

# Teachers' Perceptions of the Construction of Mathematical Concepts in Everyday Contexts

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## ABSTRACT

**Background:** As it is important to link the disciplines to real contexts, it is necessary to propose teaching-learning strategies based on diverse situations, inside or outside the mathematical field. This study considers strategies and conceptions of teaching in contextual mathematics used by basic education teachers in the Maule Region, Chile. **Objectives:** Establish beliefs of basic education teachers in this region about the use of contexts in the mathematics teaching and learning processes. **Design:** Mixed in nature, considering the collection of quantitative and qualitative data simultaneously. The descriptive research design uses closed and open questions to collect data. The closed questions, presented on a Likert scale from 1 to 5, exploring five dimensions related to the mathematical topic in context were: 1) processes, 2) skills and 3) mathematical contextualisation, 4) national curriculum and 5) favourable/unfavourable conditions to work in context. **Settings and Participants:** 99 primary education teachers from the Maule region, who guide pedagogical practices in mathematics of primary teachers in training. **Data collection and analysis:** The qualitative data were grouped into deductive categories considering the literature and the participants' answers. **Results:** The teachers' valuation of the quantitative dimensions and aspects used to build mathematical knowledge is highlighted, but they also value the exercise-type activities, which they relate mainly to mathematical concepts of numbers and operations. **Conclusions:** Mathematical practice in contexts is well valued by teachers, but not yet assumed as an adequate strategy to evaluate learning achievements.

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## **Concepciones del profesorado sobre la construcción de conceptos matemáticos en contextos cotidianos**

### **RESUMEN**

**Antecedentes:** Como importante es vincular las disciplinas a contextos reales, es necesario proponer estrategias de enseñanza-aprendizaje basadas en situaciones diversas, dentro o fuera del campo matemático. Este estudio considera estrategias y concepciones de enseñanza en matemáticas contextuales utilizadas por profesores de educación básica de la Región del Maule, Chile. **Objetivos:** Establecer creencias de profesores de educación básica en esta Región, sobre el uso de contextos en procesos de enseñanza y aprendizaje de las matemáticas. **Diseño:** De carácter mixto, considerando recolección de datos cuantitativos y cualitativos simultáneamente. El diseño investigativo, de tipo descriptivo, utiliza preguntas cerradas y abiertas para recolectar los datos. Las preguntas cerradas, presentadas en una escala de Likert de 1 a 5, explorando cinco dimensiones relativas al tema matemática en contexto fueron: 1) procesos, 2) habilidades y 3) contextualización matemática, 4) currículum nacional y 5) condiciones favorables/desfavorables para trabajar en contexto. **Contexto y Participantes:** 99 docentes de educación primaria de la respectiva Región, que guían prácticas pedagógicas en matemáticas, de profesores de primaria en formación. **Recopilación y análisis de datos:** Los datos cualitativos se agruparon en categorías de tipo deductivo considerando la literatura y las respuestas de los participantes. **Resultados:** Se destaca la valoración docente sobre las dimensiones cuantitativas y aspectos utilizados para construir conocimiento matemático, pero también las actividades tipo ejercicios, que relacionan mayoritariamente con conceptos matemáticos sobre números y operaciones. **Conclusiones:** La práctica matemática en contextos es bien valorada por los profesores, pero aun no es asumida como estrategia adecuada para evaluar logros de aprendizaje. **Palabras clave:** concepciones- contextos – profesores- conceptos matemáticos

## **Percepções dos professores sobre a construção de conceitos matemáticos em contextos do cotidiano**

### **RESUMO**

**Contexto:** Como é importante vincular as disciplinas a contextos reais, é necessário propor estratégias de ensino-aprendizagem baseadas em situações diversas, dentro ou fora do campo matemático. Este estudo considera estratégias e conceitos de ensino em matemática contextual usados por professores da educação básica na região do Maule, Chile. **Objetivos:** Estabelecer crenças dos professores do ensino básico nesta região, sobre a utilização de contextos nos processos de ensino e aprendizagem

da matemática. **Metodologia:** Mista, considerando a coleta de dados quantitativos e qualitativos simultaneamente. O design descritivo da pesquisa utiliza questões fechadas e abertas para a coleta de dados. As questões fechadas, apresentadas em escala Likert de 1 a 5, explorando cinco dimensões relacionadas ao tema matemático em contexto foram: 1) processos, 2) habilidades e 3) contextualização matemática, 4) currículo nacional e 5) condições favoráveis/desfavoráveis para trabalhar em contexto. **Contexto e participantes:** 99 professores do ensino básico dessa região, que orientam as práticas pedagógicas em matemática de professores estagiários do ensino básico. **Coleta e análise de dados:** Os dados qualitativos foram agrupados em categorias dedutivas considerando a literatura e as respostas dos participantes. **Resultados:** Destaca-se a valorização do professor sobre as dimensões e aspectos quantitativos utilizados na construção do conhecimento matemático, mas também as atividades do tipo exercício, que se relacionam principalmente com conceitos matemáticos sobre números e operações. **Conclusões:** A prática matemática em contextos é muito valorizada pelos professores, mas ainda não é assumida como uma estratégia adequada para avaliar os resultados da aprendizagem.

**Palavras-chave:** concepções - contextos - professores - conceitos matemáticos

## INTRODUCTION

Teachers' conceptions of mathematics have been of great research interest in recent years, since they have been considered one of the aspects of the affective domain for teaching, which can explain, among other aspects, students' academic performance (Gamboa, 2014).

