ISSN: 2178-7727
DOI: 10.17648/acta.scientiae.7011

Contributions of Mathematical Modelling for Learning Differential Equations in the Remote Teaching Context

Aldo Peres Campos e Lopes [a]
Frederico da Silva Reis [b]

[a] Universidade Federal de Itajubá, Instituto de Ciências Puras e Aplicadas, Itabira, MG, Brasil
[b] Universidade Federal de Ouro Preto, Programa de Pós-Graduação em Educação Matemática, Ouro Preto, MG, Brasil

Received for publication 21 Feb. 2022. Accepted after review 7 Apr. 2022
Designated editor: Claudia Lisete Oliveira Groenwald

ABSTRACT

Background: The study of the use of Mathematical Modeling as a pedagogical alternative has been an emerging research topic and it is directly linked to the importance of improving the teaching and learning of Mathematics and the development of skills. However, few researches in Mathematics Education have been dedicated to investigating the use of Modeling in the Differential Equations course and in the pandemic context. **Objectives**: Identify and analyse the possible contributions of mathematical modelling activities, in the aspects related to learning and the development of criticality in Engineering students. **Design**: The research is qualitative in its methodological assumptions, designed from the development, execution, and assessment of four mathematical modelling activities involving 1st and 2nd order ordinary differential equations. **Setting and participants**: The activities were carried out with 117 undergrad students from nine Engineering degrees at a federal university in the countryside of the state of Minas Gerais (Brazil), enrolled in the Differential Equations I course, in the 1st semester of 2020. **Data collection and analysis**: Data were collected through the activities carried out, the recording of classes taught remotely, and assessment questionnaires, being analysed through a categorisation made from the confrontation with the theoretical framework that underpinned the research. **Results**: The results allow us to state that the mathematical modelling activities carried out are rich opportunities for students' motivation and learning, allowing for a differentiated exploration of the applications of the mathematical contents involved, contributing to a critical interpretation of reality, albeit in an incipient way. **Conclusions**: Based on the research carried out, we can conclude by highlighting the importance of current and future research in mathematics education in higher education, pointing to a teaching of differential equations that breaks with the traditional model of formula tables and methods of resolution.

Corresponding author: Aldo Peres Campos e Lopes. Email: aldolopes@unifei.edu.br

Keywords: Mathematical modelling; Differential equations; Remote teaching; Engineering courses.

Contribuições da Modelagem Matemática para a Aprendizagem de Equações Diferenciais no Contexto do Ensino Remoto

RESUMO

Contexto: O estudo do emprego da Modelagem Matemática como uma alternativa pedagógica tem sido um tema emergente de pesquisa e está diretamente associado à importância de se melhorar o ensino e o aprendizado da Matemática e o desenvolvimento de habilidades. Porém, poucas pesquisas em Educação Matemática têm se dedicado a investigar o uso da Modelagem na disciplina Equações Diferenciais e no contexto pandêmico. **Objetivos**: Identificar e analisar as possíveis contribuições das atividades de Modelagem Matemática, nos aspectos referentes à aprendizagem e ao desenvolvimento da criticidade de alunos de Engenharia. **Design**: Pesquisa qualitativa em seus pressupostos metodológicos, desenhada a partir da elaboração, realização e avaliação de 4 atividades de Modelagem Matemática envolvendo Equações Diferenciais Ordinárias de 1ª e 2ª ordens. **Ambiente e participantes**: As atividades foram realizadas com 117 alunos de 9 cursos de Engenharia de uma universidade federal do interior de Minas Gerais, matriculados na disciplina Equações Diferenciais I, no 1º semestre de 2020. **Coleta e análise de dados**: Os dados foram coletados por meio das atividades realizadas, da gravação das aulas ministradas de forma remota e de questionários de avaliação, sendo analisados por meio de uma categorização feita a partir do confronto com o referencial teórico que fundamentou a pesquisa. **Resultados**: Os resultados possibilitam afirmar que as atividades de Modelagem Matemática conduzidas configuram ricas oportunidades de motivação e aprendizagem por parte dos alunos, permitindo uma exploração diferenciada das aplicações dos conteúdos matemáticos envolvidos e colaboraram para uma interpretação crítica da realidade, ainda que de forma incipiente. **Conclusões**: A partir da pesquisa realizada, podemos concluir destacando a importância das pesquisas vigentes e futuras em Educação Matemática no Ensino Superior apontarem para um ensino de Equações Diferenciais que rompa com o tradicional modelo dos formulários e métodos de resolução.

