

Mathematics teachers' learning on Exploratory Teaching: analysis of a Multimedia Case in a Community of Practice

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ABSTRACT

This article aim is to investigate what professional learning regarding Exploratory Teaching (ET) perspective was revealed by Mathematics teachers, in a Community of Practice (CoP). In a context intended to promote teachers' professional development, a multimedia resource integrating real classroom situations was used to promote discussion and learning. Qualitative research has been carried out with an audio recording of the group meetings and the written productions elaborated by the teachers. The learning related to the Exploratory Teaching perspective revealed by the teachers is associated to the actions and roles of the teacher, the student's role, classroom management and the relevance of lesson planning in teacher's practice. The involvement in the CoP favored teachers to rethink and question some of their actions during the lessons and to notice essential aspects of ET perspective and to connect them with their experiences from the classroom. The results show that the constitution of a CoP around the exploration of a multimedia case in professional development contexts can be a facilitator for the learning of its members.

Keywords: Exploratory Teaching; Mathematics Teacher Education; Community of Practice; Social Theory of Learning; Multimedia case.

Aprendizagens de Professores de Matemática com relação à Perspectiva do Ensino Exploratório: Análise de um Caso Multimídia em uma Comunidade de prática

RESUMO

Este artigo tem como objetivo investigar que aprendizagens profissionais, com relação à perspectiva do Ensino Exploratório, foram manifestadas por professoras de Matemática, numa Comunidade de Prática. Nesse contexto com intenções formativas, foi utilizado um recurso multimídia constituído por situações reais de sala de aula, para promover as discussões e as aprendizagens dos professores. Trata-se de uma pesquisa qualitativa com características de pesquisa intervenção. As aprendizagens relacionadas à perspectiva do Ensino Exploratório, manifestadas pelas professoras, estão associadas: às ações e aos papéis do professor, ao papel do aluno, à gestão da aula e à relevância do planejamento na atividade do professor. A participação em tal Comunidade de Prática permitiu às professoras repensar e questionar algumas de suas ações durante as aulas e a

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perceber (*noticing*) aspectos essenciais desta perspectiva, relacionando-os com suas experiências em sala de aula. Os resultados revelam que a constituição de uma Comunidade de Prática em torno da exploração de um caso multimídia na formação de professores podem constituir-se catalisadores de aprendizagens de seus membros.

Palavras-chave: Perspectiva do Ensino Exploratório; Formação de Professores de Matemática; Comunidades de Prática; Teoria Social da Aprendizagem; Caso multimídia.

INTRODUCTION

The constant changes in our society require the teacher to accept new roles in the teaching and learning processes, leaving aside the idea of simply transmitting knowledge and assuming the role of guiding students learning. The teachers are primarily responsible for organizing the pedagogical practice, selecting or elaborating the tasks they will propose in class, the contents that will be addressed, and finally, for creating an environment for students to engage in challenging mathematical activities (Stein et al., 2009).

According to Van Es and Sherin (2010), the institutions in which teachers are inserted are often resistant to the adoption of alternative teaching perspectives (those contraries to direct teaching), because they are not prepared to implement such perspectives. Therefore, we consider that it is necessary to offer new contexts for teachers' development in which they can take an active role in the production of their knowledge, engage in reflections that allow the attribution of meaning to their experiences and learn to teach in other ways (Sherin et al., 2009).

In this article, we analyze the learning related to the Exploratory Teaching (ET) perspective revealed by a group of teachers who participated in a professional learning program based on the constitution of a Community of Practice - CoP (Wenger, 1998). The expression "exploratory teaching" has been used to describe an approach that differs starkly from traditional teaching which is posited on knowledge transmission effected by the teacher who explains the content after which the students practice by applying the concepts and procedures that were taught. The analysis of the mathematics teacher education program, in which teachers play the role of protagonists of their learning, can promote reflections on actions considered as challenging and demanding for teachers in conducting teaching according to a ET perspective. In the program analysed in this study, we used as a tool the multimedia case *Phone Plan*¹, elaborated in the context of a research project.

In the following sections, we discuss the learning that took place in a CoP based on the exploration of multimedia cases in teacher education that assumes a ET perspective in mathematics classrooms. Next, we present the theoretical background of the study, the methodological framework, the results and the conclusions.

¹ This case is part of a multimedia resource developed in a cooperation project between the Londrina State University - UEL and the University of Lisbon - UL, funded by CNPq, which can be accessed electronically on an *online* platform through *login* and password. Available at: <http://www.rmfp.uel.br>

COP LEARNING AND EXPLORATION OF MULTIMEDIA RESOURCES

CoPs have become a fertile learning environment for teachers in the professional development process, as evidenced by several studies (see e.g. Cyrino & Baldini, 2017; Cyrino, 2016c; Graven, 2004; Nagy & Cyrino, 2014; Tinti, 2016, Warner & Hallman, 2017). A CoP may be characterized as “a group of people who interact, learn together, build relationships, and in the process develop a sense of belonging and mutual commitment” (Wenger, McDermott & Snyder, 2002, p. 34).