Researchers argue that the teachers' conceptions of mathematics are personal and subjective judgments that bias them toward the development of certain teaching practices (Leavy and Hourigan, 2018; Polly, Neale & Pugalee, 2014). Such teaching conceptions and practices causes them to interpret, decide and act accordingly (Rodrigo, Rodríguez & Marrero, 1993), which implies selecting textbooks, adopting teaching strategies and assessing the teaching-learning process according to their own beliefs. This certainly influences and encourages learning styles and objectives in quite different directions, such as those that lead, on the one hand, to rote learning and, on the other, to analytical learning. The first ones have classically guided us to a mathematics practice of repetition and application of algorithms, and the latter, to the development of superior skills.

In Mora and Barrantes (2008), the teacher transmits his own view to the students, what will influence them in a certain way of approaching the mathematics study. A relevant aspect related to beliefs rooted in an educational system is the danger of inhibiting reflection on the suitability of teaching

practices. In this regard, Seckel and Font (2015) highlight the pedagogical standard number 10 of the competency-based training model for the first years of elementary education teachers in Chile (MINEDUC, 2012, p. 17), which, among other aspects, refers to reflection on one's own practice: "Learns continuously and reflects on his/her practice and his/her insertion in the educational system". In this sense, they declare: it seems to us that the proposal of didactic analysis provided by the ontosemiotic approach allows us to guide clearly the reflective processes through six facets: the epistemic, the cognitive, the interactional, the mediational, the emotional and the ecological facet. Thus, its adoption as a reflection model would also represent an objective way of understanding the mathematics teaching and learning process, reducing the effect caused by the subjectivism to which it is exposed when it is influenced by a particular belief system.

This justifies research focusing on characterising teachers' and future teachers' conceptions before addressing proposals for innovation in the classroom (Seckel, Breda, Sánchez a& Font, 2019; Thompson, 1992), or analysing how changes in beliefs are related to effective teaching (Leikin & Zazkis, 2010).

In this analysis of the importance of understanding belief systems on mathematics and its teaching-learning processes, we highlight their complexity. Denying it would lead to simplistic proposals for teaching-learning models that would leave aside the consideration of aspects represented in the questions: How is knowledge produced? When and where is it produced? Why? Under what conditions? The answers to these questions are necessarily conditioned by teachers' and the students' belief systems at the different levels, however, a more critical and in-depth analysis of them could lead, on the contrary, to a change in these same belief systems.

Cerda, Pérez, Casas and Ortega-Ruiz (2017) present some dimensions that intervene in the teaching and learning processes of mathematics and that must be considered when deciding which could be the best way to carry them out in class, since they point out that in this process multiple factors intervene, such as, for example, the teachers training, their confidence on the subject matter, the didactics used in the classroom, their autonomy for work, parents' cultural level, the school environment and educational proposal of the institution, to name a few.

In recent years, the role of emotions in mathematics learning has also been highlighted, as Pekrun (2014) indicates, a classroom is a place of emotions,

where students express various states of mind, being emotions the ones that control students' attention, and influence their motivation to learn.

## **Mathematics in context**

The ability to solve context-based mathematical tasks is being focused by research on mathematics education (Font, 2006; NCTM, 2000; Wijaya, Van den Heuvel-Panhuizen & Doorman, 2015a). That could be explained by the importance given to the competence that students should develop to apply school mathematics to extra-mathematical contexts (Font, 2006), and because mathematics in context creates strong connections with the study of situations and solutions of real-world problems (Barbosa, 2006; Villa, 2007), making it possible, at the same time, to become a teaching strategy to approach mathematical concepts in the classroom (Bassanezi, 2002).

In Lange (1996), there are four reasons for contextualised problems to be integrated into the curriculum. They: a) simplify mathematics learning, b) develop the competencies of citizens' competencies, c) develop the general competencies and attitudes associated with problem solving and d) allow students to see the usefulness of mathematics to solve situations from other areas as well as situations from everyday life. This allows not only developing skills but also building mathematical concepts and ideas, valuing the contexts in which this mathematics is observed.

Introducing real context in a problem (Van Den Heuvel-Panhuizen, 2005; Villa-Ochoa, 2015; Bassanezi, 2002) can increase accessibility and understanding and can propose several resolution strategies. However, Bonotto (2005, 2007) observes that mathematical problem-solving practices are being consigned to classroom activities that limit opportunities for children to explore complex, disordered and real data that generate their own constructs and processes to solve authentic problems (Hamilton, 2007), since these learning experiences are rarely presented in contexts that relate mathematics from the school context to the mathematics that they apply to solve problems. Along these same lines, Verschaffel, de Corte, and Borghart (1997) provide evidence that emphasises a strong and resistant trend among future teachers to exclude real-world knowledge and realistic considerations when it comes to arithmetic verbal problems in teaching tasks, considering that these move away from learning and from the manner children must find the correct numerical ways.

Given the importance of relating disciplines and content to real situations and contexts today, the NCTM (2000) urges the development of a

teaching and learning proposal based on diverse situations, both within and outside the mathematical field; and, in this sense, they recommend that students make and investigate mathematical conjectures, evidencing different mathematical processes. For this, Verschaffel (2002) and Bonotto (2007) consider that it would be necessary to replace stereotyped problems with more realistic, literal statements, bringing reality closer to mathematics classrooms, creating opportunities for students to solve contextualised problems.

From different researchers' point of view (e.g., Bonotto, 2005, 2007; Van Den Heuvel-Panhuizen, 2005) the need to bring mathematics closer to real-world problems (Villa-Ochoa 2009) is then raised, arguing its importance and revealing its binding aspect with mathematical knowledge and its use as a tool to construct mathematical teaching processes (OECD, 2016; UNESCO, 2014). Along the same lines, Bassanezi (2002) explains that mathematics in contexts generates strong connections with the study of situations and the solution of real-world problems (Barbosa, 2006; Villa, 2007), while making it possible to become a didactic strategy to approach mathematical concepts in the classroom. Therefore, recognising the importance of mathematics in contexts to bring this discipline closer to the students, we set out to analyse the teachers' notions of the construction of mathematical concepts in everyday contexts. Knowing the conceptions of mathematics, its teaching and learning will allow us some vision on how teachers understand and carry out their work in classrooms (Benken and Brown, 2002).

