

# Helping Preservice Teachers to Reflect

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## ABSTRACT

Teacher reflection is a mechanism to evaluate and improve the educational practice. To achieve an efficient reflection, it is convenient to have guidelines that support this activity. In this paper it examines the incidence of studying basic notions of the onto-semiotic approach to develop the ability of future math teachers to reflect on teaching practices. The study was conducted during the second semester of 2018 with seven students, in three phases: first, participants reflected on the classroom practice observed in a video sequence, without intervention; second, training was given on some theoretical notions of the onto-semiotic approach; and third, a new reflective session was conducted on the continuation of a video sequence. Data analysis was focused on identifying and comparing elements referring to indicators of suitability present in both reflections. Results show that participants manage to express clearer ideas, and, mainly make justifications about their value judgments. It has been determined that, with the appropriate use of concepts from a theoretical framework of didactic analysis and guided reflection, there was a change in the nature of the reflection made by future teachers, moving from the description of events to the interpretation and analysis of situations.

**Keywords:** Preservice Teachers; Reflect; Teacher Training; Videotape Analysis; Mathematics Education, Reflexive Teacher, Teacher Reflection.

## Apoyando a los futuros profesores a reflexionar

### RESUMEN

La reflexión docente constituye un mecanismo para evaluar y mejorar la práctica educativa. Para lograr una reflexión eficiente, es conveniente contar con herramientas o pautas que apoyen esta actividad. Este trabajo tiene como objetivo presentar los resultados de una investigación sobre la incidencia del estudio básico de algunas nociones teóricas del enfoque ontosemiótico en el desarrollo de la capacidad de reflexión de los futuros profesores de matemática sobre las prácticas docentes. Se realizó con un grupo de siete estudiantes durante el segundo semestre de 2018 y se abordó mediante tres fases: los participantes realizaron una reflexión sobre la práctica de aula observada en una secuencia de video, sin intervención; luego, se desarrolló una actividad formativa sobre algunas nociones teóricas del enfoque ontosemiótico y, posteriormente, se realizó

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una nueva práctica reflexiva sobre la continuación de una secuencia de video. El análisis de los datos se enfocó en la identificación y comparación de elementos referentes a indicadores de idoneidad presentes en ambas reflexiones. Los resultados muestran que, los participantes logran exponer ideas más claras y, principalmente, realizan justificaciones sobre sus juicios de valor. Se concluye que con el uso adecuado de conceptos de un marco teórico y la reflexión guiada se puede producir un cambio en la forma de reflexión desde una descripción de eventos a interpretaciones y análisis de las situaciones.

**Palabras clave:** futuros profesores; reflexionar; formación de docentes; análisis de videograbaciones; educación matemática, profesor reflexivo, reflexión docente.

## INTRODUCTION

Teachers' task in the field of mathematical education goes beyond mastering specific content; it also requires having the capacity to mediate the teaching-learning process and to adapt and improve the classroom practice. To do so, their ability to reflect productively on what happens in the classroom is fundamental.

Appropriate reflection on classroom practice is not a simple process that can be accomplished by simply reviewing what happened in a classroom. In the case of preservice teachers, Husu, Toom, & Patrikainen (2008) indicate the importance of conducting guided reflection, using both support from a trainer and a guide that includes a system of indicators that allow critical analysis of the teaching practice (see also, Schön, 1983). Godino (2013) explains how the notion of didactic suitability within the framework of the Onto-Semiotic Approach (OSA) to knowledge and mathematical education can contribute to reflection on the practice and to the improvement of teaching.

Likewise, authors such as Aroza, Godino and Beltrán-Pellicer (2016) emphasize the importance of developing the competence for reflection on and inquiry into teaching practices in mathematics teachers from their earliest stages of training. Other studies show that, at least in some training programs in Costa Rica, there has been an interest in motivating teachers to reflect on teaching (Alpízar-Vargas & Alfaro-Arce, 2019), but this has not been observed in explicit activities in the work of teachers (Morales-López, 2017).

Studies such as those of Breda, Pino-Fan and Font (2017) and Godino, Giacomone, Font and Pino-Fan (2018) justify the importance of didactic suitability criteria as a useful tool to organize reflection and evaluation of instructional processes of preservice mathematics teachers, even in training contexts where they do not yet have explicit guidelines for evaluating their practices.

It is therefore of interest to carry out a study about the changes that the basic study of some theoretical notions of the OSA produces in the capacity for reflection of preservice mathematics teachers. This investigation attempts to 1) *characterize the capacity of reflection of preservice mathematics teachers on the instructional practice present in the video sequences in terms of suitability indicators and, 2) determine if there is a change in the type of reflection of the teaching practice of students of the Bachelor's*

and Licentiate's Degrees in Mathematics Teaching, after they have received instructions in elementary theoretical notions of the OSA.