According to Wenger (1998), a CoP is the social fabric of learning, and it is a space where to explore the negotiation of meanings as a mechanism for learning (Cyrino, 2009). Thus, learning is a consequence of a process of negotiating meanings, which involves the interaction between the processes of participation (“belonging to” or “being a member of” a CoP) and reification. Participation is a “complex process that combines doing, talking, thinking, feeling, and belonging. It involves our whole person, including our bodies, minds, emotions, and social relations” (Wenger, 1998, p. 56). Reification refers to the “process of giving form to our experience by producing objects that congeal this experience into ‘thingness’ (Wenger, 1998, p. 58). It is through this process that we project our meanings into the world, thus creating focal points around which the negotiation of meanings organizes itself. According to Garcia (2014, p.53-54),

the process of reification refers to the manifestation of our experiences in a kind of instant “portrait”. The image printed in the portrait gives visibility to the experience lived at that moment, and becomes a reference to represent it, but it cannot reveal the very experience. Although the portrait is always the same, each time we use it to talk about the experience itself, there will always be something new that will catch our attention or something that no longer seems to us as important as before, producing new meanings.

For Wenger (1998), the participation expresses the active aspect of the learning process, while reification makes it possible to establish reference points of the negotiation of meanings’ trajectory. In CoPs, participation and reification are inseparable, distinct but complementary processes that interact, and thus one does not exist without the other, but one does not replace the other, and together they are essential to learning. A change in the relationships of these processes is never neutral, as it always transforms the possibilities aiming to negotiate meanings that structure learning.

As with CoPs, several studies point to the potential of using multimedia resources, especially the video media, as a tool for teacher learning in professional development contexts (Borko et al., 2014; Calandra & Rich, 2015; Cyrino, 2016a; van Es & Sherin, 2008, 2009, 2010). These studies show that the exploration of video media can inspire teachers to reflect on the complexity of interactions in the classroom, associating them with their own pedagogical experiences. A multimedia case may consist of classroom

videos, but also of other resources such as lesson plans, students' written productions, interviews with the teachers or students and problematizing questions, which are often made available online (Cyrino, 2016a; Hatch & Grossman, 2009).

Exploring a multimedia case in teacher education can assist teachers to: improve their pedagogical thinking and reasoning, and think about the challenges surrounding the practice; engage in reflections that make sense of experiences meaningful and use their knowledge to make future decisions; promote discussions about teaching and learning, math knowledge and teaching practices (Smith et al., 2014); have contact with a another way of teaching and “to help teachers develop a vision of what is possible, a vision that may often be quite different from the kinds of teaching practices teachers typically have the opportunity to see” (Sherin et al., 2009, p. 215); develop their capacity of “*noticing*” (Rodrigues, Cyrino & Oliveira, 2018).

According to Van Es and Sherin (2008), the ability to *notice* is related to the teacher's professional view and contemplates three aspects: identifying what is important in a classroom interaction; interpreting meaningful aspects of interactions by making connections between them and the broader principles of teaching and learning they represent; and using what they know about context to reason about situations. Jacobs, Lamb and Phillip (2010) also include the action of planning responses to the teaching situations observed as part of the ability of *noticing*, and argue that, in addition to recognizing what is important in the classroom and interpreting meaningful situations, the teacher himself must decide how to respond to the challenges proposed.

Video media is also considered as a means for teachers to know alternative teaching perspectives (Sherin et al., 2009). Among them, we highlight the ET perspective, which guided the multimedia case used as a training tool in the CoP investigated in this article.

THE PERSPECTIVE OF THE EXPLORATORY TEACHING

The ET perspective has its origins in a broader *inquiry-based teaching* perspective (Artigue & Blomhoj, 2013; Oliveira & Cyrino, 2013), which refers to inquiry-centered and student-centered teaching, in which the pupil can elaborate issues, explore situations and develop their own strategies to solve them, i.e. “the students construct meanings, meaningful learning takes place in a social context, learning is supported by meaningful contexts and learning is a dialogic process” (Engeln, Euler, & Mass, 2013, p.824). According to Chapman (2013), *inquiry-based teaching* approaches are rooted in John Dewey's learning philosophy and Gordon Wells's studies on *dialogic inquiry*.

From this perspective, students can engage in working with cognitively challenging tasks² that mobilize them to build knowledge supported by understanding. Despite having

² Cognitively challenging tasks are tasks that have a high level of cognitive demand, that is, have the potential to engage students in an activity that triggers complex ways of thinking, mobilizing student independent work (Smith & Stein, 2013).

some dissemination in the international scenario (Canavarro, Oliveira, & Menezes, 2012; Engeln, Euler, & Mass, 2013), it is still a challenge for most teachers in Brazil (Cyrino, 2016b), because its implementation requires the teachers to play different roles from those they are used to perform in directive teaching. This perspective involves some aspects that are essential to its development: (i) tasks must be cognitively challenging; (ii) the management of the class, which involves both student's and teacher's roles and the way tasks are worked in class; and (iii) the communication that takes place during the completion of the task (Ponte, 2005).

Tasks should favor “mathematical reasoning about important ideas and attribute meaning to mathematical knowledge” (Canavarro, Oliveira, & Menezes, 2012, p.256); “offer students the opportunity to extend what they know and stimulate their learning” (Shimizu et al., 2010, p. 8). For Stein, Grover and Henningsen, (1996, p. 456), proposing and implementing cognitively challenging tasks in the classroom have the potential to mobilize students “to discuss their ideas with one another, where intellectual risk-taking is nurtured through respect and valuing of student thinking, and where sufficient time and encouragement is provided for exploration of mathematical ideas”.

Regarding classroom management, the teacher needs to create an environment where students can actively work, “discuss and share ideas, regardless of whether they are incomplete, confusing or even wrong” (Menezes et al., 2013, p.49). Cyrino and Teixeira (2016), inspired by Stein et al. (2008), present a framework that contemplates actions that can help the teacher when planning class, and put them into practice during the development of the class, which are: anticipating; proposing the task; monitoring students' work; selecting and sequencing students' solutions for discussion; discussing solutions and systematizing learning. It is important to highlight that these actions are not exclusive from the ET perspective and, therefore, can be contemplated in other alternative forms of teaching.