Palavras-chave: Modelagem Matemática; Equações Diferenciais; Ensino Remoto; Cursos de Engenharia.

INTRODUCTION

Differential equations (DE) can be conceived as a subject/topic that is part of differential and integral calculus. Observing several curricular structures of courses in Brazilian higher education, in some cases, the ordinary differential equations (ODE) are part of the syllabus of calculus courses (in general, Calculus II, III or IV) and, in other cases, there are specific courses for DE.

Some studies (Oliveira & Igliori, 2013) show that the introductory DE teaching has characteristics that are also verified in the teaching of calculus (Reis et al., 2019; Lopes & Reis, 2019). Such teaching has taken place, in general, through two possible "models": a model that, in a sense, we can compare to a "prescription" or "recipe book", and another model that, to a certain extent, we can associate with extra-mathematical applications.

The first model seeks to develop the discipline following the traditional sequence of ODE presentation through algorithms and formulae, which thus function as "recipes" for their resolution. In this model, it is not a priority, for example, to present in depth the various applications of ODE in other areas of science or, at least, in the exact sciences, especially in Engineering courses.

The second model requires much more "dedication" from the teacher because, in addition to the resolution methods, ODE applications gain priority through a direct presentation by the teacher or through methodologies that allow such a focus for teaching. In this sense, it is possible to address from modelling to the use of technologies and, in this way, give students a more active participation in the construction of their knowledge.

The fact is that the option for one or the other model is directly related to the issue of student learning, as demonstrated by Oliveira & Igliori (2013). The authors carried out a bibliographical survey as a way of evaluating what research in mathematics education points out problems in DE learning and what they propose as possibilities for teaching to mitigate the difficulties. They highlighted that DE teaching prioritises analytical resolutions and the algebraic manipulations involved. Students' difficulties in previous topics were also highlighted, for example, concepts of integral differential calculus or even basic mathematics. Difficulties were also associated with applications in contextualised problem-situations. To alleviate these difficulties, most of the studies analysed propose a qualitative and contextualised approach to DE through problem-situations, which aim to contribute to learning, especially if associated with the students' future area of activity, enhancing their motivation to learn. Thus, Oliveira and Igliori (2013) point to an ideal setting for DE teaching, capable of providing a balanced approach between analytical, graphical, and numerical treatment, with the use of computational resources that aid in student learning.

Other research addresses the teaching of DE for Engineering courses, highlighting the importance of working from contextualised problem-situations or the analysis of physical phenomena (Dullius, 2009; Buéri, 2019) or even with the use of mathematical modelling under different perspectives (Fecchio, 2011; Lopes, 2021). Such research was carried out under different theoretical frameworks and with different investigative focuses, and show that, in DE teaching, the prevalence of analytical resolution methods compared to the exploration of graphical interpretations, and highlights students' reluctance regarding a more qualitative treatment of DE, and the rare use of technological resources. However, all researchers highlighted the importance of DE teaching based on the contextualisation/modelling of problem-situations and/or natural phenomena for student formative outcomes, beyond the motivation for learning.

In this context, this article presents a study carried out from the perspective of DE teaching that goes beyond the traditional presentation of resolution methods and emphasises the importance of applications for the redefinition of concepts related to DE, pointing to mathematical modelling as a methodological alternative capable of providing a combination of knowledge with skills and competencies linked to the daily lives of engineering students.