## METHODOLOGY

This is a mixed study, for which we chose to collect quantitative and qualitative data concurrently or simultaneously to achieve the research objective (Guzmán, 2015). The research design is descriptive (Bisquerra, 2012), using the survey as a data collection technique (Torrado, 2016). For this survey, we carried out a specialised literature review, thus giving significance to the items that compose the instrument. Specifically, a theoretical analysis was carried out regarding the construction of context-based mathematical learning. In this instrument, I consider two sections. The first corresponds to 25 items of closed questions that aim to study the degree of agreement/disagreement that the participants express through a Likert scale from 1 to 5, where 1 corresponds to *strongly agree* and 5 to *strongly disagree*. In this way, five dimensions were explored around the subject of mathematics in context:

1) *mathematical processes*: are ways of acquiring and applying mathematical knowledge, being problem solving, reasoning and demonstration, communication, representation, and connections (NCTM, 2000).

2) *mathematical skills*: seek to develop both mathematical thinking and how to apply knowledge to solve typical mathematics problems and of other areas of knowledge through solving problems, representing, modelling, and arguing and communicating.

3) *mathematical contextualisation*: idea, opinion, way, or circumstances that allow understanding or that explain a (mathematical, non-mathematical) situation.

4) *national mathematics curriculum*: knowledge of the curriculum framework and learning contexts such as evaluation, organisation of the environment, methodological strategies, mediation strategies, and group formation.

5) *favourable/unfavourable conditions for work in context*: factors or circumstances that allow activities or experiences to be developed or not.

In the second section, the instrument considered four open questions that sought to delve into specific areas related to the topic. Subsequently, the instrument was submitted to evaluation by experts whose contributions allow the instrument to be refined to be applied.

Next, in Table 1, the dimensions explored in the closed questions section and the items related to them are observed.

**Table 1**

*Instrument characterisation: closed answers*

<b>Dimension</b>	<b>Items</b>
<b>Mathematical processes</b>	1, 6, 10, 18 and 25
<b>Mathematical skills</b>	5, 9 and 20
<b>Mathematical contextualisation</b>	7, 11, 16 and 19
<b>National mathematics curriculum</b>	2, 3, 4, 13, 14, 15, 17 and 24,
<b>Favourable/unfavourable conditions for the</b>	8, 12, 21, 22 and 23

## **Study participants**

The participants in this study were 99 elementary education teachers from the Maule Region (Chile), guiding pedagogical practices in the mathematical area of elementary school teachers in training; 75.8% of them are women, and 24.2% are men.

## **Analysis of Results**

Data analysis was carried out through the SPSS 18.0 statistical program, and the techniques used were mainly descriptive statistics of central tendency (mean, mode), dispersion (standard deviation) and asymmetry coefficient. The analyses carried out to estimate the reliability of the instrument as internal consistency show a Cronbach's Alpha of .90 for the total of the participants (N = 99).

On the other hand, for the treatment of qualitative data, categories of deductive analysis were considered after a detailed study of the existing literature and the participants' answers. Thus, for question 1, the categories of analysis correspond to the five thematic axes presented in the basic education curriculum bases (MINEDUC, 2018).

*Numbers and Operations axis:* developing the concept of numbers, mental calculation and use of algorithms.

*Patterns and Algebra axis:* relating numbers, shapes, objects, and concepts. Representing patterns in a concrete, pictorial, and symbolic way. Abstract and algebraic thinking.

*Geometry Axis:* recognising, visualising, and drawing figures. Describing the characteristics and properties of 3D and 2D figures. Understanding and describing the space structure.

*Measurement Axis:* identifying the characteristics of the objects and quantifying them, to compare and order them. Determining non-standard measures.

*Data and Probabilities Axis:* recording, classifying, and reading information arranged in tables and graphs, Probabilities.

Question 2 considers the pyramid representation of mathematics

education proposed by Alsina (2010), where the different contexts for the development of mathematical thinking as shown in Figure 1 and their frequency of use are presented: *Everyday situations, mathematisation of the environment, experiences with the body; Manipulative resources, nonspecific, commercialised, or designed materials; Recreational resources, games; Literary resources, narrations, riddles, songs; Technological resources, computer, calculator; and Books.*

**Figure 1**

*Mathematics education pyramid (Alsina, 2010)*



For question 3, the categories of analysis correspond to the skills proposed in the basic education curriculum bases (MINEDUC, 2018).

*Solving problems:* managing to solve a given -contextualised or not- problem situation without being instructed to follow a procedure.

*Arguing and communicating:* progressively establishing deductions that will allow students to make effective predictions in various specific situations.

*Modelling:* using and applying models, selecting and modifying them, and building mathematical models, identifying typical patterns of situations, objects or phenomena that you want to study or solve:

*Representing*: better understanding and operating with concepts and objects already built.

Finally, for question 4, an analysis was made based on the theory of didactic suitability (Font, Planas, and Godino, 2010), which has been useful in other investigations in which the analysis of the teachers' discourse when arguing about teaching processes has been required (Seckel et al., 2019). In other words, their discourse is categorised based on the six dimensions: epistemic, cognitive, interactional, mediational, emotional, and ecological.

## RESULTS AND ANALYSIS

This section shows the results related to the first part of the survey (25 statements), where you can see the mean values and standard deviations of the dependent variables for the total sample (N = 99), as well as the asymmetry coefficient. Subsequently, the results obtained from the open questions presented in the survey are shown.