In ET, students are invited to play an active role in working with the task, to interact with teacher and peers, to justify their strategies when solving problems, so that everyone can understand their ideas, and they must be willing to listen to the solution of the others. As for the teachers, they are the mediators of the students' learning. Thus, it is essential that they have a deep knowledge of the task they will propose to students, solving it in several ways, anticipating different strategies and representations.

Communication permeates the whole class developed in this perspective, as according to Yackel et al. (1999, p.59) “by verbalizing their thoughts through dialogue, by trying to interpret and make sense of their colleagues' verbalizations, students favor communication and consequently their learning”. Thus, the interactions and dialogue between students and between teacher and students, enable students: to engage in mathematical activities; negotiate meanings; make sense of new ideas; organize their thinking; and establish connections between mathematical ideas and build knowledge. For Smith and Stein (2013), communication must also happen in writing, graphic representations and drawings. Communication is seen as “a dialogic process, the meanings that are made by speakers and listeners or writers and readers with respect to individual

utterances are strongly influenced by the discourse context in which they occur” (Wells, 2004, p.106).

Thus, in a mathematics class from a ET perspective, the teacher is required to encourage the students to think, reason, solve problems, and communicate mathematically from their engagement with cognitively challenging tasks (Smith, 2001). Therefore, they must be prepared to deal with different (correct and incorrect) task-solving strategies, and align them with predetermined learning objectives. Although the teacher cannot anticipate everything that will happen, developing a lesson plan can help them devise actions that will assist them on not losing focus on their goals for the lesson.

RESEARCH CONTEXT

Our study was conducted in a professional development context that was characterized as a CoP, named by its members as the *Mathematics Teachers Community of Practice, Analyzing and Reflecting on Mathematics Teaching* (CoP-ProfMARE). To investigate what professional learning related to the ET perspective are revealed by mathematics teachers when analyzing a multimedia case in the context of the CoP-ProfMARE, 45 one-hour meetings were held over a period of 15 months, led by two teacher educators³. For the first eight months, the meetings were held weekly at the school where the teachers Ana⁴, Luísa and Mariana worked, in the city of Arapongas - PR. In the last seven months, the meetings took place fortnightly, in another public school in the same city and another teacher (Maisa) joined the group. The teachers taught in middle school, with 7th and 9th grades classes, with the exception of Mariana, who also taught in high school classes. All teachers were graduated in Mathematics, had postgraduate degrees in the area of Teaching or Mathematics Education.

Throughout the meetings, we used as a resource the multimedia case *Phone Plan* (Figure 1)⁵ which was available online for teachers.

³ The first author of this article and the researcher Julio Cezar Rodrigues de Oliveira.

⁴ The names of all participants are fictitious to preserve their identities. The research project was approved by the Ethics Committee of the Londrina State University. Opinion number 276.536.

⁵ Task translation: Julia wants to contract a phone plan, and, when researching, she found two companies that offered her the following plans:

Company 1: R\$0,30 per minute spoken, without any fixed fee.

Company 2: R\$0,15 per minute spoken, plus a monthly maintenance fee of R\$12,00.

If Julia speaks 110 minutes per month in average, which company offers the best price? Justify your answer.

Livia also wants to contract a phone plan. Given that she speaks 60 minutes per month in average, which company offers her the best price? Justify your answer.

For each company, find a mathematical expression that represents the amount (y) charged according to the minutes spoken (x). Represent graphically, in a same Cartesian plan, the plans of each company.

Comparing the plans of company 1 and company 2, is there any quantity of minutes spoken that will make the choice indifferent? Justify your answer.

Which plan would you choose? Why?

CASOS MULTIMÍDIA TAREFAS GEPEFOPEM PECEM CONTATO LOGOUT

Caso Multimídia 2: "Plano de Telefonia"

Introdução Antes da aula A aula Reflexão após a aula O Geogebra Colocar em prática

A tarefa

Planejamento da aula
Quadro síntese

A tarefa

Tarefa - O plano de telefonia

Júlia quer contratar um plano de telefonia, e fazendo uma pesquisa, ela encontrou duas empresas que oferecem os seguintes planos:

Empresa 1: R\$ 0,30 por minuto falado, sem nenhuma taxa fixa.
 Empresa 2: R\$ 0,15 por minuto falado mais uma taxa de manutenção mensal de R\$ 12,00.

a) Se Júlia fala em média 110 minutos por mês, qual das empresas oferece menor preço? Justifique sua resposta.

b) Livia também quer contratar um plano de telefonia. Sabendo que ela costuma falar 60 minutos em média por mês, qual das empresas oferece menor preço para ela? Justifique sua resposta.

c) Para cada empresa, indique uma expressão matemática que represente o preço (y) cobrado de acordo com os minutos falados (x).

d) Represente graficamente, num mesmo plano cartesiano, os planos de cada empresa.

e) Comparando os planos das empresas 1 e 2, existe alguma quantidade de minutos falados que tornará indiferente a escolha entre elas? Justifique sua resposta.

f) Qual plano você contrataria? Por quê?