This theoretical framework has been selected because multiple studies have supported the conclusion that reflections can be productively organized using theoretical and methodological tools of the OSA (e.g., Alpízar-Vargas & Morales-López, 2019; Morales-López, 2019; Morales-López & Font, 2019; Seckel & Font, 2020).

## **THEORETICAL FRAMEWORK**

### **The Onto-Semiotic Approach (OSA) to knowledge and mathematics teaching**

The OSA seeks to build a unified approach to mathematical knowledge and education to overcome dilemmas between different paradigms. This model “seeks to provide theoretical tools to jointly analyze mathematical thinking, the suppositions that accompany it, and the situations and factors that condition its development” (Godino, 2002, p. 5), based on the consideration of aspects or dimensions of mathematical knowledge that allow the comparison and articulation of different research approaches on teaching and learning.

Among the theoretical concepts established by the OSA is that of *practice systems*, considered as one of the possible ways of understanding the meaning of an object, which can lead to the introduction of a basic typology of meanings. Institutional meanings include the referential, the intended, the implemented and the evaluated (Godino et al., 2009).

Another concept is that of intervening and emerging objects of practice systems, in which the following typology of primary mathematical objects is proposed (Godino et al, 2009): *Linguistic elements*: terms, expressions, notations, and graphics in different types of records (written, verbal, text). *Situations-problems*: extra-mathematical applications and exercises. *Concepts*: introduced through definitions or descriptions. *Propositions*: statements about concepts. *Procedures*: algorithms, operations, calculation techniques. *Arguments*: statements used to validate or explain deductive or other propositions and procedures.

Likewise, the relationship between mathematical objects and processes is also included within the OSA, as is the description of interactions around conflicts. Regarding the latter, Godino et al. (2009) indicate that, according to the differences that arise, semiotic conflicts can be: 1) *Epistemic*, referring to the difference between institutional meanings. 2) *Cognitive*, when the difference occurs between the practices that shape the personal meaning of the same subject. 3) *Interactional*, when the difference occurs between the practices (discursive and operational) of two different subjects in communicative interaction (for example, student-student or student-teacher).

The notion of didactic suitability, which is one of the most important theoretical notions in the development of this investigation, is also considered. It is defined as follows:

The degree to which said process (or a part of it) has certain characteristics that allow it to be qualified as *ideal* (optimal or adequate) for reaching adaptation between the personal meanings achieved by students (learning), and intended or implemented institutional meanings (teaching), taking circumstances and resource availability into account (environment) (Godino, Giacomone, Batanero and Font, 2017, p. 101).

The notion of didactic suitability is broken down into six suitability types: epistemic, cognitive, affective (emotional), mediational, interactional and ecological. The notion of didactic suitability can be explained as a theoretical tool that makes it possible to go from explanatory descriptive didactics to a normative didactics that favors or allows efficient intervention in the classroom. These six types of suitability are detailed in the methodological framework.

Finally, Godino et al. (2006) distinguish six types of processes and sample trajectories:

- *Epistemic trajectory*, which is the distribution over time of the teaching of components of the implemented institutional meaning. These components (problems, actions, language, definitions, properties, arguments) occur in a certain order.
- *Teaching trajectory*, which refers to the distribution of educational tasks/actions throughout the instructional process. Teaching functions include planning, motivation, homework assignment, regulation, evaluation and investigation.
- *Learning trajectory*, which involves the distribution of actions performed by students (one for each student). The potential types of states or functions of the student in an instructional process are acceptance, exploration, interpretation, formulation, argumentation, reception of information, demand for information, exercise, and evaluation.
- *Mediational trajectory*, which represents the distribution of the technological resources used.
- *Cognitive trajectory*, which refers to the development of students' personal meanings through time.
- *Emotional trajectories*, which concern the distribution of emotional states through time (attitudes, values, effects and feelings) of each student with respect to mathematical objects. (Godino et al., 2006, p. 47).

## **Model of didactic-mathematical knowledge and teacher's competences (DMKC)**

The theoretical Didactic-Mathematical Knowledge and Competences (DMKC) model has been developed as part of the OSA; initially, it consists of a system of categories. This model is described in different works as a theoretical-methodological tool that allows its users to characterize and develop competencies of mathematics teachers that are required in professional practice (Godino, Giacomone, Font and Pino-Fan, 2018).

The DMKC considers two key competences that a mathematics teacher must have, to approach the teaching process successfully: on the one hand, mathematical competence and, on the other, competence in analysis and didactic intervention. This second competence has to do with the “design, application and evaluation of the teacher's own and other's sequences through didactic analysis techniques” (Breda et al., 2017, p. 1897). The general competence of analysis and didactic intervention consists of five sub-competencies (Godino et al., 2016; Godino et al., 2017). This investigation focuses on the competence of *analysis and assessment of didactic suitability*.

### **Reflective practice**

The concept of reflective practice in the context of the teaching practice refers to a continuous interaction between thought and action. It is important that teachers reflect on their teaching practice constantly to identify problems, and to transform and improve it (Schön, 1983).