In Table 2, we can see that teachers on average value the *mathematical processes* by agreeing strongly with most of the statements (items 1, 10 and 18). These results are confirmed by the modal value and the asymmetry coefficient, which is generally positive asymmetric. Items 6 and 25, which refer to the use of connections and deductive processes as the only ones that matter for learning school mathematics, present a negative asymmetry coefficient. This reveals that teachers do not recognise connections as a tool to build learning. The literature indicates that mathematical processes are tools to analyse the various contexts critically and pick those that are most effective in planning and managing competency-based mathematical activities (Alsina, 2012; 2019).

**Table 2**

*Mathematical processes*

Indicators	Avg.±D.E	Mode	Asymmetry C.
<b>The planning and implementation of activities in context foster the development of mathematical processes.</b>	1.4 ± 0.8	1.0	2.8
<b>The use of connections helps students to understand mathematics,</b>	1.4 ± 0.8	1.0	-2.3

<b>enabling them to create and use their own representations.</b>			
<b>The connection between school mathematics and real or everyday world problems is essential.</b>	1.2 ± 0.5	1.0	4.0
<b>The processes of “mathematical modelling” in the classroom allow us to understand the role that the student’s own contexts play in school mathematics.</b>	1.6 ± 0.8	1.0	1.3
<b>Deductive processes are the only important ones for learning school mathematics.</b>	3.5 ± 1.2	4.0	-0.6

In relation to the items that are related to the dimension *skills to develop mathematical learning* in contexts, as shown in Table 3, on average, the teachers strongly agree with the statements presented in the instrument, while acknowledging that students develop mathematics skills in everyday problems, focused on real contexts, allowing them to better represent math ideas. Teachers recognise that the development of skills leads students to have a more active and participatory attitude, allowing them to develop learning, being quite aligned with what is defined in the curriculum bases (MINEDUC, 2018), pointing out that skills are necessary for the student to discover, explore and build knowledge.

**Table 3**

*Skills to develop mathematical learning*

<b>Indicators</b>	<b>Avg.±D.E</b>	<b>Mode</b>	<b>Asymmetry C.</b>
<b>Mathematical skills are best developed in a meaningful context for students.</b>	1.3 ± 0.7	1.0	3.1
<b>Students better develop their skills when math problems are presented in a real context.</b>	1.3 ± 0.7	1.0	3.1

<b>During the mathematical modelling process, students can use different representations to solve context problems.</b>	1.5 ± 0.8	1.0	2.3
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In Table 4, we can observe that, on average, teachers recognise that students construct mathematical learning more easily when in real contexts, also creating a positive attitude in the teaching and learning process. They agree strongly with the statements given in the instrument. These results are confirmed by the mode value and the asymmetry coefficient, which is generally positive asymmetric. These results are related to what was exposed by Alsina (2016), who points out that the use of mathematics in non-exclusively school contexts are tools that favour the motivation, interest or meaning of mathematics, contributing to the formation of more competent people mathematically.

**Table 4**

*Mathematical contextualisation*

<b>Indicators</b>	<b>Avg.±D.E</b>	<b>Mode</b>	<b>Asymmetry C.</b>
<b>The application of mathematical ideas in extra-mathematical contexts and situations favours student learning.</b>	1.4 ± 0.6	1.0	2.2
<b>Students' understanding is boosted if problems are constructed from contexts that are familiar and close to them.</b>	1.2 ± 0.61	1.0	3.5
<b>The use of varied contexts such as daily, cultural, and technological contexts promotes the student's involvement and motivation in learning mathematics.</b>	1.3 ± 0.61	1.0	2.3

**The historical-cultural contexts are appropriate to design tasks that allow students to generate critical thinking and mathematical knowledge.**

1.7 ± 0.8      1.0 and 2.0      1.4

About the indicators related to the dimension *national mathematics curriculum*, as shown in Table 5, the teachers recognise that students make sense of mathematics when the activities are linked to their lives and interests, to everyday contexts, which favours the construction of personal interpretations of actual situations and formulate them as significant mathematical problems. Along these lines, the teachers, on average, strongly agree with most of the statements presented in the instrument. These results are confirmed by the mode value and the asymmetry coefficient, which is generally positive asymmetric. However, the teachers evaluated negatively the indicators that speak of the evaluation only through written tests; and that examples, problems and activities proposed for teaching and learning mathematics in the classroom should not consider the culture, history and heritage, as can be observed in the negative asymmetric distribution shown in Table 4. This contradicts what was assumed by English and Gaingsburg (2016) when stating that school mathematics curriculum must train citizens to apply mathematics to daily life problems in social, work and interdisciplinary situations; apply various didactic approaches to respond to the needs of all students; use evaluation methods effectively.

**Table 5**

*National mathematics curriculum*

Indicators	Avg.±D.E	Mode	Asymmetry C.
<b>Mathematical learning should be evaluated only through written tests.</b>	4.4 ± 0.9	5.0	-1.8
<b>In its annual programming, it always considers activities that involve mathematics work in contexts of daily life, games, and culture.</b>	1.8 ± 0.9	2.0	1.6
<b>The examples, problems and activities proposed for teaching and</b>	4.4 ± 0.9	5.0	-2.2

**learning mathematics in the classroom should not consider the culture, history, and heritage.**

**Mathematical problems in real contexts allow students to construct personal interpretations of actual situations and formulate them as meaningful mathematical problems**

1.3 ± 0.6      2.0      2.9

**Students gain a better understanding of the mathematical content when faced with situations that allow them to problematise, discover variables and their relationships, argue, and use different mathematical expressions.**

1.4 ± 0.6      1.0      2.4

**For students to make sense of mathematics in the initial phase of learning, it must be personally and socially linked to their life.**

