

Attitudes towards probability and its teaching in prospective mathematics teachers from Chile and Spain

Felipe Ruz^{1a}
 Elena Molina-Portillo^{1a}
 Claudia Vásquez^{2b}
 José M. Contreras^{1a}

^a Universidad de Granada, Facultad de Ciencias de la Educación, Departamento de Didáctica de la Matemática, Granada, Spain

^b Pontificia Universidad Católica de Chile, Departamento de Matemática, Santiago, Chile

Received for publication on 31 Jul. 2019. Accepted after review on 19 Feb. 2020.

Designed editor: Claudia Lisete Oliveira Groenwald

ABSTRACT

Background: Promoting a positive attitude towards teaching statistics and probability has become a shared challenge for teachers' training today. **Objectives:** In this study we examined validity and reliability of Attitudes' Scale towards Probability and its Teaching (ASPT) given by Estrada, Batanero, and Díaz. **Design:** Following a quantitative methodology. **Setting and Participants:** The attitudes of a purposive sample of 126 prospective Chilean and Spanish mathematics teachers were analysed using ASPT scale. **Data collection and analysis:** through an Exploratory Factor Analysis. **Results:** they were organized into two parts. While the first part focused on the psychometric characteristics of the scale, the second one emphasised on specific attitudes of the sample. **Conclusions:** The findings revealed a positive attitude towards the scale. It was viewed ASPT to be an instrument with good characteristics to explore attitudinal aspects of teachers.

Keywords: attitudes, probability, teaching, teachers' training.

Atitudes em relação à probabilidade e seu ensino em futuros professores de matemática do Chile e da Espanha

RESUMO

Contexto: Promover uma atitude positiva em relação ao ensino de estatística e probabilidade tornou-se um desafio compartilhado para a formação de professores hoje. **Objetivos:** A validade e a confiabilidade da Escala de Atitudes em relação à Probabilidade e seu Ensino (ASPT), dada por Estrada, Batanero e Díaz, foram examinadas neste estudo. **Design:** metodologia quantitativa. **Ambiente e participantes:** as atitudes de uma amostra intencional de 126 futuros professores de matemática chilenos e espanhóis foram analisadas usando a escala ASPT. **Coleta e análise de dados:** por meio de uma Análise Fatorial Exploratória. **Resultados:** estes foram organizados em

Corresponding author: Felipe Ruz Angel, felipe.ruz.angel@gmail.com

duas partes. Enquanto a primeira parte focou nas características psicométricas da escala, a segunda enfatizou atitudes específicas da amostra. **Conclusões:** Os resultados revelaram uma atitude positiva em relação à escala. Considerou-se o ASPT um instrumento com boas características para explorar aspectos atitudinais dos professores.

Palavras-chave: atitudes, ensino, formação de professores, probabilidade.

INTRODUCTION

Currently, statistics and probability are considered to be a fundamental knowledge to face effectively the challenges of the 21st century (Batanero & Borovcnik, 2016; Ben-Zvi, Makar, & Garfield, 2018; United Nations, 2015). In effect, the requirements of today's society have reinforced the need for critical educated citizens, or data consumers, to actively participate in debates using arguments based on numerical evidence, promoting the development of a more democratic society (Ben-Zvi & Makar, 2016). The accredited role for probability in the development of critical thinking (Everitt, 1999) and its applicability in different fields of knowledge, has promoted the incorporation of probability into all school grades of mathematics curriculum in many countries (Scheaffer, Watkins, & Landwehr, 1998). Its study from an early age "provides an excellent opportunity to show students how to mathematize, and how to apply mathematics to solve real problems" (Godino, Batanero, & Cañazares, 1997, p.12). As a result, it has required new demands and challenges for teachers in charge of this task. Those requirements involve diverse knowledge, not only at a didactic and disciplinary level, but also in their disposition and interest in the teaching of these topics (Batanero, 2013). However, literature has shown the lack of qualification of teachers and their non-readiness to tackle this work, reflecting a current problem to these professionals (Batanero, Burrill, & Reading, 2011; Groth & Meletiou-Mavrotheris, 2018).

In this context, it is necessary to investigate the possible causes of this situation from an affective perspective, and the present study could contribute to broadening the knowledge on this subject. To this effect, the affective domain is assumed from McLeod's perspective like a "wide range of beliefs, feelings and moods, which are generally considered as more than the pure domain of cognition" (McLeod, 1992, p. 576). The term "affects" is used in a general sense, and emotions, beliefs and attitudes as their main descriptors. In addition, because of the absence of consensus in the field to define attitude (Philipp, 2007), we took the Batanero's perspective in which that notion is considered "a mental construction, no directly observable, it must be inferred from the assessment on a scale of attitudes or the observation of the subjects' behavior" (Batanero, 2009, p. 6).

On the other hand, if a teacher does not value a statistics and probability topic, he feels that he is not prepared to teach it or dislike it, he will not achieve effective teaching to his students (Estrada & Batanero, 2015). For this reason, it is important to give value and reinforce the affective component since teachers' training. This faces a challenge to identify the attitudes of prospective teachers in relation to the topics that should teach, so that, teacher trainers could use this knowledge to promote its improvement during their initial teacher training (Veloo & Chairhany, 2013). In consequence, we examined

validity and structure of Attitudes' Scale towards Probability and its Teaching (ASPT) given by Estrada, Batanero, and Díaz (2018) in a sample of 126 prospective Spanish and Chilean mathematics teachers. In what follows, we stated the theoretical background and the methodology used to achieve this goal. Afterwards, we detailed the results obtained, and concluded with the discussion and findings.

THEORETICAL BACKGROUND

The framework that supports this study is established on the basis of previous research on attitudes towards statistics, and towards statistics and its teaching; and attitudes towards probability and its teaching.