ESTABLISHING A MATHEMATICAL MODELLING PERSPECTIVE

Biembengut (2016) uses the term "modelling" to refer to modelling in education, which uses the core of the modelling process in teaching and learning. According to the researcher:

> Modelling is a teaching method with research in school boundaries and spaces, in any subject and stage of schooling: from the initial years of elementary school to the end of higher education, and also in continuing education or postgraduate courses. (Biembengut, 2016, p. 177)

For Biembengut (2016), in higher education we can use physical and/or symbolic modelling. The decision of which modelling to use depends on a few factors such as number of students in a class, subject content, and students' prior experience with modelling.

This modelling perspective is one among many others. The perspectives of mathematical modelling "have in common, among their objectives, the use of mathematics for the study of real problems or situations" (Araújo, 2002, p. 31).

Biembengut (2016) uses three steps for modelling: 1) Perception and apprehension; 2) Comprehension and explanation; 3) Signification and expression. For each step, subdivisions are made to better understand the modelling. The researcher also highlights the need for us to know "how, when, and how much to approach each content, integrate content with a purpose" (Biembengut, 2016, p. 209). While considering problems of interest to students, Biembengut's (2016) approach helps to better understand the contents covered and, generally, is a first step in research.

As we will detail later, a subdivision of the modelling steps will occur (which we will call "steps") for better understanding of the students.

We also highlight the research by Klüber (2012), who analysed national authors with a strong presence in the field of mathematical modelling in mathematics education. He concluded that:

> Mathematical modelling shows itself in a multifaceted way due to the theoretical assumptions made in terms of knowledge, science, mathematics, and mathematics education. The plurality of these conceptions, sometimes contradictory among authors, indicates the stay of the search to understand mathematical modelling beyond these particularities. (Klüber, 2012, p. 13)

This conclusion shows us that, based on this plurality of conceptions of mathematical modelling, we must adopt a modelling conception to guide us in carrying out research. We reaffirm, then, our option for the concept of mathematical modelling brought by Bassanezi (2002) and conceived from an educational perspective by Biembengut (2016).

It is also worth noting that the stage of signification and expression highlighted above is, for Bassanezi (2002), the expression of the real situation studied using a succinct and symbolic language. In the context of DE teaching, Burghes and Borrie (1981) add that solutions must lead to an understanding of the real problem, which precedes predictions and/or decision-making. Thus, DE teaching will not be centred on algebraic formulas, techniques, and manipulations, but also on understanding the real problem considered.

Finally, the authors mentioned above emphasise that, if there is no such "connection with reality", an activity performed may no longer be characterised as a modelling and impairs students' perception of the mathematics role in solving actual industrial/business or everyday problems.

CONTEXTUALISING THE RESEARCH

The research was carried out in the 1st academic semester of 2020, in the last month of classes (June), with students from the Differential Equations I course, taught by the first author of this article, which is part of the curricular structure of the third period of the nine Engineering degrees offered by the Federal University of Itajubá (Unifei), Itabira Campus – MG: Environmental Engineering, Computer Engineering, Control and Automation Engineering, Materials Engineering, Mobility Engineering, Production Engineering, Health and Safety Engineering, Electrical Engineering, and Mechanical Engineering.

This compulsory course has a total workload of 64 class-hours, and its syllabus is comprised by the following topics: first order ODE; ODE of order greater than or equal to two; series solutions; Laplace transforms and ODE systems. Among the objectives of the course, "identifying and solving problems involving differential equations" stands out.

The basic bibliography adopted had as main reference, initially, the classic book by Zill and Cullen (2001) and later, Santos (2017), authored by Prof. Dr. Reginaldo Santos, Ph.D., from the Mathematics Department of the Federal University of Minas Gerais (UFMG), available free of charge on the author's *website*, which facilitated student access due to the impossibility of accessing the university's physical library.