1.4 ± 0.7      1.0      2.7

**Students' understanding can be boosted if the student finds the new mathematical content in a familiar context.**

1.4 ± 0.8      1.0      2.3

**In mathematics teaching, it is essential to know the students and their interests so that a relevant link between the mathematical content and their reality is achieved.**

1.3 ± 0.6      1.0      2.4

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In the dimension *favourable/unfavourable conditions for the work in context*, as shown in Table 6, we observed that teachers strongly agree with the assertions provided in the instrument, which is confirmed by the modal value and the asymmetry coefficient, which is generally positive asymmetric. For Wijaya, Van den Heuvel-Panhuizen, and Doorman (2015b), teachers must analyse their own role in students' learning processes critically, since the teachers are responsible for offering them opportunities to learn to solve

context-based tasks. Teachers, then, share this view. They recognise that the construction of mathematical knowledge is favoured when problems are posed in real, familiar contexts, which allows students to connect school mathematics with the real world, finding meaning and utility in mathematical learning.

**Table 6**

*Favourable/unfavourable conditions for the work in context*

<b>Learning mathematical knowledge in family contexts favours the transfer and application to different contexts.</b>	1.4 ± 0.6	1.0	1.6
<b>Presenting problems in real contexts promote better skill development of mathematical thinking in students.</b>	1.3 ± 0.7	1.0	2.6
<b>Problems that articulate contextualisation situations allow school mathematics to be connected to the real world.</b>	1.4 ± 0.6	1.0	2.3
<b>Problems posed in real contexts expand intuitive thinking and form deductive and logical thinking.</b>	1.5 ± 0.7	1.0	1.8
<b>Problems in which the data and the unknowns are clearly specified, and the solution paths are easily inferable allow students to develop their own resolution strategies better, favouring the construction of mathematical ideas.</b>	2.6 ± 1.2	2	0.6

As mentioned above, the second section of the instrument included open-ended questions, prepared to collect more extensive and detailed information on the subject under study. Hence, question 1 (see Figure 2) sought to identify the type of mathematical activities that teachers propose to build mathematical learning, considering the fair as a learning context, considering for its analysis and categories the axes of content: *number and operations; patterns and algebra; geometry; measurement; data and probability*. Table 7 shows the distribution of the content axes, linked to the activities proposed by the teachers.

## Figure 2

### Open question N° 1.

1. La foto muestra un aspecto de una feria pública. Describa algunas actividades de aprendizaje matemático que pudiera realizar con sus estudiantes en este contexto.



The answers provided by the participants show that 87% of their activity proposals are related to the *Numbers and Operations* axis. This result is explained by the presence given in the mathematics curriculum bases (MINEDUC 2012), where 44 learning objectives are presented to work this axis during the first four years of elementary education.

An example of activity in this axis is seen in what was expressed by participant 4 (P4), who manages learning situations to internalise knowledge related to number writing and place value. However, in P4's proposal we do not recognise experiences with a sense of usefulness, as students need to be encouraged in everyday contexts, where they can soon see the practical usefulness of a mathematics that can seem arid and without applications.

the following activities can be carried out with the prices of fruits and vegetables: Write in words the price of vegetables and fruits. Through the money, they can count by 10's, by 100's and by 1,000's. Identify the place value of the digits of a number. Additively compose and decompose numbers up to 10,000. (P4)

Regarding the *patterns and algebra* category, which obtained 34% of activity proposals, the literature (Blanton & Kaput, 2005; Kaput, 2000; NCTM, 2000) recognises the importance of encouraging in class processes and ways to recognise mathematical patterns, relationships, and properties in an

environment where students are valued to explore, model, predict, discuss, and argue. In P42's proposal, we found a possible suggestion of pattern activity, where P42 observes that the shape of the fruits and vegetables corresponds to their own nature and that it is a characteristic that allows their classification through a system of patterns. For example, the shell of pineapples has a spiral distribution that, in number, is always the same, i.e., it has a standard of distribution.

analyse the shape of fruits, vegetables, and boxes. (P42)

An example of activity related to *algebra* can be seen in P80's proposal, which involves making an algebraic model that relates the final value of the purchase of a product with the amount of the same in the respective purchase.

in the supply context, percentage variations can be applied by applying discounts and calculating the final value of a product. (P80)

Concerning the *measurement* category, which reaches 23%, the curriculum bases (MINEDUC, 2018) indicate that students can select and use the appropriate unit to measure time, capacity, distance, and weight, using specific tools according to what is being measured. An example of this is what P94 proposes by presenting an interesting problem from the point of view of the variables, since it relates the measurement of two magnitudes (weight and quantity), implying a level of complexity in the proposal.

calculate the amount of fruit, such as an apple that may be per kilogram of it. If a tomato box weighs 500 grams and there are 33 tomatoes, and each tomato weighs 125 grams, how many kilograms does a box of tomatoes weigh? (P94)

Regarding the *data and probabilities* category, which reaches 20% of the proposals, the literature indicates that there has been an important change in the elementary education curriculum of many countries with the incorporation of probability contents in the last decade (Parraguez, Gea, Díaz-Levicoy, & Batanero, 2017). Furthermore, the importance of statistical tables and graphs seems to be understood from the curriculum, since they are the central element of the statistical culture (Pino, Díaz-Levicoy, & Piñeiro, 2014). In this sense, P16 recognises the work with data tables, being a simple representation and less complex than the frequency tables. In this case, P16 proposes that count tables can be associated with fruit prices.

data tables and percentages in relation to fruit prices. (P16)

Finally, in relation to the *Geometry* category, 9% of activities are proposed by teachers. An example is found in P43's proposal, where the respondent points out that geometric forms can be seen in the shapes of the fruits. In these ideas, an interesting level of abstraction is recognised when thinking that it is possible to *sense* that bodies and geometric figures can be identified in fruits and vegetables, where, for example, an orange can be thought of as a sphere that could be inscribed in a cube. A small problem inspired by this exercise would be to identify properties of bodies with vertices as the tangent points. Barrantes, Balletbo, and Fernández (2014) point out that the problem solving methodology in geometry teaching-learning must be based on the learning of geometric concepts through tasks immersed in a real-life context, in which situations and problems that arise are not necessarily perfectly finished and exemplary.