Previous research about attitudes towards statistics, and towards statistics and its teaching

Attitudes towards statistics have been a topic of growing interest among researchers and statistical educators for more than 70 years (Bendig & Hughes, 1954). This field of research has become a target of educational reform regarding its teaching (Tishkovskaya & Lancaster, 2012). Consequently, a large number of studies emerged that assessed students' attitudes in relation to academic performance and attitudinal changes generated before and after instruction (Carmona, 2004; Nolan, Beran, & Hecker, 2012.)

Among the most common instruments used are: (1) One-dimensional scale, Statistics Attitude Survey (SAS) from Roberts and Bilderback (1980), (2) Two-dimensional scale Attitudes Toward Statistics (ATS) from Wise (1985), (3) The multidimensional scale Attitudes Toward Statistics (EAE, in its Spanish acronym) from Auzmendi (1992), and (4) The Surveys of Attitudes Toward Statistics (SATS-28; Schau, Stevens, Dauphinee, & DelVecchio, 1995; and SATS-36: Schau, 2003) which have an initial structure of four factors (SATS-28), and six at the end (SATS-36). These scales over time have been combined with each other as they share some aspects such as feelings towards the subject, their professional utility and personal value, as well as cognitive competence, self-confidence, and perceived difficulty towards statistics.

Despite the lack of research on the attitudes of teachers (both pre-service and in-service teachers) towards statistics and its teaching (Estrada, Batanero, & Lancaster, 2011; Groth & Meletiyou-Mavrotheris, 2018), two main trends can be identified: (1) The use of an instrument mentioned above (as ATS in Onwuegbuzie, 1998; SATS-28 in Estrada, 2002; Nasser, 2004; SATS-36 in Hannigan, Gill, & Leavy, 2013; Zientek, Carter, Taylor, & Capraro, 2011), or (2) to propose a new one which also considers the attitudes toward teaching of the subject, such as the scale called Attitudes Toward Statistics and its teaching (EAEE in its Spanish acronym) from Estrada (2002). The EAEE scale has been used in many countries like Spain (Estrada, Batanero, & Fortuny, 2004), Portugal (Martins, Estrada, Nascimento, & Comas, 2015), and Perú (Aparicio & Bazán, 2006). Years

later, Estrada et al. (2018) propose an evolution of the EAEE scale applied to probability, called Attitudes' Scale towards Probability and its Teaching (ASPT). However, in the next section this scale will be explored, since it was the main theoretical support of the instrument used in this study.

Onwuegbuzie (1998) applied ATS scale on 222 American teachers. The findings reported relatively less positive attitudes than others researches carried up to that point. Nasser's (2004) work, which applied SATS-28 on 162 prospective teachers from Israel, reported weak correlations (from 0.11 to 0.28) between performance and scores according to each component of the scale. Estrada (2002), applied a Spanish translated version of SATS-28 scale on 367 prospective Spanish teachers. The results showed a generally positive attitude but low correlations (0.09 - 0.26) between the scores in the scale and some questions of the Statistical Reasoning Assessment test of Garfield (2003). Zientek et al. (2011) evaluated the effect of attitudes on the performance of 95 prospective American primary school teachers through SATS-36 scale. Their study showed correlations between moderate (0.492 in the affective component) and low (0.142 in the difficulty component). Making use of the same instrument (SATS-36), Hannigan et al. (2013) analyzed the relationship between attitudes and conceptual statistics understanding in 104 future Irish secondary school mathematics teachers. Low correlations (-0.02 - 0.19) were found among the score obtained in the Comprehensive Assessment of Outcomes in Statistics test (DelMas, Garfield, Oms, & Chance, 2007) and the scale.

Several authors propose new approaches to study teachers' affective aspect in statistics education. Begg and Edwards (1999) analyzed the attitudes and beliefs about statistics teaching in 34 primary school prospective teachers through personal interviews, measurement scales and concept maps. Their findings revealed negative attitudes of the participants toward statistics, who preferred teaching old topics, which they considered to be safe, rather than the new ones proposed by the program, such as probability and the use of technology. Estrada (2002) developed the EAEE scale for teachers. This scale includes 25 items considering the interaction between six components. These components are organized according to (1) anthropological and (2) pedagogical aspects. The anthropological aspects are related to the usefulness of statistics, its training and multidisciplinary. It includes components such as: (1.1) social (perception of the value of statistic in society); (1.2) educational (the interest in learning and teaching statistics), and (1.3) instrumental (the perception of the usefulness of the statistics in other areas). The pedagogical aspects comprise three traditional components as: (2.1) affective (personal feelings about statistics); (2.2) cognitive (conceptions and beliefs about learning statistics), and (2.3) behavioral (tendency to use statistics).

In this context, Estrada et al. (2004) applied the EAEE scale in 140 Spanish teachers (66 in-service and 74 pre-service). Their work indicated a positive global attitude, as same as in the other components of the scale. Aparicio and Bazán (2006) used the EAEE scale to assess the relationship between attitude and performance of 87 Peruvian teachers at the beginning and end of a professional development program. The authors highlight that the course increased teachers' attitudes, and a positive correlation (from 0.07 at the

beginning and up to 0.22 at the end) was found between the score in the scale and the performance in the program. Martins, Estrada, Nascimento and Comas's (2015) analysis of the attitudes of 1098 Portuguese teachers from 1st and 2nd stage in primary school revealed a slightly positive general attitude.