In mid-March 2020, due to the Covid-19 pandemic, Unifei officially adopted the exceptional work regime (RTE) system. Classes were then held remotely through Google Meet and communication with students mainly took place during these classes and through weekly forums available on the Moodle platform, a virtual teaching and learning environment that has already been adopted by the institution for several years, with faculty having a choice of using the platform or not.

Weekly, at the scheduled times for the two in-person classes in the course, classes were taught remotely. Students' attendance was a little lower than during face-to-face classes, remaining between 50% and 75%.

The conduction of mathematical modelling activities was done through Google Meet, during class hours. As the workload of two weekly classes was not enough to carry out all the modelling activities, the groups met outside class hours to complete them. In addition, extra classes were scheduled to discuss students' queries, before completing the assignments. The records of the modelling activities were logged on the Moodle platform.

Through Google Meet, students presented the models produced, highlighting the issues they deemed most relevant. Some students preferred to do the presentation in advance and record it. Afterwards they sent the file to the teacher. On the day and time of the presentation, the recorded file was transmitted to all students via videoconference. This was done to reduce internet connection problems, as indeed occurred with some students.

DESCRIBING MODELLING ACTIVITIES

The mathematical modelling activities were carried out in two Engineering classes. Class T1 had a total of 52 students and class T2 had 65 students. Groups of four to six students were formed, with groups being organised freely among peers. Nine groups were formed in T1 and eleven in T2.

All groups performed the mathematical modelling activities, that is, none of the groups gave up. The activities were divided into two blocks. Each block contained two activities students should develop. Thus, each group performed an activity in the first block, related to 1st order ODE, and another in the second, related to 2nd order ODE, i.e., each group performed two activities in total. The themes of the mathematical modelling activities were:

1st block: 1A) Absorption of alcohol in the body and risk of accidents

1B) Modelling the adequacy of a diet

2nd block: 2A) Consumer purchasing behaviour

2B) Modelling the spread of an epidemic

According to Bassanezi (2002, p. 45), “the formulation of new or interesting problems is not always a very simple activity for a mathematics teacher”. Because of this caveat, we conducted a survey to better reconcile certain topics of differential equations to interesting problems. Traditional differential equations books have basically the same problems and applications. Many of these applications may not be interesting or fail to make a connection with the professional future of engineering students or their daily lives. The usual applications are: radioactive decay, bacterial colony growth, electrical circuits, spring-mass system, among others (Zill & Cullen, 2001). Therefore, we searched some articles and other non-traditional books to get problems that could be of interest to our students. According to Bassanezi (2002, p. 45): “There is a survey of possible study situations which should preferably be comprehensive so that they can raise questions in various directions”. Thus, we considered that the chosen topics could arouse the interest of students, which seemed reasonable to us, since the issues are part of the daily lives of practically all of them.

Importantly, some authors such as Bassanezi (2002) and Klüber (2012) suggest that students choose the topics, so that they also feel responsible for the learning process. However, due to the limitation of time, the uncertainties regarding the novelty of remote learning and the need to adapt the topics covered in the activities to the syllabus, unfortunately, we could not follow this recommendation. Thus, after defining the models to be worked on in the activities, we presented the topics to the students and let them choose, after a discussion with their group, which activities they would develop in each block.

To carry out mathematical modelling activities, we adapted the eight steps for ODE applications in physical phenomena described in the book authored by Prof. João Bosco Laudares from the Pontifical Catholic University of Minas Gerais (PUC Minas) and others (Laudares et al., 2017, p. 98) and thus defined the following didactic steps:

Step 1: Mathematisation of the physical law

Step 2: Solving the differential equation of the model

Step 3: Initial or boundary conditions

Step 4: Replacement of given constants

Step 5: Calculations requested in the problems (explain what is requested)

Step 6: Mathematical model of the phenomenon (found equation)

Step 7: Model graphics

Step 8: Synthetic description of the phenomenon

Step 9: Analysis of the model equation

Step 10: Critical review of the model

Due to the characteristics of some problems in mathematical modelling activities, some steps were subdivided into some sub-items, to facilitate understanding and resolution.