Figure 1. Phone Plan Multimedia Case Fragment. (<http://rmfp.uel.br>)

This case presents a mathematics lesson⁶ developed from the ET perspective with a first-grade high school class, in a public school in Brazil, in which students had the opportunity to explore the concept of function, especially in the case of affine functions, with or without using the *GeoGebra* software. The case is organized into six sections that contemplate different media, and guide the teachers' exploration on the online platform: (1) *Introduction of the multimedia case*, which contains information about the school, the class, the teacher of the case (teacher Loreni) and indications on the use of the case; (2) *Before class*, with the statement of the *Phone Plan* task, the teacher's lesson plan, excerpts from the audio of the interview with the teacher before class and questions that problematize what was presented (problematizing questions); (3) *The class*, which integrates the video excerpts⁷ allowing to observe teacher and students' actions in the classroom, the students' written productions with the solutions of items a, b, c of the task and problematizing questions related to the episodes and the ET perspective and synthesis table (for teachers to register the teacher's actions discussed as they explored the case); (4) *Post-class reflection*, presents excerpts of the audio interview with the teacher after the class, problematizing questions regarding the interview and the framework for ET (Cyriano & Teixeira, 2016), (5) *GeoGebra*, consisting of four video excerpts related to

⁶A more in-depth account of this class is found in Baldini (2016).

⁷ These videos are related to the ET phases. There is a video for the "proposition and task presentation" phase; five for the "class development" phase; four for the "collective discussion" phase and two for the "systematization" phase.

some task items developed with the use of this software, problematizing questions related to these episodes and the use of *GeoGebra* as a resource to accomplish task items (d, e, f) and excerpts of the audio interview with the teacher after the class, and (6) *Putting it into practice* appears as an invitation to teachers to plan and implement a lesson from the ET perspective.

RESEARCH METHODS

During the CoP-ProfMARE meetings, the teacher educators developed a shared action in the organization of the tasks⁸, in order to mobilize the teachers to an active role in the construction of their knowledge. They acted as members of the Cop in the preparation of the tasks, in the dynamization of the group, seeking to support them in their formation process, and as researchers, in the effort to broaden their knowledge and understanding in relation to the theme investigated.

Throughout the development of CoP-ProfMARE, the different media available in the case (lesson plan, excerpts from class videos, interviews, problematizing questions, students' written productions) were explored by the teachers. The discussions that took place at Cop-ProfMARE were audio recorded and transcribed so that the teachers' statements could be captured in their original form, maintaining the integrity of the dialogues. However, in transcribing some episodes, we corrected errors of subject-verb agreement, prepositions, vices of the language, taking care not to alter what they meant to say.

For the analysis of the information collected, we used: the transcripts of the episodes that show the negotiations of meanings in group meetings (GM); the written productions elaborated by the teachers (WP); and the researcher's field journal (FJ) notes. In order to identify the learning revealed by the teachers regarding the ET perspective, we made a thorough reading of the information contained in these instruments, adopting an interpretative analysis, pattern identification and coding.

In this study, we organized the analysis based on the teachers' negotiations of meanings during the exploration *The Class* section in the multimedia case, according with the four phases of a class from the ET perspective: *proposition and presentation of the task, task development, collective discussion and systematization*. To identify the information described in the analysis, we used the fictitious name of the person providing the information, followed by a code that identifies the instrument (GM, WP or FJ) and the date it was obtained. Then, we discuss the learning related to the ET perspective, manifested by the teachers, associated with the teacher's actions and roles, the student's role, the classroom management and the class planning.

⁸ In the CoP-ProfMARE we solved and analyzed the Phone Plan task and others tasks proposed by members of the CoP.

COP-PROFMARE NEGOTIATION OF MEANINGS IN THE EXPLORATION OF THE 'PHONE PLAN' MULTIMEDIA CASE

The exploration of the Phone Plan multimedia case occurred sequentially, aiming to allow the teachers to elaborate their learnings about the ET perspective. However, during the exploration, the teachers were free to resume the other sections and subsections when they felt it necessary. Considering that the negotiation of meanings is a learning mechanism, the work with the multimedia case happened as follows: first the teachers explored the media individually, for example, they analyzed an episode of the *proposition and presentation phase of the task* (*The Class* section) and, at the next meeting, they socialized and shared their ideas and reflections with the other CoP participants, holding a collective discussion. The teacher educators had access to the answers elaborated by the teachers to the problematizing questions before the meeting to socialize the ideas, so they could analyze them and elaborate inquiring questions for the moment of the *collective discussion*.

Next, we present the negotiation of meanings triggered by CoP-ProfMARE in the exploration of the section *The Class* multimedia case, focused on the four phases of the class.

Negotiation of meanings regarding exploration of *The Class* section

The class section is organized around the phases of a class from the perspective of the ET, namely: *task proposition and presentation*, *task development*, *collective discussion and systematization*. In analyzing and discussing the first phase, the teachers highlighted an issue regarding the reading of the task's statement.

Mariana: Most of the times I read. Because usually students do not read it, and they already say they do not know how to do it. Sometimes I don't read it and tell them, "Today I won't read it, I want you to try to understand on your own" But I can't wait, I end up reading and telling them what to do.

Ana: So, we get the students used to expect for us to do the reading. We don't give them the time to think and they know it, so they don't. I think that is wrong.

Luisa: I read too. But when I saw this "task proposition and presentation" video, I wondered whether in the interview [the teacher] Loreni had said that she would do the reading or not. I went back to the interview and what caught my attention was that she justified that she would ask a student to read, precisely so that she did not emphasize some keywords in the task. Because that's it, we have this habit of reading and highlighting the keywords in the task.

(GM, session 17)

Reading the assignment and highlighting the keywords in the task's statement seems to be a common practice of these teachers, allowing us to infer that they often underestimate their students' ability to read and interpret the proposals. Ana recognizes

that this action can prevent students from thinking for themselves. By reading the assignment, emphasizing the keywords, the teacher can unknowingly eliminate the challenging aspects of the task and thus prevent the student from developing complex ways of thinking (Stein et al., 2009).