Previous studies related to attitudes towards Probability and its teaching

To promote a positive attitude among teachers is a shared challenge for the teaching of statistics today (Tishkovskaya & Lancaster, 2012). Because students' attitude is one of the main factor to learn about a subject, teachers' trainers need to identify the prospective teachers' attitudes in relation to the subject they will teach. They also need to use that knowledge to promote the improvement of those attitudes (Veloo & Chairhany, 2013). Consequently, we are interested in obtaining information about the attitudes towards probability and its teaching, through the ASPT scale (Estrada & Batanero, 2015; Estrada et al. 2018). This scale considers seven components, grouped into three dimensions:

1. Components towards Probability: Appreciation, cognitive perception, and tendency to action towards probability. It considers three components:
 - 1.1 Affective component towards Probability (AP): Subject's feelings, positive or negative, towards probability content.
 - 1.2 Cognitive Competence towards Probability (CCP): Self-perception of the intellectual capacity towards probability content.
 - 1.3 Behavioral component towards Probability (BP): Tendency to use probability tools when appropriate.
2. Components towards didactic aspects of Probability: Appreciation, perception of didactic capacity and tendency to action towards the teaching of probability. It considers three components:
 - 2.1 Affective component towards Teaching probability (AT): Personal feelings, positive or negative, towards the teaching of probability content.
 - 2.2 Didactic Competence towards Teaching probability (CT): Prospective teachers' perception of their own ability to teach the probability content.
 - 2.3 Behavioral component towards Teaching probability (BT): Asses tendency to didactic action in the teaching of probability content.
3. Appreciate the content and its teaching component: Appreciation towards the content and teaching of probability, which considers one component:
 - 3.1 Value towards Probability and its Teaching (VPT): The value, usefulness and relevance that the teacher gives about probability content in personal and professional life.

This scale was applied by Estrada et al. (2018) on 232 Spanish primary school prospective teachers. An average score of 102.6 points out of a possible range between 28 and 140 points was obtained, thus indicating a mostly positive attitude. Also, they provided evidence of adequate reliability (Cronbach's alpha of 0.892) and a structure of seven factors that explained 67.99% of the variance of the model. In the same way, Alvarado, Andaur, and Estrada (2018) explored the attitudes of 70 in-service and 51 pre-service Chilean mathematics teachers towards probability and its teaching. They concluded a positive attitude in both groups, although slightly better with in-service teachers than pre-service teachers. Meanwhile, Vásquez, Alvarado, and Ruz (2019) analyzed the attitudes of 124 prospective Chilean elementary teachers towards statistics and probability, and its teaching. Results showed that the attitudes towards statistics and its teaching were slightly better than the ones towards probability.

The present study aims to examine the validity and reliability of ASPT scale, and also increase the results of previous research providing information about this aspect (attitudes) of the affective domain. For this sake, a sample of prospective mathematics teachers from Chile and Spain was selected. Their selection was justified by the fact that the school curriculum of both countries (Ministry of Education Chile 2009; 2015; and Ministry of Education, Culture and Sports Spain 2014; 2015) emphasize the need to deepen into the probability study. Indeed, their curriculums include topics like the modelling of random phenomena through probability distributions and concluding with an introduction to statistical inference.

METHODOLOGY

This research is carried out under the quantitative paradigm with a descriptive scope (Hernández et al. 2014) because we started from a previous instrument (ASPT scale), and we have specific background of its implementation.

The study was conducted in a non-probabilistic sample of 126 prospective mathematics teachers, who were organized into two groups according to their country of residence (Spain and Chile). The first group (G1) consisted of 84 prospective Spanish teachers (35 women and 49 men aged between 21 and 50 years old). They held a Master's degree in compulsory secondary and bachelor education from a Spanish university. The second group (G2) included 42 prospective teachers from a Chilean university (21 women and 21 men aged between 19 and 38 years old). All participants have taken all probability courses scheduled in their initial training. In addition, implementation took place in a special session in the middle of second semester of 2018, where all participants had voluntarily and individually completed the scale.

The instrument used was ASPT Scale for teachers (Estrada et al. 2018) as shown in Appendix 1. For each of the seven components, participants had to answer four questions: two in affirmative and two in negative sense. In total, 28 items were provided. A five-step Likert scale was used, moving from Strongly Disagree (1), to Strongly Agree (5) with

a midpoint which expressed Indifference (3). The scores obtained in those items with negative sense (marked with * in Appendix 1) were reversed. This allowed us to define a serie of variables as a result of the sum between the different items of each component, and the total score (see Table 1).

Table 1
Variables analyzed according to ASPT components

Component	Total items	Range
Affective towards Probability (AP)	1, 5, 16, 27	4 – 20
Cognitive Competence towards Probability (CCP)	6, 8, 17, 22	4 – 20
Behavioral towards Probability (BP)	2, 7, 15, 18	4 – 20
Affective towards the Teaching of Probability (AT)	9, 21, 26, 28	4 – 20
Didactic Competence towards the Teaching of Probability (CT)	3, 10, 14, 23	4 – 20
Behavioral towards the Teaching of Probability (BT)	11, 20, 24, 25	4 – 20
Value towards Probability and its teaching (VPT)	4, 12, 13, 19	4 – 20
Attitudes towards Probability and its Teaching (APT)	All	28 – 140

Given the quantitative nature of previous variables, we started analyzing the total score (APT variable) in the global sample. On this basis, we provided detailed information about internal consistency (instrument reliability) and construct validity as a result of a factor analysis. For reliability, the Cronbach’s alpha coefficient was used as an indicator that the scale produced the same results each time it was administrated to the same person in similar conditions. The closer the value is to 1, the higher its internal consistency will be. The Factor Analysis was Exploratory (EFA) (Morales, 2013; Lloret-Segura, Ferreres-Traver, Hernández-Baeza, & Tomás-Marco, 2014). Finally, we carried out a descriptive study of the different components, identifying if results tended towards a positive, negative or neutral (indifferent) attitude, and analyzing if there are significant differences between sampling groups scores. Statistical significance was set at $p < 0.01$.

RESULTS AND ANALYSIS

Research outcomes have been organized in the following sections: validity and reliability; attitudes of the participants and differences according to groups that comprise the sample.