It is usual for people to reflect on some of their actions from time to time, and in the field of education this occurs more frequently because teachers must constantly evaluate the effectiveness of their class plan and proposed activities.

However, reflection on a single episode of the task of teaching does not necessarily make the teacher a “reflective practitioner” (Shön, 1983). As Perrenoud (2004) notes, true reflective practice implies that this attitude becomes habitual, and that analysis and action occur regardless of the obstacles that arise.

In pedagogy, reflective practice involves a series of decisions that can be made at different times during the teaching process, although it is obvious that reflecting and making decisions during class development can be difficult. Therefore, it is most convenient to carry out reflection on the pedagogical work carried out retrospectively (Perrenoud, 2004). In the case of OSA, which is the theoretical framework used in this investigation, reflective practice is considered to be a formative strategy to develop mathematical didactic knowledge.

## **Guided reflection and the ontosemiotic approach**

Analyzing the teaching practice requires knowledge, mastery and application of appropriate conceptual and methodological tools. To engage in reflective practice, it is useful for teachers to be familiar with a method or guidelines that allow them to identify relevant aspects of the teaching process, and to organize and analyze the information available. The OSA is especially useful in this context, providing theoretical elements to analyze various dimensions and aspects that should be considered in the mathematics teaching and learning processes; likewise, it

[...] tries to operationalize the notions of mathematical practice, epistemic and cognitive configuration, didactic configuration, normative dimension and didactic suitability by means of “guidelines” for the recognition of mathematical objects and processes, didactic interactions, norms and meta-norms that support and restrict the processes of study, and for the assessment of their didactic suitability (Godino and Batanero, 2009, p. 4).

Godino and Batanero (2009) state that reflection on different aspects and moments of the practice should be carried out with the support not only of a trainer in his or her role as a tutor, but also of an aid that addresses critical aspects of the practice and provides a structure to assist throughout the practice. In addition, among the instruments of didactic analysis they include a guide for assessment of didactic suitability, which allows the evaluation of the study process in each of the dimensions involved (epistemic, cognitive-affective, instructional and curricular). The last one will be especially important in carrying out reflective practice and, in particular, in achieving the objectives of the present investigation.

### **Use of videos as an instrument for reflection (among preservice teachers)**

Several authors (Borko, Jacobs, Eiteljorg and Pittman, 2008; Climent and Carrillo, 2007; Kleinknecht and Schneider, 2013; Rosaen et al., 2008) have made use of video clips or video sequences of a class episode to assist in carrying out a didactic analysis of the teaching practice, given the advantages of being able to observe a scene of interest several times.

This tool makes it possible to highlight aspects of the teaching practice that a teacher might overlook in the course of the lesson (Borko et al., 2008).

It has also been shown that teachers who watch videos of the practices of other teachers tend to express emotions such as disapproval, as well as suggest changes or alternatives to improve the practice of the teacher being observed, while when viewing a video of their own practice, they tend to be more descriptive and critical of their activity, although in less depth (Kleinknecht and Schneider, 2013).

Therefore, the use of class episodes to carry out reflection can be regarded as a very useful tool for teacher training, since it provides a space for students to observe and analyze particular situations in a class, and to make value judgments and propose alternatives that improve the teaching exercise.

## METHODOLOGY

This investigation studies the impact of knowing theoretical notions of OSA on the capacity of reflection on teaching practices in preservice mathematics teachers of the Bachelor's and Licentiate's Degrees in Mathematics Teaching of the Universidad de Costa Rica. The study uses a qualitative methodology, seeking to understand a social phenomenon from the perspective of the participants.

The methodological design is a case study, since an attempt was made to learn about the situation of a specific group without intending to generalize the results; in addition, analysis of the data focused on a phenomenon selected by the researchers, regardless of the number of scenarios or participants (McMillan and Schumacher, 2005).

The study was carried out with a group of students in the course MAB 504 of the previous curriculum (1995-2016) of the Bachelor's and Licentiate's Degrees in the Mathematics Teaching program, the Guided Research Seminar I of the II Cycle 2018 of the Universidad Nacional. The group consisted of a total of seven individuals, who shared characteristics such as: studying at the fifth level of the previous curriculum of the Bachelor's and Licentiate's Degrees in Mathematics Teaching, having studied only at the Universidad Nacional, and having passed the Teaching Development and Practice course, in which they had the opportunity to work as teachers for approximately eight weeks. In addition, they had between one and three years of experience in some type of teaching activity.

Two questionnaires were used for collecting the information. Questionnaire 1 contained a series of questions adapted from Font (2018), which refer in general to suitability criteria. The question: *Have mathematics been taught with acceptable quality?* is related to epistemic suitability. *Do you consider that students have learned from the proposed assignments?* with cognitive suitability. *Do you consider that temporary resources, materials, ICT, etc. have been used adequately?* with mediation suitability. *Do the tasks and the way in which they are conducted promote student involvement?* with emotional suitability. *Is interaction in the class adequate, and does it allow students to resolve their difficulties?* with interactional suitability. *Are contents consistent with the curriculum and are they useful for the student's insertion in society and in the labor market?* with ecological suitability. Questionnaire 2 is about adaptation of the "Guideline for analysis and assessment of didactic suitability of the mathematics teaching and learning processes" designed by Font (2015).