Geometry, geometric figures, and bodies. Identify bodies and geometric figures in fruits and vegetables. (P43)

Table 7 shows the frequency and percentage distribution presented by each of the indicators related to the axes of mathematical content linked to the activities proposed by the teachers. We can observe that most teachers propose activities related to the *numbers and operations* axis (87%). Then, 34% of the proposed is linked to the *patterns and algebra* axis. Concerning the *measurement and data and probabilities* axes, we see that the activities proposed are 23% and 20%, respectively, while only 9% propose activities related to the *geometry* axis.

**Table 7.**

*Axes of mathematical content linked to the activities proposed by the teachers*

Contents	Frequency (n=99)	Percentage
<b>Numbers and operations</b>	87	87
<b>Patterns and Algebra</b>	34	34
<b>Geometry</b>	9	9
<b>Measurement</b>	23	23
<b>Data and probabilities</b>	20	20

Regarding question 2, the exploration of the answers sought to account for the importance that teachers give to a list of resources to develop mathematical thinking, and this reveals the importance that they give to mathematics in context in the early grades. Participants were presented the question as shown in Figure 3.

**Figure 3**

*Open question N° 2*

2. En esta representación piramidal ordene el listado de recursos que a continuación se identifican, como importantes y necesarios para desarrollar el pensamiento matemático (Alsina, 2010). Ordene la información en orden ascendente, colocandó en la base de la pirámide lo que consideras más importante y en la cúspide lo menos importante.

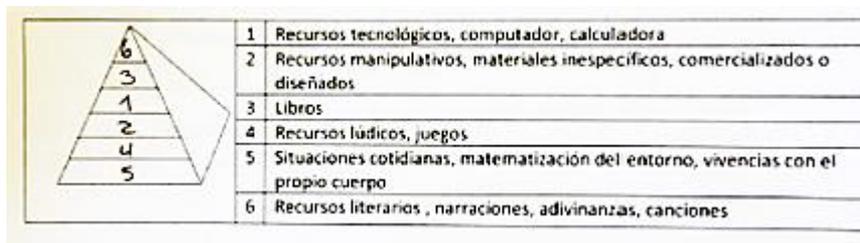
	1	Recursos tecnológicos, computador, calculadora
	2	Recursos manipulativos, materiales inespecificos, comercializados o diseñados
	3	Libros
	4	Recursos lúdicos, juegos
	5	Situaciones cotidianas, matematización del entorno, vivencias con el propio cuerpo
	6	Recursos literarios , narraciones, adivinanzas, canciones

Table 8 shows the indicators of the pyramid for mathematics education (Alsina, 2010) and the assessment made by teachers based on importance, where everyday situations correspond to the base of the pyramid and the indicator books is located at the top.

The first indicator of *everyday situations, mathematisation of the environment, experiences with their own body* was identified as the main resource by 60 Ps, considering that to start any mathematical learning, one must start with the student’s daily life. Similarly, the curriculum bases reveal that starting from everyday situations or problems allows students to develop capacities to make sense of the world and to act in it. (MINEDUC, 2018, p. 214). An answer that considers this the main indicator is shown in Figure 4, posed by P20.

#### Figure 4

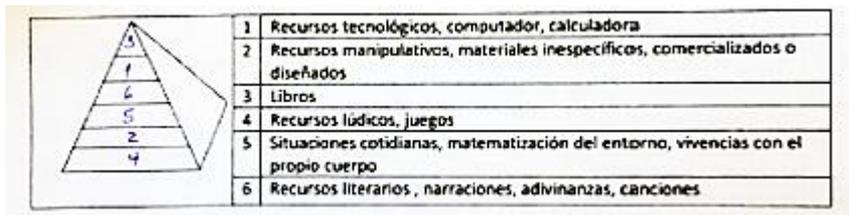
Answer to open question N°2. Everyday situations, mathematisation of the environment (P20)



In relation to the indicator *manipulative resources, nonspecific materials*, 26 Ps coincide with what the author proposed, defining it in the second place, valuing manipulative material to build mathematical learning. The literature (Clements, 1999; Rosli, Goldsby, & Capraro, 2015) argues that by using manipulative materials students can reflect on their actions and explore the concepts by themselves; teachers must help students to “see” the mathematical relationships between materials and abstract symbols. Figure 5 shows an example of a response from P4.

#### Figure 5

Answer to open question N°2. Manipulative resources, nonspecific materials (P4)

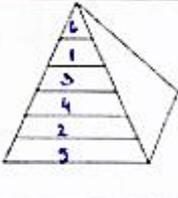


The indicator *recreational resources, games*, was considered in the third level of the pyramid by 26P. The literature (Castro, Menacho, & Velarde, 2019; Chamoso, Durán, García, Martín, & Rodríguez, 2004) explains that recreational activities, like mathematics, have educational purposes, and that the application of didactic games based on meaningful focus improves learning

achievement in the area of mathematics. Figure 6 shows an example of P55's response.