Validity and reliability

On the overall results, average scores per item ranged from 2.67 (item 21) to 4.64 (item 12) points (Appendix 1). In all cases, 96.8% of responses were exceeded with respect to the total sample. However, to analyze the total score (APT), we only considered those

cases where no answer was omitted, which corresponded to 102 of the total. These results ranged from 62 to 140 points with an average of 104.39 points and a standard deviation of 14.67 points. The mean score was higher than 84 points (indifference state because in all cases option 3 was chosen), which generally implies a positive attitude towards probability and its teaching. Regarding reliability, we obtained a Cronbach's alpha of 0.879. This value is considered more than adequate (Pedhazur & Pedhazur, 1991). Table 2 shows Alpha values for each variable defined in Table 1, and the number of valid answers. The values oscillate between usual ranges for this type of studies (Carmona, 2004; Nolan et al., 2012), although slightly lower for the component of cognitive competence towards probability (CCP).

Table 2
Cronbach's Alpha for the analyzed variables

Measure	AP	CCP	BP	AT	CT	BT	VPT
n	123	119	121	123	120	120	120
Alpha	0.605	0.446	0.602	0.622	0.736	0.689	0.725

Subsequently, the Bartlett's sphericity test was applied, obtaining a p-value of 0.000 (Chi-square approximate 1251.01). This allowed rejecting the null hypothesis which stipulated that the correlation matrix is the identity, i.e. the items of the scale are correlated with each other. In this way, we calculated Kaiser-Meyer-Olkin (KMO) sampling adequacy measure to determine the degree of joint relationship between the questions. The value of 0.753 was obtained, which suggests the suitability of the Exploratory Factor Analysis carried out [EFA](Morales, 2013; Lloret-Segura et al. 2014).

The EFA was carried out using the unweighted minimum squared method because the scores of each item were not normally distributed. The results showed that seven factors explained 63.67% of the total variance of the model. Although it is not a usual topic of an exploratory analysis, we wanted to provide information on the Item-Factor relationship from a correlation coefficient between them. In addition, to have a better understanding of the results, we decided to work with the rotated matrix components using the varimax criteria. In simple words, this latter consists of maximizing the variance of the coefficients that define the effects of each factor in the analyzed items (see Appendix 2). In Table 3, we synthesised the four items with the highest correlation within every factor, and sorted them in descending order of magnitude. In addition, we included a third column about theoretical components to which each one of them belongs.

Table 3
Items with the highest correlation for each factor

Factor	Items with greater proportion	Theoretical classification of the items
1	5, 1, 26, 27	AP = 1, 5, 27; AT = 26
2	14, 23, 22, 3	CT = 3, 14, 23; CCP = 22
3	17, 13, 25, 19	VPT = 13, 19; CCP = 17; BT = 25
4	15, 2, 7, 4	BP = 2, 7, 15; VPT = 4
5	24, 4, 19, 20	VPT = 4, 19; BT = 20, 24
6	8, 6, 10, 22	CCP = 6, 8, 22; CT = 10
7	18, 16, 12, 5	AP = 5, 16; VPT = 12; BP = 18

From the previous results, we associated the first factor with AP component, and grouped three of the four items which were considered theoretical in it. The same happened with the second, fourth, and sixth factor in relation to the CT, BP, and CCP components, respectively. However, the third and seventh factors are not grouped mostly in some component. The fifth was related to the behavioral (BT) and value (VPT) components towards teaching probability. Furthermore, we classified items 4, 5, 19, and 22 in more than one factor, and left items 9, 11, 21, and 28 unassigned. When we delve into the results for each one, we observed that items 9 and 11 were correlated in greater extent with factor 1, while those in the 21st and 28th positions were correlated with factor 2. In other words, they were mainly related to affective aspects toward probability (AP), and the pedagogical principles (didactics) toward its teaching (CT). This supports the initial grading in the affective component towards the teaching of probability (AT). The resulting factor do not group the four items for each proposed theoretical component, but rather reflect relationship among them. As Estrada et al. (2018), we recognize the need to collect more data to verify the validity of the initially proposed seven-factor model.

Subsequently, we grouped the results according to the variables defined in Table 1, and we computed the correlation coefficient between them. Results are shown in Table 4. It is worth mentioning that we completed only the upper triangular matrix of correlations. This was made on purpose to avoid having overloaded information in the table because it was symmetrical.

Table 4
Pearson's correlation between the different components of attitudes

Variable	AP	CCP	BP	AT	CT	BT	VPT	APT
AP	1	0.411*	0.393*	0.663*	0.412*	0.482*	0.310*	0.767*
CCP		1	0.351*	0.547*	0.565*	0.314*	0.285*	0.679*
BP			1	0.286*	0.342*	0.417*	0.475*	0.628*
AT				1	0.655*	0.439*	0.366*	0.819*
CT					1	0.267*	0.337*	0.713*

Variable	AP	CCP	BP	AT	CT	BT	VPT	APT
BT						1	0.544*	0.669*
VPT							1	0.609*
APT								1

* Represents significant correlations to 0.01

In general, we can notice that all previous correlations were significant. Related variables with the affective component toward probability (AP) and its teaching (AT) were those with the greatest influence on the total score (APT). While the strength of the relationship was less towards the value component (VPT). However, we note as considerable the association between the seven components of the ASPT scale, and their total score (APT). For the others, we can conclude that the affective component towards teaching probability (AT) correlates mostly with the affective components towards the content (AP) and the didactic competence towards its teaching (CT), with a weak interaction between the BP-AT, CT-BT, and CCP-VPT components (all less than 0.3).

Attitudes of the participants

In what follows, we evaluated if there were differences between results obtained according to each component, and the theoretical value of indifference (12 points of having answered 3 in the four items for each component, or 84 in the total score). We also studied whether or not there were differences according to both sampling groups (G1 and G2). In Table 5, we present the average score and standard deviation of each component, along with the results of contrasting the normality assumption by the Kolmogorov Smirnov test (KS).