Data collection activities were organized in three phases. Questionnaire 1 was applied in the first phase, and it consisted of open questions, without guidance; the second

phase consisted of a training activity in which the suitability indicators were very briefly explained; and the third phase consisted in the application of Questionnaire 2.

An analysis of participation was carried out to systematize the results, using the theoretical foundations of the OSA as categories of analysis. The analysis of didactic suitability used the following dimensions or suitability criteria (Godino and Batanero, 2009), and descriptors associated with each of the suitability criteria proposed by Font (2015) were used as categories of analysis in the study. Table 1 shows the adaptation for the present investigation of the descriptors originally presented in Font's study.

Table 1  
Suitability components and indicators (Adapted from Font, 2015)

Components and descriptors of each suitability	Indicators
Epistemic [IE]	(1) Errors; (2) Ambiguities; (3) Diversity of processes; (4) Representativeness of procedures, definitions and properties in understanding the mathematical concept being studied; (5) Language
Cognitive [IC]	(1) Prior knowledge; (2) Curricular adaptation to individual differences; (3) Evaluation
Interactional [II]	(1) Teacher-student interaction; (2) Interaction between students; (3) Autonomy for the student to explore, formulate and validate the object of study; (4) Formative evaluation
Mediation [IM]	(1) Material resources; (2) Number of students, class schedule and conditions; (3) Proper use of time
Affective [IA]	(1) Interests and needs; (2) Attitudes; (3) Emotions
Ecological [IG]	(1) Adaptation to the curriculum; (2) Intra- and inter-disciplinary connections; (3) Didactic innovation

## RESULTS

To better understand the impacts of reflection, the results of an analysis of descriptors associated with each of the suitability criteria observed in the reflections of the participants is presented, based on the responses to the two questionnaires applied. In addition, a tabular comparison of participation in each of the questionnaires is shown. Questionnaire 1 consists of open questions without guidance, while Questionnaire 2 is the instrument applied after the training, where the indicators of suitability are explicitly indicated.

### Epistemic suitability

#### *Questionnaire 1*

In response to the question related to epistemic suitability, *Have mathematics been taught with acceptable quality?*, most of the participants presented a brief description of

what was observed in Episode 1, highlighting elements such as the problem situation or the moment in the class in which it occurred, but without mentioning elements that relate to indicators of epistemic suitability.

However, two of the participants did identify a mistake made in the presentation of the concept of a logarithmic function; that is, they included elements of IE1 in their reflection. In one of the cases, presence of the error was mentioned in a quite synthesized way; the other participant indicated in greater detail his assessment, including identifying errors in both written and verbal records.

Student A states: *“There were some errors regarding the lack of mathematical symbology; but they do not affect understanding of the exercise. An error that does affect understanding is the indication of an incorrect value of the base of the logarithm; in addition, the set of the Real Numbers is also mentioned as a range, although it is the codomain... The argument of the logarithmic function is not mentioned.”*

It should be noted that although this participant identifies the errors, his assessment in this regard is confusing, since he states that the lack of symbology has no implications regarding the understanding of the exercise. Thus, he manages to point out errors, but describes them inaccurately without analyzing the implications.

### *Questionnaire 2*

In the second intervention, a change in the reflection of the participants related to epistemic suitability is evident. For example, everyone managed to identify the presence of errors from the mathematical point of view (IE1), while in the first intervention only two of them included this element in their reflection. The answers to the question *Are problems given which are mathematically incorrect?* coincide in identifying an error in the concept of a logarithmic function.

In addition, regarding the questions related to the ambiguities indicator (IE2), participants give an evaluative judgment on the lack of clarity of the concepts and procedures addressed. They state:

Student A: *“The table does not correspond to a function, and in addition the logarithmic function is poorly defined.”*

Student B: *“In the definition of the logarithmic function it was said that the base must be greater than one and non-zero, but it can also be a number between zero and one. It is called drawing to the graphic*

Student C: *“The symbol for existence is used incorrectly.”*

On the question, *Are the definitions and procedures clearly addressed?* it should be noted that two of the participants believed that the procedure was clear, based on the lack of doubts by students. However, this assessment is not necessarily correct, since the reasons for not expressing doubt could be different – for example, not understanding linguistic elements, concepts or the same procedure covered in the class, which, as indicated in the theoretical framework, are considered as primary mathematical objects.

Regarding the question, *Are there incorrect statements, or is the way they are presented incorrect?* participants mentioned again the error that arose when defining the concept of a logarithm. It should be mentioned that according to their reflections, they did not perceive a difference between the identification of an error made in a procedure or in a concept, since the answers on these elements were the same.