**Figure 6**

Answer to open question N°2. Recreational resources, games (P55)

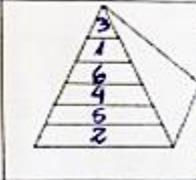


1	Recursos tecnológicos, computador, calculadora
2	Recursos manipulativos, materiales inespecificos, comercializados o diseñados
3	Libros
4	Recursos lúdicos, juegos
5	Situaciones cotidianas, matematización del entorno, vivencias con el propio cuerpo
6	Recursos literarios, narraciones, adivinanzas, canciones

Regarding indicator *literary resources, narrations, riddles, songs*, 16 Ps locate this indicator in the place granted by the author. For Mari and Gil (2006) and Marín (2013), stories are a good resource for developing mathematical competence, bringing mathematics closer to the child's reality. Figure 7 shows an example of P59's response.

**Figure 7**

Answer to open question N°2. Literary resources, narrations, songs (P59)

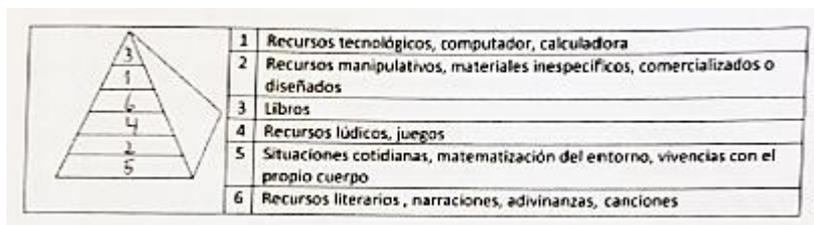


1	Recursos tecnológicos, computador, calculadora
2	Recursos manipulativos, materiales inespecificos, comercializados o diseñados
3	Libros
4	Recursos lúdicos, juegos
5	Situaciones cotidianas, matematización del entorno, vivencias con el propio cuerpo
6	Recursos literarios, narraciones, adivinanzas, canciones

The indicator *technological resources, computer, calculator*, 22 Ps located it in the place proposed by the author. The correct use of technology is one of the principles formulated by the NCTM (2000, p. 24), which states that: "Technology is essential in learning and teaching mathematics. This medium can positively influence what is taught and, in turn, increase student learning". Figure 8 shows an example of P60's response.

**Figure 8**

Answer to open question N°2. Technological Resources (P60)



Finally, for the indicator *books*, 33 Ps place it at the top of the pyramid, coinciding with the model proposed by the author. For Díaz-Levicoy, Batanero, Arteaga, and López-Martin (2015), the textbooks constitute a didactic resource with a long tradition in teaching and learning of different subject matters, being recognised as mediators between the content of the curriculum and the students, which should be used occasionally (Alsina, 2010; Santaolalla, Gallego, & Urosa, 2017).

**Figure 9**

Answer to open question N°2. Books. (P11)

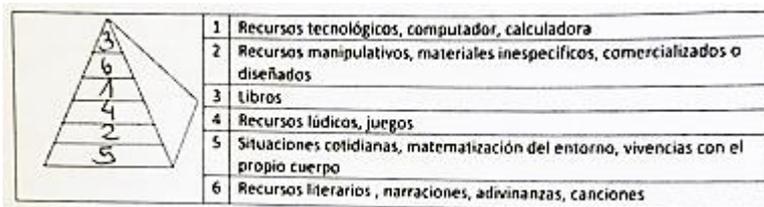


Table 8 shows that most participants (60 out of 99) believe that *everyday situations that allow the environment to be mathematised* are fundamental to building mathematical knowledge. The *manipulative resources* were identified in the second position, with 26 participants (out of 96 responses), which is consistent with what was proposed in the pyramid (Alsina, 2010). The *recreational resources* were ranked third by 26 participants, as the author shows. In relation to *literary and technological resources*, 16 and 22 respondents identify them in fourth and fifth places, respectively. The indicator *books* was placed at the top of the pyramid by 33 participants. Finally, note that

only one participant identified the different indicators in the correct place on the pyramid.

**Table 8**

*Order in the pyramid representation of mathematics education resources*

Indicators of the mathematics education pyramid	Pyramid position					
	1st	2 <sup>nd</sup>	3rd	4th	5th	6th
Everyday situations, mathematisation of the environment, experiences with their own body.	60	15	7	5	3	6
Manipulative resources: nonspecific, commercialised, or designed materials.	13	26	29	10	11	7
Recreational resources: games.	11	35	26	16	7	4
Literary resources: narrations, riddles, songs.	5	7	5	16	32	30
Technological resources: computer, calculator.	3	6	18	31	22	16
Books.	4	7	11	18	21	33

For the exploration of the data provided in question 3 (see Figure 10), the skills proposed by the basic education curriculum bases proposed in Chile were considered as categories of analysis (MINEDUC, 2018, p. 217).

**Figure 10**

*Open question N° 3.*

3. ¿Qué tipo de habilidades pueden desarrollar los estudiantes con este tipo de actividad?

Ubicados en la Plaza se genera una discusión en torno a las siguientes preguntas:

¿Qué elementos geométricos observa en este sitio?  
 ¿Qué medidas tiene la pileta?, ¿Qué elementos considera para hacer su aseveración?  
 ¿Las baldosas de la plaza tiene forma poligonal? ¿Qué polígono observa? ¿Cuál es la medida de sus lados?, ¿Cuál es la medida de sus ángulos?  
 ¿Puede encontrar en la plaza algún fenómeno o proceso que implique variación de algún objeto? Si lo hay ¿cuáles son sus variables?  
 ¿Hay elementos cuyo análisis pueda desarrollar habilidades de tipo aritmético?  
 ¿Hay elementos cuyo análisis pueda desarrollar habilidades de tipo algebraico?

Respuesta:

Table 9 shows the skills considered by the participants when answering the question. *Arguing and communicating* presented the highest frequency,

with 43.43%. Then we see the category *represent* with 37.37%. The skills of *solving problems and modelling* were less considered by the teachers, with 35.35% and 28.28%, respectively. Only 8 participants did not answer this question.