Table 5
Summary statistics of analyzed variables

Variable	n	Statistics		KS Test	
		Average	D.E.	Statistic Z of KS	p-value
AP	123	14.18	3.18	0.875	0.428
CCP	119	13.87	2.74	1.207	0.108
BP	121	14.87	3.01	1.238	0.093
AT	123	14.05	3.16	1.126	0.158
CT	120	14.35	3.25	1.417	0.036
BT	120	15.56	3.02	1.130	0.156
VPT	120	16.62	2.63	2.279	0.000*
APT (Total)	102	104.39	14.67	0.688	0.731

* Represents significant results to 0.01

Based on the results, normality assumption was rejected only for the VPT variable. Consequently, nonparametric procedures were used for its analysis. Later, delving into those variables with normal behavior, we present in Table 6 the T-test results for independent samples which compare if differences between each average score and the indifference position (12 points for each component and 84 for the total) were equal to zero. For each case, 95% confidence interval (CI) are shown intended to investigate its tendency.

Table 6
Contrasts and Intervals regarding the indifference position in normal variables

Variable	T Test independent samples		CI (95%) for differences	
	Statistic	p-value	Lower	Higher
AP	7.610	0.000*	1.612	2.746
CCP	7.427	0.000*	1.368	2.363
BP	10.477	0.000*	2.326	3.410
AT	7.198	0.000*	1.485	2.612
CT	7.926	0.000*	1.763	2.937
BT	12.894	0.000*	3.012	4.105
APT (Total)	14.038	0.000*	17.511	23.274

* Represents significant results to 0.01

With a significance level of 0.01, we reject the null hypothesis that the mean score in each case corresponds to indifference. Moreover, when observing confidence intervals for the mean differences, we note that in all of them their lower limit is greater than 2 in each component and greater than 17 in the total score. We conclude that the participants of this study expressed mostly positive attitudes towards probability and its teaching, at least in the analysed variables. On the other hand, with respect to the one where the normality assumption fails, in Table 7 we present some summary statistics along with the results of comparing if the median score is equal to 12 points through the Wilcoxon rank test.

Table 7
Contrasts and statistics regarding non-normal variable indifference

Variable	Wilcoxon Test		Percentiles		
	Null Hip.	p-value	25	50	75
VPT	The median of VPT is equal to 12	0.000	15	17	18.75

Therefore, with a significance of 0.01, we reject the null hypothesis that the median scores of the VPT variable was equal to 12 points. In addition, when we observed percentiles, we noticed that more than 75% of the participants assigned scores over 15

points for the VPT variable (see percentile 25 in Table 7), which, as in the previous case, allows us to conclude that the attitudes declared, in this case, were mostly positive too.

Finally, we studied whether or not there were differences between the analyzed variables of both sample groups (G1 and G2). Therefore, given the sample sizes of each group, we used the nonparametric Mann Whitney U test for independent samples to test whether the distribution of the scores obtained in each variable was the same in both groups. We rejected the null hypothesis only for the BT component, and in the other cases we observed non-significant differences.

DISCUSSION

This study explored attitudes towards probability and its teaching in a sample of 126 prospective mathematics teachers from Chile and Spain. In this regard, participants generally declared to have a significantly positive attitude in the seven components of the Attitudes' Scale towards Probability and its Teaching (ASPT). Same with in the total average score, who distances more than 17 points on average from the theoretical value of indifference (Table 7). This situation is consistent with previous studies where mild or moderately positive attitudes towards statistics or probability are reported (Estrada, 2002; Estrada et al. 2004; Hannigan et al., 2013; Martins et al., 2015; Zientek et al., 2011; Alvarado et al., 2018; Vásquez et al., 2019). However, these studies are opposed to the ones of Begg & Edward (1999) and Onwuegbuzie (1998) which report mostly negative attitudes of in-service and pre-service teachers. In specific terms, it is interesting to note that item 21 ("I am concerned about answering questions about probability of my students") is the one with the lowest average, and item 12 ("Probability is useless") is the one with the greatest. However, in both cases the items are expressed in a negative sense. Then, in item 21 our participants do feel worried about adequately addressing questions about probability of their students. The same with item 12, where our prospective teachers highlight the usefulness of probability.

Despite the fact that there is a clear tendency to declare positive attitudes towards the content and its teaching, we are concerned that content knowledge about probability is still insufficient for these prospective teachers (Ruz, Molina-Portillo, Contreras, in press). Therefore, an interesting projection would be to deepen the reasons why they have such a positive attitude towards probability and its teaching, even when their content knowledge is poor, as it was analyzed by Martins et al. (2012) with Portuguese teachers.

Regarding the characteristics of the analyzed instrument, we highlight an adequate internal consistency (Cronbach's $\alpha = 0.879$) and a structure with seven factors that explain 63.67% of the total variance of the factorial model. Likewise, four of these factors correlated in greater magnitude with the items of the Affective components (AP, factor 1), Cognitive Competence (CCP, factor 7) and Behavioral components towards the content (BP, factor 4), as well as the Didactic Competence towards teaching probability (CT, factor 2). Regarding the other three factors, we observed in one of them (factor

5) a greater interaction between the items of the components Behavior (BP) and Value towards the content and its Teaching (VPT). However, there is no clear information of the remaining two. In synthesis, we can notice that results did not directly group the four items for each proposed theoretical component, but reflected relationships between them. Thus, we agree with Estrada et al. (2018), because this study had a modest sample size, we accept that we have to gather more observations to check the validity of the seven factors model, originally proposed. Yet, we appreciate the presentation of these results as we expect that they will be a reference for an upcoming implementation. Therefore, we conclude that exploration work must be continued in order to confirm a factorial structure of the scale which we project as a challenge for its future application.

Finally, we consider that the ASPT scale is an instrument with good characteristics with which it is possible to explore aspects of the affective domain, as attitudes, in teachers. This would allow, among other things, to monitor the evolution of attitudes before and after the instruction, or to relate them with other variables such as content knowledge.

AUTHORS' CONTRIBUTIONS STATEMENTS

All authors conceived the presented idea. F.R. and C.V. collected the data, and contributed with funding. F.R. and E.M. analysed the data. F.R. prepared the initial draft. All authors discussed the results and contributed to the final version of the manuscript. J.C. supervised the project.