Regarding the *development phase of the class*, after analyzing the episodes and answering the problematizing questions, the participating teachers of the CoP pointed out some actions of the protagonist teacher in the multimedia case and highlighted the relevance for this phase of the lesson plan she had elaborated.

Mariana: Loreni spent all the time coming to the [different] groups, supporting ... , as she said before, "monitoring" the students.

Ana: [...] She didn't give the answer to the students. She was questioning them. She assisted students, monitored, relied on their ideas to make them reflect or going forward on the task.

Luisa: She encouraged the students to do the task. She asked questions to see how the students were thinking, and if they were on a coherent line of thought, if they were attending to what the exercise was asking for. She would give no answers, she monitored the work in the groups.

Mariana: Now it's clear why she mentioned in the plan the strategies that the students could conceive, so she already had an idea of what she was going to find. Because then the plan is a basis for you to teach the lesson. We can't plan everything, though; there are things beyond our control.

(GM, session 18)

The teachers identified some of Loreni's actions during the class development phase and added the term "monitor" to the CoP-ProfMARE repertoire. Mariana acknowledged (*reified*) the importance of the lesson plan and argued that it serves as support to the teacher, although it is not possible to predict everything that will happen (Smith & Stein, 2013). They realized that, in the lesson plan, it is relevant to record different ways of solving the task, probable questions to be asked to the students and anticipating possible questions from the students; they emphasized that teacher Loreni did not answer the students' questions directly, but sought to understand the way how they were thinking and used questioning to help them solve the task so as to value the ideas developed by the students (Chappin, O'Connor, & Anderson, 2003).

In the development of CoP-ProfMARE, Mariana assumed that she had difficulty managing the students' work on the task without giving them the answers and to give them the time they needed to work out the solution, and also that she interfered and performed the task for them.

Mariana: Guys, I have a hard time with it. It's bad, because some students do like to think, to develop the task, and suddenly I give them the answer. Then the student says: " teacher, it is not interesting anymore, you did it already, didn't you?!". But it's hard to wait for everyone to finish the task in the room; if you do it, you won't progress.

Teacher Educator: But do we need to wait for everyone to finish?

Ana: For most of them, we should, but for all of them, it is difficult. I think the teacher must evaluate the moment to start correcting the solutions. That is why Loreni has set a time for each phase in the planning.

Lúisa: I think this issue of time is very cool. By monitoring the groups, you are aware of their progress on the task.

Ana: If you monitor the groups, you already know what is going on. You (refers to Mariana) are still in conflict with the traditional class, what we are seeing here has nothing to do with a traditional class.

(GM, session 20)

By explaining her weaknesses, Mariana showed confidence in the other members of the CoP. Her actions did not give their students the opportunity to take responsibility for their own learning, which is an essential concern in *inquiry-based teaching* perspectives (Chappin, O'Connor & Anderson, 2003). The teachers recognized the importance of establishing in the lesson plan a period of time for student to accomplish the task (Smith & Stein, 2013) and, generally, a period of time for each phase of the lesson. Providing adequate time for students' work on the tasks is critical, since "by offering too much time to solve the task, the student can disperse, but when insufficient time is given, this work can be compromised" (Stein et al., 2009, p.16).

Developing teaching from an ET perspective can pose a number of obstacles to the teacher, as it requires unfamiliar skills from the ones needed in traditional classes (Sullivan et al., 2015), such as: supporting the students in their autonomous work, using questions that mobilize them to think about and justify their ideas, and not validate their answers. In exploring the multimedia case, the teachers carefully observed the situations presented, making them meaningful, thus contributing to the development of their capacity of *noticing* (Van Es & Sherin, 2002). In general, we can infer that they realized that teaching in a different way from the traditional approach is possible, and began to adopt in their practice new actions such as: monitoring students' work without solving the task for them, and giving student enough time to work on the task. This can be observed through the account of Mariana's experience with a new task she proposed in her class.

Mariana: I gave them the "Flight-of-the-geese" task for the 7th grade. I made the reading, and asked if anyone had any questions. They have worked in groups. I walked around the room assisting the groups, holding my tongue not to give the answer or help the students too much. They talked to each other, and I kept watching, questioning. If anyone needed, I would help. And I kept holding myself as not to interfere too much. And you know, the class was a success! It was super cool! The students liked it. I just didn't get to the systematization. But I asked some students to present their work, and I asked questions. (GM, session 20)

Teacher Mariana began to reflect on her pedagogical practice and felt challenged to implement a class from the perspective of ET. In each meeting, she showed to be

increasingly excited about her experiences in the classroom. At one of the CoP meetings, Mariana made the following statement:

Mariana: I am finding this [experience of] thinking about the way we teach very interesting. I usually work a lot with exercises, lots of content, and keep my practice only on the drill. I tell them how to do it, I don't let the students to find out [the solutions] for themselves. And all this [in the CoP] is making me think. What we see here is that the students must be active, and the teacher just needs to be attentive to check if they are on the right track, or to help them get to a path. (GM, session 28)

Exploring the Phone Plan multimedia case helped teacher Mariana reflect on her way of managing teaching and learning processes. She began to rethink and change her role in the classroom, reifying the idea that students can work in groups and are able to solve the task without her direct intervention. By sharing her experiences, Mariana encouraged the other CoP teachers to implement classes from the ET perspective. So, Luisa shared her experience with a new task in the classroom.