The last 2 steps were aimed at the students' critical perception of the model conceived by the group. To stimulate critical discussion of the model, we've added steps 9 and 10 to the original script. According to Laudares et al. (2017, p. 98): "This structure can be considered a pattern to be followed, with changes occurring according to the type of problem to be solved".

We emphasise that we can also associate the ten steps defined with the three modelling steps established by Biembengut (2016) as follows: 1) Perception and apprehension: Steps 1 and 2; 2) Understanding and explaining: Steps 3 to 7; 3) Signification and expression: Steps 8 to 10.

Each mathematical modelling activity started with a problem question to stimulate students. Then, a brief introductory text contextualised the problem, providing some data clues. From there, the groups proceeded to the resolution of the models. As a last task, each group presented their resolution to the other students.

We then go on to describe modelling 1B.

MODELLING THE SUITABILITY OF A DIET

We start with the following *Problem*: How to create a mathematical model that involves daily energy expenditure and daily energy intake (food) to predict the best fit and evolution of a diet for a given person?

What controls body weight is the balance between two factors: energy intake and energy expenditure. When the balance is positive, the body will have excess energy that will be stored as fat, while when this balance is negative, the stored fat is used to supply the body with the necessary energy (Chin et al., 1992). Energy restriction is the most efficient and traditional treatment for obesity. However, care must be taken regarding the basal metabolic rate (Abdel-Hamid, 2003; Newby, 2019). Another factor to consider is the importance of physical activity as an effective means of decreasing excess weight or maintaining a healthy weight (Chin et al., 1992; Newby, 2019).

Starting with Step 1, students recognised that two variables are important: daily food intake and energy expenditure. A possible model (Charalampos, 2004) is given by the following ODE:

$$\frac{dB}{dt} = A(C - aB)$$

where:

- B is the body weight in Kg.
- A is the dietary conversion factor which is equal to $\frac{10}{322168}\,\mathrm{Kg/KJ}$ (or $\frac{1}{7700}\,\mathrm{Kg/Cal}$).
- C is the daily energy intake rate, measured in KJ/day.
- a = 167.36 (KJ/Kg) /day (or 40 (Cal/Kg) /day) is a constant that represents an average of energy expenditure.
- t is time, usually measured in days.

Thus, we move on to Step 2. However, this model can be improved by adding the exercise component (already bringing forward a possibility for Step 5).

To build a diet model that involves some physical exercise, we must consider the factors that play a role in determining body weight, which are the daily rate of energy intake, given by *C*, and the daily rate of energy expenditure *a* which, according to Charalampos (2004), ranges from 146.44 to 188.28 (KJ/kg) /day (or from 35 to 45 (Cal/kg) /day). Thus, the rate of change in body weight $\frac{dB}{dt}$ will be proportional to C – (40+d)B, where *d* is obtained from exercise. Thus, we must have the following first-order differential equation:

$$\frac{dB}{dt} = A(C - bB),$$

where b = 40+d must be determined, considering the dietary conversion factor $A = \frac{dB10}{322168}$ Kg/KJ (Mackarness, 1988).

Steps 3 to 6 involve the analytical algebraic part of solving the ODE and include possible particularities. We will comment on these steps in what follows.