DATA AVAILABILITY STATEMENT

The data supporting the results of this study will be made available by the corresponding author, F.R., upon reasonable request.

REFERENCES

- Alvarado, H., Andaur, G., & Estrada, A. (2018). Actitudes hacia la Probabilidad y su enseñanza: un estudio exploratorio con profesores de Matemática en formación y ejercicio de Chile [Attitudes towards Probability and its Teaching: An exploratory study with Chilean mathematics teachers in training and exercise]. *Revista Paradigma*, 39(2), 36-64.
- Batanero, C. (2013). La comprensión de la probabilidad en los niños. ¿Qué podemos aprender de la investigación? [Understanding of Probability in children. What can we learn from research?]. In J. A. Fernandes, P. F. Correia, M. H. Martinho, & F. Viseu, (Eds.) *Atas do III Encontro de Probabilidades e Estatística na Escola*. Braga: Centro de Investigação em Educação. Universidade Do Minho.

- Aparicio, A., & Bazán, J. (2006). Actitud y rendimiento en Estadística en profesores peruanos [Attitude and performance in Statistics in Peruvian teachers]. In G. Martínez (Ed.), *Acta Latinoamericana de Matemática Educativa* (pp. 644–650). México DF: CLAME.
- Auzmendi, E. (1992). *Las actitudes hacia la matemática/estadística en las enseñanzas medias y universitaria. Características y medición* [Attitudes towards mathematics/statistics in middle and tertiary education. Features and measurement]. Bilbao: Mensajero.
- Batanero, C. (2009). Retos para la formación estadística de profesores [Challenges for Statistical teacher training]. *II Encontro de Probabilidade e Estatística na Escola* (pp. 1–23). Braga: Universidade do Minho.
- Batanero, C. y Borovcnik, M. (2016). *Statistics and Probability in High School*. Rotterdam: Sense Publishers.
- Batanero, C., Burrill, G., & Reading, C. (Eds.). (2011). *Teaching Statistics in school mathematics: Challenges for Teaching and teacher education. A Joint ICMI/IASE Study*. New York: Springer.
- Begg, A., & Edwards, R. (1999). Teachers' Ideas About Teaching Statistics. In *Proceedings of the 1999 combined Conference of the Australian Association for Research in Education and the New Zealand Association for Research in Education*. Melbourne: AARE/NZARE.
- Ben-Zvi, D., & Makar, K. (2016). International Perspectives on the Teaching and Learning of Statistics. In D. Ben-Zvi, & K. Makar, *The Teaching and Learning of Statistics* (pp. 1-10). Cham: Springer.
- Ben-Zvi, D., Makar, K., & Garfield, J. (Eds.). (2018). *International Handbook of Research in Statistics Education*. Cham: Springer. <http://doi.org/10.1007/978-94-010-0462-6>
- Bendig, A., & Hughes, J. (1954). Student attitude and achievement in a course in introductory statistics. *Journal of Educational Psychology*, 45(5), 268–276. <http://doi.org/10.1037/h0057391>
- Carmona, J. (2004). Una revisión de las evidencias de fiabilidad y validez de los cuestionarios de actitudes y ansiedad hacia la estadística [A review of evidence on reliability and validity of attitude and anxiety questionnaires towards statistics]. *Statistics Education Research Journal*, 3(1), 5–28.
- DelMas, R., Garfield, J., Ooms, A., & Chance, B. (2007). Assessing Students' Conceptual Understanding after a First Course in Statistics. *Statistics Education Research Journal*, 6(2), 28–58.
- Estrada, A. (2002). *Análisis de las actitudes y conocimientos estadísticos elementales en la formación del profesorado* [Analysis of attitudes and elementary statistical knowledge in teacher training] (Doctoral Thesis). Universitat Autònoma de Barcelona.
- Estrada, A., & Batanero, C. (2015). Construcción de una escala de actitudes hacia la probabilidad y su enseñanza para profesores [Building an Attitudes towards Probability and its Teaching Scale for Teachers]. In C. Fernández, M. Molina, & N. Planas (Eds.), *Investigación en Educación Matemática XIX* (pp. 239–247). Alicante: SEIEM.
- Estrada, A., Batanero, C., & Díaz, C. (2018). Exploring Teachers' Attitudes Towards Probability and Its Teaching. En C. Batanero y E. Chernoff (Eds.), *Teaching and Learning*