Luisa: It was fine, but it was funny. I gave them the task, asked a student to do the reading, and explained the class that they should write down as they thought. Then, I was monitoring the groups, asking questions, [when] the student asked something, I would return with a question... until I arrived in a group and a student asked a question, then I made another one aiming to have her looking at the task and realizing what was going wrong. The student insisted on the question and I insisted on returning to her with another question, until another student in the group said: "Can't you see that the teacher herself can't answer it, that is why she isn't answering you?!" ... I explained that I didn't want to give the answer because I wanted them to think by themselves. I asked another question and let the group think. I got away, then I came back, but I saw that they were not very much comfortable with it. ... I think only with time we gain experience with this way of teaching [but] I feel it was very productive. (GM, session 24)

Luisa's account of her experience corroborates the claim that students, just like some teachers, do not know how to deal with alternative forms of teaching (Chapin, O'Connor, & Anderson, 2003). Students are so used to the traditional mode, in which the priority is the repetition of procedures and memorization, that they mistakenly associated the teacher's action of questioning them with a probable lack of knowledge. Smith and Stein (2013) argue that it is complex for the student to work in a way he/she has never experienced before and it is therefore essential that the teacher gradually builds a classroom culture that privileges the ET perspective, assisting students to assume different roles from those they had previously played in directive teaching (Engeln, Euler, & Mass, 2013).

Luisa's statement about the need to gain experience with this form of teaching is in line with the idea that the transition from directive teaching to teaching from the ET perspective can only happen gradually, so that teachers and students feel comfortable with it (MacGraw, 2002). Since this perspective represents a new and complex classroom situation for teachers and students, it takes time for them in making a transition to such a new classroom culture.

Therefore, the teachers legitimized CoP-ProfMARE as a formative space for learning, and the ET perspective as a way of teaching that can be implemented in their classes. In her notebook, Luisa stated that joining the group allowed her to learn many "things" about the teaching profession, namely how to act and to deal with certain difficulties in the classroom. She also emphasized that as the other CoP members had more classroom experience, this allowed her to share doubts and anxieties during the meetings.

In exploring the *collective discussion phase*, before analyzing the episodes, we asked the teachers to select and sequence students' solutions for the Phone Plan task and write about the importance of this phase of the class. Ana and Mariana wrote that the intention is to:

Compare the different students' strategies and helping students who did not reach the solution to find a path, understanding the task. (Ana, WP, session 19)
Show the different strategies used by the students. Discussion [phase] represents one more chance to know what the student is thinking, if they really reveal understanding. The ones who did not understand, through the colleague's explanation and also through the teacher's questions may come to understand. Therefore, I think collective discussion is a very important phase in Exploratory Teaching. (Mariana, WP, between sessions 19 and 20)

For these teachers, the purpose of the *collective discussion* phase was to compare students' solutions. However, they indicated other aspects that can be explored during this phase, such as helping students who failed to carry out the task to understand it. Mariana highlighted an important feature of this phase, "the discussion is one more chance to know what the student is thinking" (WP, between sessions 19 and 20). However, it is essential that the *collective discussion* phase is not limited to a mere presentation of students' solutions, but that the teacher mobilize students to "learn to communicate their ideas by making public their thinking and to evaluate their own mathematical ideas and those of others" (Smith & Stein, 2013, p.1).

The teachers pointed out that the use of questions by the teacher to conduct this phase of the class helps students clarify what they did and highlight aspects of the task that had gone unnoticed (Sullivan & Mousley, 2001). Regarding the action of selecting and sequencing the students' solutions, the teachers stated that this was the most "difficult" action of the enterprise, as they had doubts in choosing the ones that could be the most

appropriate for the discussion. After exploring the episodes, we discussed what they analyzed.

Luisa: [...] The collective discussion phase is intended not only to compare the different solutions, but to make connections between them, highlighting the important concepts to the class. Motivate students to recognize different procedures.

Ana: After we chose the productions, I realized the difficulty I had, but now I realized that if I had looked at the lesson objectives in the plan, it might have been easier to choose the productions, because the whole lesson is revolving around those goals.

Luisa: She (Loreni) chose the different strategies thinking about the goals. In fact, Loreni had to focus on the class objectives all the time, both in the plan design phase, in the class development, now in the collective discussion.

Ana: That's why she couldn't choose simple any solution. [...] she needed solutions that would help to discuss the concept of function. If she had chosen a students' solution that did not match what she wanted, it made no sense. How was she going to do the systematization? [Thus] You could not establish the concept of [affine] function. (GM, session 24)

The participating teachers recognized the importance of the teacher's practice to focus on the objectives of the lesson when selecting and sequencing students' solutions for *collective discussion*. Ana's reliance in CoP members allowed her to expose her difficulty in choosing students' productions, justifying that she did not consider the lessons objectives. For Smith et al. (2014), it is not important if there is a right way to select and sequence students' solutions, as these actions depend largely on the goals that the teacher outlined for the class. Luisa wondered if the *systematization* phase could not happen simultaneously with the *collective discussion phase*.

Luisa: In the collective discussion phase, can we no longer systematize together?

Ana: I think it can be together and it can be separated. I think it depends on the teacher. I think the systematization is mostly the teachers' part.

Mariana: I think it makes no difference in doing both phases together. But the teacher needs to be very attentive in that. (GM, session 24)

During the analysis and discussion of this episode within the CoP, the teachers reified their idea of the *collective discussion* phase, understanding that it goes beyond the comparison of the different strategies (Sullivan & Mornane, 2014). They evidenced that, in this phase, the teacher needs to value the students' ideas, making them responsible for what is being studied. For Smith and Stein (2013, p.2), "students learn when they are encouraged to be the authors of their own ideas and when they are held accountable for reasoning and understanding key ideas", something that the *collective discussion* can provide for students.