In the case of the use of metaphors and explanations appropriate for the educational level, also within IE2, most of the participants did not identify problems, and indicated that the teacher's behavior was correct. However, one respondent noted that the use of metaphors contributed to students remembering a concept that was studied, although this does not necessarily imply that they understood the mathematical object. Comments included:

Student A: *“Suitable language for the level is used.”*

Student B: *“The metaphors used are suitable for the concept to be remembered, however, the students do not remember the mathematical object.”*

Regarding the diversity of processes indicator (IE3), three of the participants were able to identify processes such as argumentation and connections within the learning trajectory (concerning processes and sample trajectories), which is different from the first reflection, where these processes are omitted. They mention:

Student B: *“Yes, it is possible to argue when asked why zero and negative numbers have no image, and to establish connections when asked what happens to the exponential function.”*

Student C: *“Yes, it allows argumentation by students.”*

Student D: *“I can identify argumentation and connections.”*

Participants' answers to the questions related to the representativeness indicator (IE4) were related to the epistemic trajectory, discussing mathematical objects such as concepts and propositions. In addition, in one of the participations it was possible to observe an assessment in which the balance between IE4 and IM3 (time investment) is considered, since the convenience of the number of problems developed with respect to time investment is mentioned. It was noted, for instance, that:

Student D: *“Given the amount of time it took; I think it would not be suitable to include more examples than those that were given”*

Although some of the answers to the question: *Are the problems used representative?* differ, it is relevant to mention that at least some reflections regarded the problem as not representative. However, this is one of the model problems proposed in the study program of the Ministry of Public Education of Costa Rica (MEP), and one could assume an ignorance of said program or that in effect it is not representative, although the program suggests it. Students D and C mentioned:

Student C: *“The problem is not representative; it is very simple and could cause errors in the elaboration of contextualized problems.”*

Student D: *“The problem is typical, not representative.”*

Finally, regarding the IE5 (Language) indicator, all participants managed to identify linguistic mathematical objects (verbal and written) such as notations, graphics and concepts. It is important to note that the participants clearly detected these objects in the educational process, and that possibly they were equally clear to participants during their first reflection, although they were not actually mentioned until they were given a guide to reflection which explicitly indicates that it is important. Some of the answers are:

Student A: *“For the concept of logarithmic function, they use graphics, tables, definitions and mathematical language.”*

Student C: *“Verbal, symbolic and graphic representation is used for the logarithmic function when the value of  $a$  is greater than zero. Also, graphic representation for the concept of the asymptote.”*

## **Cognitive suitability**

### *Questionnaire 1*

In response to the question, *Do you consider that students have learned from the proposed assignments?* participants included in their response a brief description of what was observed, but none of the indicators of cognitive suitability are mentioned there. Some observations were:

Student A: *“I think so, because they have seen a usefulness of the logarithmic function. It requires more examples and practice.”*

Student B: *“I think not, because this is just the introduction.”*

Only one of the participants made a retrospective reflection which considered elements of IC1, such as previous knowledge. This shows that intended meanings had a manageable level of difficulty, since the student presented ideas organized according

to what happened in the class episode related to trying to reach a conclusion about components of meaning. This student mentioned:

Student C: *“within the tasks I observed in the video fragment, the exponential equations with different bases required the use of logarithms to be able to be solved, and the calculation of logarithms required using a calculator; in these tasks, students were able to work at least at an introductory level.”*

### *Questionnaire 2*

In the second reflection, indicators of cognitive suitability that were absent in the first reflection were included in the responses. In the case of prior knowledge (IC1), for instance, most participants referred to mathematical objects as arguments, propositions and concepts which are part of the student's previous knowledge. Responses to the question *Do students have the prior knowledge necessary for study the topics presented?* included:

Student B: *“The basic concepts of a function and an exponential function have already been studied, which allows connections to be made.”*

Student C: *“They managed to remember the exponential function and the analysis of graphics of functions.”*

On the other hand, in the case of the learning indicator (IC3) in the teaching trajectory, half of the participants identified an evaluation method that allows monitoring of the appropriation of students' knowledge, consisting of asking questions about the work being done. The other participants stated that an evaluation method could not be identified. This, at least, could be linked to the teacher's evaluation of understanding of the mathematical object (having the institutional meaning implemented and evaluated).

## **Mediation suitability**

### *Questionnaire 1*

In the question related to mediation suitability, *Do you consider that temporary resources, materials, ICTs, etc. have been used adequately?* all participants mentioned elements considered in IM1 in their descriptive responses.