- Stochastics, ICME-13 Monographs* (pp. 313–332). Cham: Springer. http://doi.org/10.1007/978-3-319-72871-1_18
- Estrada, A., Batanero, C., & Fortuny, J. M. (2004). Un estudio comparado de las actitudes hacia la estadística en profesores en formación y en ejercicio [A comparative study of attitudes towards statistics in teachers and prospective teachers]. *Enseñanza de las ciencias*, 22(2), 263–274.
- Estrada, A., Batanero, C., & Lancaster, S. (2011). Teachers' attitudes towards statistics. En C. Batanero, G. Burrill y C. Reading (Eds.), *Teaching Statistics in School Mathematics: Challenges for Teaching and Teacher Education. A Joint ICMI/LASE Study* (pp. 163–174). New York: Springer. <http://doi.org/10.1007/978-94-007-1131-0>
- Everitt, B. S. (1999). *Chance rules: An informal guide to probability, risk, and statistics*. New York: Copernicus Springer-Verlag.
- Garfield, J., & Ahlgren, A. (1988). Difficulties in Learning Basic Concepts in Probability and Statistics: Implications for Research. *Journal for Research in Mathematics Education*, 19(1), 44–63.
- Garfield, J. (2003). Assessing Statistical Reasoning. *Statistics Education Research Journal*, 2(1), 22–38.
- Godino, J. D., Batanero, C., & Cañizares, M. J. (1997). *Azar y Probabilidad. Fundamentos didácticos y propuestas curriculares* [Chance and probability: Educational foundations and curricular proposals]. Madrid: Síntesis.
- Groth, R., & Meletiou-Mavrotheris, M. (2018). Research on Statistics Teachers' Cognitive and Affective Characteristics. En D. Ben-Zvi, K. Makar y J. Garfield (Eds.), *International Handbook of Research in Statistics Education* (pp. 327–355). Cham: Springer. http://doi.org/10.1007/978-3-319-66195-7_10
- Hannigan, A., Gill, O., & Leavy, A. (2013). An investigation of prospective secondary mathematics teachers' conceptual knowledge of and attitudes towards statistics. *Journal of Mathematics Teacher Education*, 16(6), 427–449. <http://doi.org/10.1007/s10857-013-9246-3>
- Hernández, R., Fernández, C., & Baptista, P. (2014). *Metodología de la investigación* [Investigation Methodology]. México DF: McGraw-Hill.
- J. Pedhazur, E., & Pedhazur, L. (1991). *Measurement, Design, and Analysis*. Hillsdale, New Jersey: Lawrence Erlbaum Associates Inc.
- Lloret-Segura, S., Ferreres-Traver, A., Hernández-Baeza, A., & Tomás-Marco, I. (2014). El análisis factorial exploratorio de los ítems: una guía práctica, revisada y actualizada. *Anales de psicología*, 30, 1151–1169.
- Martins, J. A., Estrada, A., Nascimento, M. M., & Comas, C. (2015). Actitudes hacia la Estadística de los Profesores: Un camino a recorrer [Teachers' Attitudes towards Statistics: A way to go]. In J. M. Contreras, C. Batanero, J. D. Godino, G. Cañadas, P. Arteaga, E. Molina-Portillo, ... M. López-Marín (Eds.), *Didáctica de la Estadística, Probabilidad y Combinatoria 2* (pp. 101–107). Granada: University of Granada.
- Martins, J. A., Nascimento, M. M., & Estrada, A. (2012). Looking back over their shoulders: A qualitative analysis of Portuguese teachers' attitudes towards statistics. *Statistics Education Research Journal*, 11(2), 26–44.

- McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 575–596). Reston, Virginia: NCTM.
- Ministerio de Educación Chile. (2009). *CURRICULUM. Objetivos fundamentales y contenidos mínimos obligatorios de la Educación Básica y Media* [Curriculum. Fundamental objectives and mandatory minimum contents of Basic and Middle Education]. Santiago: MINEDUC.
- Ministerio de Educación Chile. (2015). *Bases curriculares 7° a 2° medio* [7 to 10 grade curricular bases]. Santiago: MINEDUC.
- Ministerio de Educación Cultura y Deporte. (2014). Real Decreto 126/2014, de 28 de febrero, por el que se establece el currículo básico de la Educación Primaria [Royal Decree 126/2014, of February 28, which establishes the basic curriculum of Primary Education]. *Boletín Oficial del Estado*, (52), 19349–19420.
- Ministerio de Educación Cultura y Deporte. (2015). Real Decreto 1105/2014, de 26 de diciembre, por el que se establece el currículo básico de la Educación Secundaria Obligatoria y del Bachillerato [Royal Decree 1105/2014, of December 26, which establishes the basic curriculum of compulsory Secondary Education and Baccalaureate]. *Boletín Oficial del Estado*, (3), 1–35.
- Morales, P. (2013). El Análisis Factorial en la construcción e interpretación de tests, escalas y cuestionarios [Factor Analysis in the construction and interpretation of test, scales and questionnaires]. *Universidad Pontificia de Comillas*, 46.
- Nasser, F. (2004). Structural Model of the Effects of Cognitive and Affective Factors on the Achievement of Arabic-Speaking Pre-service Teachers in Introductory Statistics. *Journal of Statistics Education*, 12(1).
- Nolan, M. M., Beran, T., & Hecker, K. G. (2012). Surveys assessing students' attitudes toward statistics: A systematic review of validity and reliability. *Statistics Education Research Journal*, 11(2), 103–123.
- Onwuegbuzie, A. J. (1998). Teachers Attitudes toward statistics. *Psychological Reports*, 83, 1008–1010. <http://doi.org/10.2466/pr0.1998.83.3.1008>
- Philipp, R. A. (2007). Mathematics teachers' Beliefs and Affect. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257–315). Charlotte, NC: NCTM & Information Age Publishing.
- Roberts, D. M., & Bilderback, E. W. (1980). Reliability and validity of a Statistics Attitude Survey. *Educational and Psychological Measurement*, 40, 235–238. <http://doi.org/10.1177/001316448004000138>
- Ruz, F., Molina-Portillo, E., & Contreras, J. M. (in press). Exploring knowledge about probability content in prospective mathematics teachers. *Boletín de Estadística e Investigación Operativa (BEIO)*.
- Schau, C., Stevens, J., Dauphinee, T., & Del Vecchio, A. (1995). The development and validation of the survey of attitudes toward statistics. *Educational and Psychological Measurement*, 55(5), 868–875. <http://doi.org/10.1177/0013164495055005022>
- Scheaffer, R. L., Watkins, A. E., & Landwehr, J. M. (1998). What every high-school graduate should know about statistics. In S. P. Lajoie (Ed.), *Reflections on statistics:*

Learning, teaching and assessment in Grades K-12 (pp. 3-31). Mahwah, NJ: Lawrence Erlbaum.

Tishkovskaya, S., & Lancaster, G. A. (2012). Statistical Education in the 21st Century: a Review of Challenges, Teaching Innovations and Strategies for Reform. *Journal of Statistics Education*, 20(2), 1–24.

United Nations. (2015). *Transforming our world: The 2030 Agenda for Sustainable Development*. UN. https://unctad.org/meetings/en/SessionalDocuments/ares70d1_en.pdf. Accessed April 6, 2018.

Vásquez, C., Alvarado, H., & Ruz, F. (2019). Actitudes de futuras maestras de educación infantil hacia la estadística, la probabilidad y su enseñanza [Attitudes of future teachers of childhood education towards statistics, probability and its teaching]. *Educación Matemática*, 31(3), 177-202.