**Table 9**

*Skills identified as main by teachers*

<b>Skills</b>	<b>Frequency (n=99)</b>	<b>Percentage</b>
<b>Argue and communicate</b>	43	43.43
<b>Modelling</b>	28	28.28
<b>Represent</b>	37	37.37
<b>Solving problems</b>	35	35.35
<b>Does not answer</b>	8	7.92

Finally, with question 4, we can say that 87 of the 99 participants answered. Generally, their responses revealed that when they argue about the importance of working with contextualised problems in the mathematics classroom, they mainly refer to cognitive-type (76% of the participants) and emotional-type arguments (67% of participants) and, to a lesser extent, epistemic, interactional, mediational, and ecological arguments are observed (with 17, 6, 1 and 9%, respectively).

On the other hand, 53% of the participants argue only with one of the six dimensions (or categories) with which their discourse was analysed. 37% argue using two of the dimensions, 9% use three dimensions, and 1% use 4 of the dimensions.

Here are some units of analysis that show results in each of the categories:

The student is a manipulator of mathematical objects and can model the situation according to their convenience. They develop strategies and skills to better solve similar problems. They develop the four essential skills (solving problems, modelling, representing, arguing and communicating). (P79, epistemic dimension).

To achieve significant learning, where they relate the new

information to what they already have, generating skills, allowing themselves to reconstruct said information (new and that which they have). (P5, cognitive dimension).

Use materials for skills. (P13, mediational dimension)

Better socialisation and teamwork are made possible. (P49, interactional dimension)

It is important to work with contextualised mathematical problems close to the students, since they need to know the usefulness that can be given to the learning they are acquiring and to be able to use it in their daily life, outside the classroom. Therefore, if the students are involved in the learning and are presented with mathematics in a real context, they can be motivated, and teaching-learning process can become more effective. (P4, emotional dimension)

That students can transfer academic knowledge to real life and in the future from school to the workplace. (P56, ecological dimension)

## CONCLUSIONS

Considering the results, teachers believe mathematical processes are relevant to contextualise mathematical activities while facilitating their comprehension, also allowing students to create and use their own representations. According to Alsina (2012), those processes highlight the ways to acquire and use mathematical knowledge. The combination of mathematical content and processes favours new views that emphasise not only the content and the process, but –and especially– the relationships established between them.

Likewise, teachers affirm that students develop their skills better when mathematical problems are presented in a real and meaningful context, with mathematical skills being an element that guarantees a mathematically competent student. Along these lines, Chamorro (2003) states that a mathematically proficient student should develop a conceptual understanding of mathematical notions, properties and relationships, procedural skills, strategic thinking and communication skills, mathematical argumentation, and problem solving.

On the other hand, when teachers are asked about the context as facilitators of mathematical learning, they recognise context as an element that gives meaning so that students can construct mathematical knowledge, which makes it easier if the problems are constructed from familiar or close contexts. Likewise, they value varied contexts, as they encourage more the student towards mathematics learning. The NCTM (2000) urges the development of a teaching and learning proposal based on real situations and diverse contexts, both within and outside the mathematics field, bringing reality closer to mathematics classes, creating opportunities for solving contextualised problems (Bonotto, 2007; Verschaffel, 2002).

In relation to the curriculum, teachers recognise that it is essential to know the interests of the students, in this way, different types of problems, examples and activities that consider social, cultural, recreational, and gaming contexts can be sought to achieve mathematical learning. According to Colmenares (2009) and Castro, Menacho-Vargas, and Velarde-Vela (2019), it is necessary to look for several practical, recreational, and novel strategies and activities to make it possible to capture the student's attention, as to put in evidence a mathematics competence. However, despite recognising the favourable and positive aspects of considering the elements detailed above, when teachers are asked about evaluation as the main instrument to measure learning achievements, they take a more radical position, considering that mathematical learning should only be assessed through written tests. Alsina (2016) points out that mathematical competence is evidenced in how students face and solve problematic situations permeated by mathematical processes, which act as vehicles for knowledge. Therefore, each of these elements allows collecting evidence to assess students' mathematical competence.

Regarding the favourable conditions for working mathematics in contexts, teachers indicate that mathematical problems in real, familiar contexts that articulate situations in contextualisation promote mathematical work that favours the transference and application to school mathematics. They also suggest which types of problems should be taken into account to build mathematical learning. They also recognise the particularity of each context and how it makes meaning for the students, allowing them to build models to explain mathematical concepts or ideas.

Likewise, when teachers can describe some mathematics learning activities they could carry out with their students in a proper context, they widely favoured those related to the numbers and operations axis, where writing the price of fruits, identifying the largest values, and calculating prices

are the most frequent activities observed. Blanton and Kaput (2005) recommend a school environment where students are valued to explore, model, make predictions, discuss, argue, check ideas, and practice different skills.

Finally, we can conclude that although teachers positively value the context, recognising it as an important tool to develop skills and build ideas, concepts and meanings of mathematics, as it allows the student to face real problems, assume an active role in their teaching and learning process, improve their attitude and motivation, the proposal of learning experiences in a fair context is reduced to problems of basic operations, reading numbers, counting the number of elements, identifying forms, knowing the monetary system, and estimating magnitudes.

Therefore, it is necessary to change the guidance in teachers' mathematical practice to form citizens who can apply mathematics in daily life problems, in social and in work situations (English and Gaingsburg, 2016) where students can make sense and use mathematics.

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All authors (M.S.S., M.S.S., J.P.F., R.G.E., and C.C.M.) participated in all stages of the research process, as well as in the creation, writing and correction of the article in an equivalent way.

## **DECLARATION OF DATA AVAILABILITY**

The data supporting the results of this study will be made available by the corresponding author M.S.S., upon reasonably previous request.

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