Student B: *“It was apparently planned to use these resources and materials; in addition to calculate logarithms, it was necessary to use the calculator.”*

Student C: *“No; the teacher does not use ICTs. She teaches a lecture class using a whiteboard, and the use of the board is a bit messy; writing an incomplete example, and then writing the subject on the other side of the board. I may have been better to use a calculator.”*

In addition, some respondents included comments on time management in the class, observed in their responses, which correspond to the indicator IM3. They mentioned:

Student C: *“Time management was not observed.”*

Student D: *“The time devoted to the development of the subject is appropriate, the use of a calculator is noticeable, and the topic allows it; perhaps some historical element could have been included.”*

### *Questionnaire 2*

In the second reflection, most participants considered the teaching trajectory, including elements of IE3 such as time availability and use during the class. Regarding the question, *Is the time available sufficient for teaching the intended meanings?*, three students indicated that the time is not sufficient, although one of them mentioned the lack of understanding of the meanings intended during the lesson, and stated that this depended on assessing whether the way in which available time was used was adequate or not. Comments included:

Student A: *“It depends on the meanings that are intended to be taught, but for the introduction of the topic it is fine.”*

Student B: *“It took time, because there were not many examples.”*

Student C: *“I feel that she hurries a little bit because she has little [time] to cover the contents.”*

In addition, regarding the question *Is there an adequate investment of time for more difficult contents?* two participants stated that there was, although in the reflections there was no justification of what contents were to be considered the most difficult, and why this was so.

It should be noted that in both reflections on mediation suitability, participants considered elements of IM1 and IM3 (material resources and time investment), although in the second reflection they are considered in more detail. For example, in the second reflection, one of the participants considered the use of analogies and metaphors, in addition to material and technological resources.

## **Affective suitability**

### *Questionnaire 1*

In the question, *Do the tasks promote student involvement?* which is related to affective suitability, all participants considered elements of IA2 and IA3, referring to attitudes and emotions. For example:

Student B: *“Yes, the teacher asks students about solution strategies and the results that they get when performing procedures, and they actively participate.”*

Student C: *“Yes, because the students were the ones who worked on the introductory problem, and because formulating questions motivated them to participate.”*

Student D: *“Yes, the teacher asked questions about the subject and raised questions that challenged the students.”*

### *Questionnaire 2*

In the second reflection on affective suitability, responses show a consideration of the three indicators: interests (IA1), attitudes (IA2) and emotions (IA3). For example, although their opinions differ, half of the participants considered that there were tasks that were interesting to students and motivated them, while others indicated otherwise, noting that in reality few students seemed to be interested in the subject. In response to the question *“Are the selected tasks interesting to the students?”* they stated:

Student B: *“I feel that they are motivated to work. They also performed the proposed tasks.”*

Student C: *“The content has not been linked to any everyday topic for students. The teacher poses an introductory problem using the transversal axis of financial mathematics. It encouraged student participation, clarified doubts, was addressed well, and students seemed to understand.”*

Then, considering indicator IA2, three participants identified the opening of spaces for student participation, and that the teacher encouraged them to solve and answer the exercise. However, one of the participants stated that in most cases, participation and motivation for the same segment of the class was promoted – that is, the questions were always addressed to the same students:

Student C: *“Sometimes I feel that she only lets the same students speak.”*

It should be noted that all reflections refer to the teaching trajectory, and that although all participants included elements of affective suitability in their reflections, their assessments differ when considering the promotion of participation in situations of equality.

## **Interactional suitability**

### *Questionnaire 1*

In response to the question, *“Is interaction in the class adequate, and does it allow resolving students’ difficulties?”* the participants included descriptions of the teaching and learning trajectories, since they mention motivational work by the teacher and it is

also possible to identify the indicators II1 and II2 within the reflections, referring to the interaction between teachers and students, and between students. For example:

Student B: *“The teacher gets involved in the work students are doing and her management gives them confidence to raise and resolve doubts.”*

Student C: *“Students are in a silent environment, allowing classmates to pay attention and clarify the doubts that are raised. The way in which the lesson is presented also allows work in groups in which doubts are clarified and at the end of the explanation individual doubts are resolved.”*

### *Questionnaire 2*

In the second reflection, three of the participants mentioned elements of indicators II1 and II2. For example, on the question, *“Does the teacher provide an adequate, clear and well organized presentation of the subject?”* an evaluation is made about the need to develop explanations of the concepts in greater detail, and even about the sequential organization of the class:

Student A: *“A sequence is followed, but some concepts were ambiguous, such as the basis of the logarithm.”*

Student B: *“If it had been more organized, a space to graph the exponential function would have been provided and  $x = 1$  would have been included in the table of values from the beginning.”*

Student C: *“An explanation of the theory and further explanation of the concept of a logarithm is missing.”*

Regarding the question, *“Are conflicts in meanings among students recognized and resolved?”*, three participants mention appropriate interaction by the teacher to resolve conflicts in meaning held by students, stating:

Student C: *“The teacher interacts correctly in the process of clarifying doubts and in moments when students participate.”*

However, all these reflections focus on the attitude of the teacher in responding to specific questions that arise during the video sequence, leaving aside other moments in which silence or mistakes could be clearly interpreted as indicating a conflict in the understanding of the concept being studied.