Veloo, A., & Chairhany, S. (2013). Fostering students' attitudes and achievement in probability using teams-games-tournaments. *Procedia-Social and Behavioral Sciences*, 93, 59–64. <http://doi.org/10.1016/j.sbspro.2013.09.152>

Wise, S. L. (1985). The development and validation of a scale measuring attitudes toward statistics. *Educational and Psychological Measurement*, 45, 401–405. <http://doi.org/10.1177/001316448504500226>

Zientek, L. R., Carter, T. A., Taylor, J. M., & Capraro, R. M. (2011). Preparing Prospective Teachers: An Examination of Attitudes Toward Statistics. *Journal of Mathematical Sciences & Mathematics Education*, 5(1), 25–38.

Appendix 1. Attitudes' Scale towards Probability and its Teaching (ASPT scale) for teachers and analysed sample outcomes

Item	N (% total)	Mean	S.D.
1. I enjoy classes where probability contents are explained.	126 (100)	3.41	1.12
2. I use information about probability when making decisions.	125 (99.2)	3.69	1.14
*3. It will be difficult for me to teach probability lessons.	125 (99.2)	3.54	1.13
4. Probability contents help to understand today's world.	125 (99.2)	3.94	0.89
5. I like probability, it is a topic that has always interested me.	125 (99.2)	3.31	1.27
6. Probability contents are easy.	125 (99.2)	3.22	1.03
*7. I have never used probabilities outside a scientific context.	124 (98.4)	3.57	1.24
8. I master the main probability contents.	122 (96.8)	3.51	1.03
9. I am sure that I would like to teach probability content in school.	126 (100)	3.53	1.14
10. I think I will know how to detect and correct mistakes and difficulties of students about probability subject.	123 (97.6)	3.71	1.04
*11. I will only teach probability content if I have time left after the other topics.	126 (100)	3.87	1.18
*12. Probability is useless.	124 (98.4)	4.64	0.74
*13. Probability is not as valuable as other mathematics areas.	124 (98.4)	4.11	1.02
14. It will be easy for me to design probability assessment activities.	124 (98.4)	3.27	1.14
15. I use probability in daily life.	124 (98.4)	3.47	1.06
*16. I feel intimidated by probability data.	125 (99.2)	3.79	1.08
*17. Probability is understood only by scientists.	124 (98.4)	3.81	1.17
*18. I avoid reading information where probability terms appear.	125 (99.2)	4.19	0.99
19. Probability knowledge helps students to reason critically.	125 (99.2)	3.94	0.88
20. More time should be devoted to teaching probability in the first levels of education.	126 (100)	3.64	1.05
*21. I am concerned about knowing to answer my student's probability questions.	123 (97.6)	2.67	1.34
*22. I don't feel ready to solve any probability problem.	125 (99.2)	3.29	1.25
*23. I think I will not be able to prepare teaching resources for probability content lessons.	125 (99.2)	3.78	1.04
24. When appropriate, I will use probability in the others mathematics curriculum areas.	123 (97.6)	3.89	0.86
*25. If I could eliminate any subject from the mathematics curriculum it would be probability.	123 (97.6)	4.15	1.10
*26. I have no interest in teaching probability content, even if they appear in the curriculum.	126 (100)	4.05	1.12
*27. I don't like solving probability problems.	125 (99.2)	3.65	1.23
28. As a future teacher, I think I will feel comfortable teaching probability content.	126 (100)	3.77	0.99

Appendix 2. Varimax rotated factor matrix for the 7-factor model

Item	Factor						
	1	2	3	4	5	6	7
1	0.785	-0.024	0.017	0.039	0.031	0.021	-0.174
2	0.276	0.163	-0.136	0.706	0.294	-0.049	0.047
3	0.127	0.629	0.099	-0.137	0.366	0.156	0.122
4	-0.009	0.039	0.299	0.447	0.517	-0.090	0.067
5	0.808	-0.019	-0.137	0.094	0.067	-0.027	0.241
6	0.090	0.199	-0.100	-0.101	0.229	0.640	0.048
7	-0.077	0.089	0.298	0.674	-0.167	0.076	0.102
8	0.204	0.285	0.056	0.201	0.065	0.735	0.056
9	0.616	0.515	0.111	-0.144	0.248	0.148	-0.040
10	0.231	0.603	0.029	0.116	-0.044	0.409	-0.037
11	0.549	0.133	0.335	0.159	0.034	0.085	0.097
12	0.196	0.306	0.428	0.074	0.221	-0.360	0.322
13	0.087	0.024	0.699	0.020	0.284	-0.122	0.150
14	0.257	0.735	0.012	0.192	0.051	-0.201	0.039
15	0.259	0.052	0.025	0.800	0.165	0.083	0.019
16	-0.117	0.368	0.098	-0.012	0.046	0.104	0.760
17	0.023	0.034	0.732	0.029	-0.232	0.006	0.019
18	0.147	0.067	0.097	0.128	-0.013	0.013	0.831
19	0.166	0.090	0.466	0.274	0.426	0.105	-0.022
20	0.418	-0.367	0.246	0.097	0.394	0.302	0.239
21	-0.190	0.535	-0.032	0.040	0.013	0.251	0.103
22	0.111	0.645	0.112	0.117	-0.207	0.348	0.152
23	0.031	0.663	0.016	0.119	-0.018	0.001	0.202
24	0.110	0.056	-0.057	0.082	0.725	0.168	-0.015
25	0.542	-0.064	0.641	0.105	0.066	0.102	0.042
26	0.755	0.232	0.281	0.113	0.056	0.049	0.027
27	0.657	0.281	0.122	0.233	0.054	0.228	0.034
28	0.469	0.597	0.042	0.000	0.260	0.232	-0.040

Note: The four major correlations per factor are yellow highlighted while those case where their maximum weight between factors doesn't correspond to theoretical components are calypso (blue) stressed.