The teachers also recognized that, in the *collective discussion phase*, although the teacher and students need to work together, at this point it is up to the teacher to have a “more intense and careful” participation, making the necessary interventions (Stein et al., 2008). This is why it is essential, at this stage, that the teacher maintains a balance between his/her actions and those of the students. Mariana drew attention to the student’s posture during this phase, that should be respectful among peers (Sullivan & Mornane, 2014), so that she is able to organize her ideas and to communicate them. For Borko et al. (2014), the discussions that take place in the classroom are intentionally constituted through students’ thinking, which is another challenging task for teachers.

As for the *systematization phase*, the teachers associated it with the actions of summarizing, synthesizing and summing up what was studied. It is a moment when the teacher systematizes the concepts, ideas, or procedures involved in the class, allowing students to consolidate their learning.

Teacher educator: In your opinion, what is the systematization phase?

Mariana: To summarize, to sum up everything that was seen in class.

Ana: Sum up all the ideas they came up with. Unravel all content.

Mariana: Systematization is an organization of ideas, from thinking to something more general.

Ana: Using what was studied in any context, if the students understand they can use that concept in another context. For me, it is the greatest difficulty that students have. Using what they have learned in other tasks.

Luisa: It’s our fault, because we stick only to tasks that are not connected to anything. Only algorithm repetition. So, it is actually hard for the students to take what they have learned to other tasks. (GM, session 25)

In the *systematization phase*, it is recommended that the teacher use the resolutions explored in the *collective discussion phase* where relevant “mathematical ideas are brought to the surface, contradictions are exposed, and understandings are developed or consolidated” (Smith & Stein, 2013, vii). At this time, the teacher can help the student establish connections with previous learning, and show the possibility of employing the knowledge constituted in other contexts by using examples or new tasks. Using what you have learned in other situations, according to Ana, is the hardest part for students. Luisa stated that the teachers themselves are responsible for that difficulty for only privileging tasks with low level of cognitive demand (Stein et al., 2009). Therefore, it is important to propose tasks that allow different connections to be established (with other contexts, with previous and future learning).

After exploring the episodes of the *systematization phase*, we asked the teachers if it was possible for teacher Loreni to have performed this phase without going through the *collective discussion*. Mariana answered she could have, but then the *systematization phase* would not have been so satisfactory. Luisa argued that when the teacher does not make the *collective discussion*, this can be compared to when they give the students a task, and at the end of the class they simply correct it, as in traditional teaching. The students

would probably assume only the teachers' solution and as she states "some will erase what they did, thinking they are wrong" (Luísa, WP, between sessions 28 and 29) and their thinking would not be valued (Sullivan et al., 2015). Luísa reflected on the collective discussion phase, highlighting the dialogical characteristic of the ET perspective: "the collective discussion phase is what differentiates this perspective from the others, because at this moment the student builds their knowledge by dialoguing with the teacher and with the peers" (Luísa, WP, between sessions 28 and 29).

The teachers pointed out some significant actions of the students in the *collective discussion* phase, such as participating actively; establishing connections between the students' solutions and the targeted content; organizing their ideas to be clear about what they have done; justifying their solutions and explaining their thinking so they can communicate what they have learned. They realized (reified) that the *systematization* phase is not just about summarizing the content, but formalizing it, having as main support the ideas that were discussed in the *collective discussion* phase.

PROFESSIONAL LEARNING REVEALED BY TEACHERS

As they became acquainted with the ET perspective through the exploration of the multimedia case, the teachers had the opportunity to rethink and question some of their actions and to perceive essential aspects of this perspective, namely regarding the teacher's actions and roles, the students' role, the class management and how important are class planning and after-class reflection. This enabled them to develop other ways of looking at the teaching activity. For Ball and Cohen (1999), when approaching work produced in a classroom different from their own, teachers can learn from the practice of others and, consequently, reflect on their own practice.

Regarding the *teacher's actions and roles*, the teachers showed some reifications such as: choosing tasks that allow different connections to be established; monitoring and supporting students' autonomous work; asking questions carefully enough not to eliminate the challenging aspects of the task; promoting interaction between students; fostering the development of complex forms of mathematical thinking; encouraging the student to evaluate and explain their thinking regarding their task-solving strategy; not answering students' questions directly or validating their answers; encouraging students' mathematical communication; selecting and sequencing student's solutions for the collective discussion phase; orchestrating the mathematical discussions that occur during class development in small groups; establishing connections between the mathematical ideas present in the different students' solutions; promoting the systematization of learning; generalizing what was studied to any given context. We infer that these actions and roles were the teacher's learnings. First, because they were not part of their repertoire and practice. Second, because they have gone beyond the meeting sessions context and reached their classrooms (Van Es & Sherin, 2002). As they began to share their classroom experiences under the ET perspective, they legitimized it as a way of teaching that allows students to build knowledge with understanding.

In analyzing the episodes of class, teachers realized that their actions and decisions can influence their students' learning and that their role during class is to support the students' work, mediate interactions, and provide them with the opportunity to make sense of mathematical ideas. They reified the idea that the teacher's role is not to deliver information, but to be a mediator of the learnings that takes place in the classroom. They became aware that it is not necessary to tell everything to students, but to guide their work with the task, encouraging them to explain their thinking without intervening too much.