## Ecological suitability

### *Questionnaire 1*

With regard to the question, *Are contents consistent with the curriculum and are they useful for the student's insertion in society and in the labor market?* three of the participants point out elements such as adaptation to the curriculum (IG1) and intra-disciplinary connections (IG2).

For example, one of the participants considered that the methodology used by the teacher in the video was consistent with the methodology suggested by the MEP programs, and also mentioned the social and labor usefulness of the problem and possible interdisciplinary connections.

Another of the participants focused her reflection on the compliance of the activities in the course with certain guidelines established within the curriculum, as well as the use of a contextualized problem:

*Student C: "Yes, thanks to the transversal axis of compound interest, because it is a subject that is very common in daily life of Costa Ricans when it comes to loans. In addition, the lesson responds to the curriculum requested by the MEP for problem solving."*

One of the reflections contains an evaluative judgment about the problem addressed in the class observed, although lack of knowledge of the curriculum in these subjects is also mentioned, so it is possible that for this participant it was more difficult to assess the suitability of the instruction process presented.

### *Questionnaire 2*

The three indicators related to ecological suitability – intra- and interdisciplinary connections and didactic innovation – were addressed in the reflections on this topic. In responses to the question *"Are the contents related to other mathematical contents or contents of other disciplines?"* participants mentioned concepts that were previously studied, according to the curriculum. For instance:

*Student C: "Yes, with other mathematical contents such as functions, analytical geometry and graphic representations."*

In reflecting on the question *"Is educational innovation evident?"* a variety of positions were mentioned. On the one hand, some considered that innovative practices were not present because the class depended on lectures; on the other hand, others

considered using a problem to structure the class as innovation. However, in the light of the methodological proposal of the MEP mathematics programs, this practice should be considered as standard in classrooms.

It should also be noted that the majority of participants indicated having between 1 and 3 years of experience teaching in a formal system, so they should be familiar with the style of teaching used during the class recorded in the video.

### Comparative summary of reflections

Table 2 shows a comparative summary of the elements identified by participants related to each of the indicators of each suitability criterion.

Table 2  
Comparative summary of the suitability indicator elements identified by participants

Suitability	Indicators	Questionnaire 1	Questionnaire 2
Epistemic	Mistakes	The minority identified errors in definition.	Linguistic errors are identified in the written definition of a logarithm.
	Ambiguities	None have indicated observing ambiguous processes or definitions.	Moments where ambiguities occur were not explicitly noted.
	Diversity of processes	The diversity of processes is not included in any reflection.	It was noted that processes of argumentation and connection are present during classes.
	Representativeness	This has not been considered as part of the analysis.	A mostly positive evaluation was made of the representativeness of the exercises and the number of examples, although in answers to subsequent questions, some respondents indicated that there were few examples.
Cognitive	Language	No observation was made about this indicator in the descriptions provided in the first reflection.	In all reflections, students identified different notations and representations of the same mathematical object.
	Previous knowledge	Only one of the participants includes a description with elements related to epistemic trajectory.	In their reflections, all participants cited some of the previous knowledge that the student should have to solve the proposed exercise.
	Curriculum adaptation to individual differences	<i>[Not observed in the video]</i>	<i>[Not observed in the video]</i>
	Evaluation	This indicator was not considered in the first reflections.	Some identified a type of formative evaluation, which consists of asking students verbally about concepts, algorithms and definitions.

Suitability	Indicators	Questionnaire 1	Questionnaire 2
Interactional	Teacher-student interaction	Everyone assessed this type of interaction positively.	The majority assessed this type of interaction positively.
	Interaction between students.	<i>[Not observed in the video]</i>	<i>[Not observed in the video]</i>
	Autonomy for the student to explore	<i>[Not observed in the video]</i>	<i>[Not observed in the video]</i>
Mediation	Material resources	The use of traditional materials in a lecture class is noted.	The use of traditional materials in a lecture class is noted.
	Number of students	<i>[Not observed in the video]</i>	<i>[Not observed in the video]</i>
	Classroom hours and conditions	<i>[Not observed in the video]</i>	<i>[Not observed in the video]</i>
	Use of time	Although not explicitly mentioned, it could be inferred that time management is appropriate.	Most respondents consider that there is no good time management. Elements such as the number of exercises in a given period of time are mentioned.
Affective	Interests and needs	None of the reflections mention elements related to the interests and needs of students.	Some participants believe that no students are interested in the class.
	Attitudes	Some reflections mention elements of this indicator.	The majority of participants consider that efforts are made to improve the attitude of students.
	Emotions	All participants indicate that teachers motivate students well.	The majority of participants believe that the teacher works to motivate students. However, one of the participants indicates that there is no equality, because the questions are always addressed to the same group of students.
Ecological	Curriculum adaptation	An adequate class curriculum is seen, in terms of the methodology suggested by the MEP	The majority of participants state that there is adequate development of the class curriculum, although one participant mentions lack of knowledge in this area.
	Intra- and interdisciplinary connections	All participants agree that the problem addressed in the class allows interdisciplinary connections.	Most participants agree that the problem addressed in the class allows interdisciplinary connections, and some mention intradisciplinary connections. However, one respondent states that it is not totally adequate for addressing concepts.
	Didactic innovation.	Ideas about class innovation are not made explicit.	Participants do not identify innovative practices in the development of the lesson.

