.

Regarding the cognitive aspects evidenced, through the components and indicators of the epistemic and cognitive tools it was possible to establish that whereas the Problem Situation, Languages, Logical Reasoning and Reading/Interpretation components reached a high degree of suitability, the Rules and Relationships were identified with a medium degree of suitability. Nevertheless, the Argument and Analysis/Synthesis components obtained a low degree of suitability, largely due to the students' lack of habit in performing so complex activities, as well as their difficulty in expressing verbally what they were calculating or representing.

Thus, we understand that the OSA analysis tools used in this study are resources that, when applied to examine a content and its series of activities, enable us to look into the mathematical knowledge and into the way the teaching and learning process may develop. Moreover, when articulated with the other dimensions of didactic -emotional, interactional, mediating and ecological dimensions- suitability, it constitutes a powerful apparatus for the constitution of educational projects.

AUTHOR'S STATEMENT OF CONTRIBUTIONS

V.N and C.T.K conceived the idea presented. V.N carried out the activities and collected the data. Both authors discussed the results and jointly prepared the final version of the article.

DATA AVAILABILITY STATEMENT

Data supporting the results of this study will be made available by the corresponding author, V.N, upon request.

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