Regarding *the student's role*, they realized that, from the perspective of the ET, they need to play an active role; be willing to do different work; provide justifications for their solution strategies; learn to communicate their ideas in a way that makes them co-responsible for their learning and for validating their own and their peers' thinking; respect the work of both peers and teacher; use what they have learned in other tasks; and establish connections with previous learning.

As for *classroom management*, the teachers' learning focused on the following aspects: organizing the students (group work) and maintaining a classroom dynamic that fosters students' learning and time management for the task and for each phase of the lesson. The teachers reified that the way to organize the students for the development of the class, favoring a collective work, makes a difference in the way they understand and solve the tasks proposed. The analysis of the episodes of the *class development* phase (*The Class* section) presented interactions between students in small groups and this allowed them to look more closely at this type of work. They realized and recognized the potential of this type of work for the development of students' autonomy and their ability to communicate mathematically. Gradually they began to experiment working with their students in this way.

Thinking about time management for students' work with the task was another aspect reified by the teachers. They realized that they did not allow enough time for the students to accomplish the task and thinking, in this respect, it was not relevant to them. The idea that the teacher should let students work on the task until everyone finishes it may cause them to disperse, perform other activities, and then the task may become meaningless. This situation may jeopardize the discussion of the task later. They reified this idea from the analysis of the episodes about the *class development* phase (*The Class* section) and when they implemented tasks from the ET perspective throughout the activities. Luísa points out that the action of monitoring students' work allows the teacher to know, be aware of what is being done, so managing time is important for the class to have a proper pace.

The teachers highlighted the *importance of planning*, with emphasis on the definition and clarity of the objectives of the class and the task, so that in the *collective discussion* phase the teacher uses them as a guide for selecting and sequencing the students' solutions for *collective discussion* and for aspects to be highlighted in the *systematization* phase; registering different ways of solving the task; anticipating students' possible difficulties and possible questions to be asked them; setting a time for the task at each phase of the

class. Being attentive to the objectives supports the teacher in elaborating questions that can be made to the students during the class.

By analyzing the multimedia case in the *collective discussion* phase (*The Class* section), the teachers realized that the lesson portrayed in the case had a *common thread*, outlined by the objectives previously established by the teacher. This fact became more noticeable when they selected and sequenced the students' solutions for this phase, as they realized that it would not be possible to choose simply any resolution because there was a specific purpose for the class: to build the concept of function.

By exploring the episodes of the *class development* phase (*The Class* section), the teachers made sense of the lesson plan. They realized that the students' solutions questions and possible misconceptions could be anticipated in the plan and, therefore, they would have subsidies to assist them in these situations. They recognized the potential of this resource to support the teacher in class management when aligned with the objectives set for the class.

The learning presented in this section was sustained through shared repertoire, mutual engagement, sharing of experiences and negotiation of meanings developed at CoP-ProfMARE.

CONCLUSIONS

We infer that the exploration of the Phone Plan multimedia case at CoP-ProfMARE enabled the teachers' learning, as discussed above, since it allowed them to: (i) develop the *noticing* ability, as they identified important aspects from the ET perspective and interpreted them using their classroom knowledge and experience, and acting on these situations; (ii) come to realize that it is possible to teach differently from the directive teaching they are used to; (iii) be (co) responsible for their professional learning, as they were encouraged to explain and justify their ideas; (iv) enter the classroom world without being in the teaching position at the moment, and thus being able to look at the specific classroom interactions that might otherwise be overlooked in their routine (Sherin & Van Es, 2002); (v) make connections between the situations analyzed and their own practice; (vi) interpret the complexity of the classroom, with time to reflect more deeply on interactions that occurred in the classroom and events they observed; and (vii) make a self-evaluation of their actions in the classroom in connection with the classroom situations they have analysed.

The professional learning that took place became evident as they shared their experiences, reported on their difficulties and weaknesses, made explicit different questions that could be asked to students during task development, and implemented the practice of "writing" in more detail what they would work on with the students. In the context of CoP-ProfMARE, through the exploration of the multimedia case, the teachers could also engage together in the analysis of a real classroom situation to share experiences and repertoires, to reflect, discuss and build professional knowledge from their practice and

the practice of the others. We observed that the relationships of trust and mutual respect the teachers established throughout the meetings allowed them to openly recognize what they did not know and share their weaknesses. Teachers need to recognize and assume their vulnerabilities to be able to develop their ability to cope with uncertainty, classroom diversity, the need to think and tolerate periods of “clutter” in the classroom.

The use of the Phone Plan multimedia case proved to be a catalyst for the learning constituted in the CoP. From this exploration, the teachers were mobilized to reflect on their role, the students’ role, their practice and their actions, and they could recognize the complexities of dealing with discussions and the different strategies in the classroom. Thus, we suggest that multimedia cases can be explored in professional development contexts where teachers can interact frequently, share experiences, reflect on their profession and learn together.

ACKNOWLEDGMENTS

We would like to express thanks to all the participants for their engagement in this study, the National Council of Technological and Scientific Development (CNPq/Brazil) for the scholarship conferred to Cristina Cirino de Jesus (doctoral) and for the grant conceded to Márcia Cristina de Costa Trindade Cyrino (Research Productivity). This study was partially supported by CNPq/Brazil and Araucaria Foundation.

AUTHORS’ CONTRIBUTIONS STATEMENTS

C.C.J, M.C.C.T.C and H.M.O. conceived of the presented idea, developed the theory, adapted the methodology to this context, created the models, and performed the activities. C.C. J collected the data. All authors analysed the data, discussed the results and contributed to the final version of the manuscript.

DATA AVAILABILITY STATEMENT

Data supporting the results of this study will be made available by M.C.C.T.C upon reasonable request.

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