Without a doubt, the students managed to mention more elements, and this is explicitly because they have been provided with an order and sequencing about what is important to observe (this is natural in guided reflection).

When comparing the results of the first and second reflections, the main finding is that there is a change in the type of reflection by participants on the situations observed: the depth and complexity of the analyses have increased due to using the second instrument, which is based on guided, organized and directed reflection using the suitability indicators of the ontosemiotic approach.

For example, in the second reflection the participants identify elements such as previous knowledge, use of different notations for the same mathematical object, or the diversity of processes, which according to current theory and the curriculum, play a fundamental role in the instructional process.

Likewise, it should be noted that indicators such as efficient use of time (in the case of mediation suitability) or evaluation (within cognitive suitability) were considered by the majority of participants only in their second reflections, although they are indicators that are usually considered in lesson planning.

It can also be seen that both reflections focus primarily on the teaching trajectory, and to a lesser extent on elements of the epistemic and mediational trajectories. For example, the participants included a greater quantity of elements related to indicators of affective suitability, but having to do with the teacher and the handling of the class. Finally, it is worth mentioning that although some of the elements or conditions considered in the second reflections are different, the fact that preservice teachers ask themselves about their presence or absence, and their convenience or lack, allows them to evaluate classroom practice, and perhaps think of ideas for improving the teaching-learning process.

## CONCLUSIONS

Regarding the first objective, it was shown that participants did not originally have a clear idea about what they should observe when evaluating an instructional process. In addition, there were few tools to engage in reflective practice; in most cases, reflections involved unclear ideas without justification, and were often simply based on beliefs.

For example, in the case of cognitive and epistemic suitability, three of the preservice teachers who participated in the study focused their attention on the class model being developed and the teaching trajectory, mainly provided a general description of the observed events and did not engage in analysis or propose improvements in the instructional process.

In addition, the study on affective and interactional suitability types showed the interest of participants in assessing the types of interaction between the teacher and the students, as well as the influence this has on the motivation and level of student

participation during class, but this was done in general terms without a clear justification of participant judgments.

Regarding the objective of determining the impact of instruction about elementary OSA theoretical notions on reflections on teaching practice, it was possible to identify a change in the depth of the analysis performed by participants, since most of them identified many more elements of interest in mathematical education, and their presentation of their ideas and justifications was clearer and was done in a more orderly fashion.

For example, linguistic elements, representations or processes present during the instruction activities were not mentioned in the first reflection, but they were discussed after the training activity and the use of a guide or guideline to carry out teaching reflection. It is clear that students were aware of these elements and their importance in the instructional process, but they did not actually reflect on them until they were provided with guidance about the importance of doing so (guided reflection).

In addition, in the second reflection, some participants managed to identify situations in which individual needs of the students were presented (viewing the students as persons who think and learn) (Schoenfeld and Kilpatrick, 2008). For example, although some participants judged the attitude and motivation of the majority of students observed in the video in positive terms, others observed in greater detail some cases where lower motivation and interest was shown – i.e., some of the participants made a more detailed analysis of individual differences.

In summary, it has been determined that the appropriate use of a theoretical framework of didactic analysis and guided reflection had an impact on the type of reflection that preservice teachers participating in this research performed, from the description of events (first instrument) to the explanation and analysis of situations (second instrument).

The use of strategies such as the one used in this study, with the support of video sequences, could be considered for incorporation in the teacher training process, since it would provide trainee teachers with an introduction to reflecting on their teaching practice. The reflective practice also represents an effective tool to improve teaching, since it is possible to constantly evaluate the strategies used in the teaching-learning process.

### **Limitations of the investigation**

This study is descriptive and considers a sample of students from a specific curriculum; therefore, the results obtained provide insight only into the chosen sample, and it is not possible to generalize or extrapolate them to other study programs.

Another limitation is the method used for recording information about the reflections of participants, which was done in writing. A deeper investigation, with a wider scope, more time, and the use of other instruments could further clarify teacher knowledge and the way situations are analyzed.

Finally, a limited amount of time was available to instruct participants about the theoretical model used. More extensive and detailed training would allow participants to have a better understanding of the scope that the application of the OSA theoretical and methodological tools can have.

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## AUTHORS' CONTRIBUTIONS STATEMENTS

Both authors (D.A.R. and Y.M.L.) participated equally in all stages of the research process, as well as in the creation, writing and correction of the article.

## DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by corresponding author D.A.R., upon reasonable request.

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