The ODE involved is linear and of the 1st order. A solution can be reached through the integrative factor (Zill & Cullen, 2001). Considering the exercise factor, the solution is the following function:

$$B(t) = \frac{C}{b} + \left(B_0 - \frac{C}{b}\right) e^{-Abt},$$

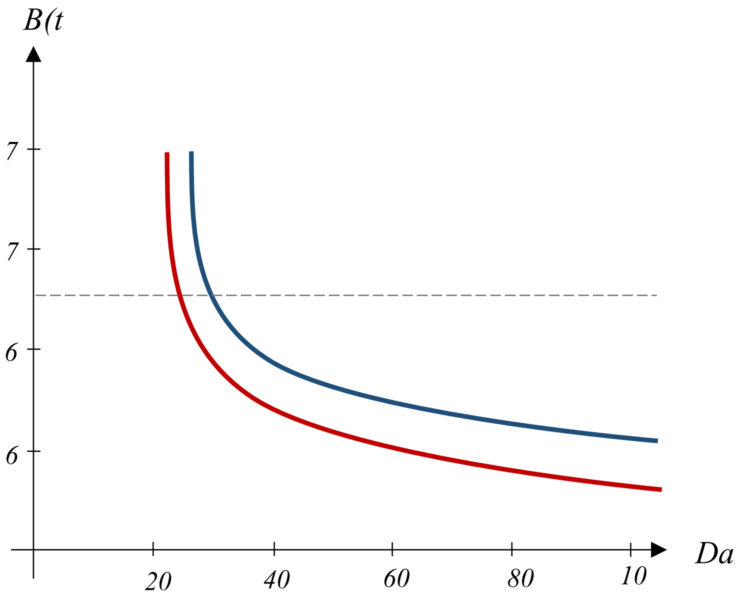
In this function we see the constants A, C and b which are as we established before. The value of $B_0 = B(0)$ is the person's initial weight, that is, how much they weighed at the beginning of the diet.

To determine the initial condition, we suggested that each group picked a member to simulate weight loss. In this way, the groups found the necessary values (student's weight and daily energy intake) to substitute in the initial equation and find the solution based on them.

The model graph, that is, the solution function (Step 7) was obtained considering the data of the chosen group member. Therefore, an estimate of what his or her weight loss would be like was obtained. Figure 1 below illustrates what the weight loss curve would look like, that is, the variation of B(t) with time t (assuming an initial weight of 90 kg). We outline the cases: with and without the exercise component.

Figure 1

Comparison between models (with and without physical exercise)



The example considered in Figure 1 was of a person who was 1.70 m tall and weighed 90 kg (Charalampos, 2004). The chosen daily energy intake was 10460 KJ/day (or 2500 calories/day), which is normal for an active adult male (Fitrakis, 1985). The ideal weight for this person is around 68 kg, according to the BMI (Body Mass Index). In the figure, the ideal weight is shown as a dashed line (the horizontal line $y = 68$). The blue curve refers to the first model, while the red curve is for the improved model, that is, the model with the exercise. Using the first model, which is the initial diet, the subject will reach his ideal weight after 313 days. After adding the exercise component, the subject can reach his ideal weight after just 240 days.

Students observed that physical activity helps in weight loss, improving the model. Student Juliana, for example, said at one of the meetings: "I think this model is valid because it relates exercise and diet".

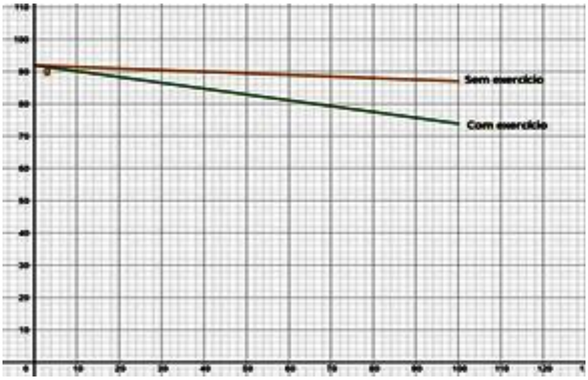
As in the case with the exercise b is higher, over time the exponential term of the solution will cause the solution B to decrease faster than the case without the exercise, as illustrated in Figure 1. Consequently, the ideal weight and body weight limit are reached in less time. Walking at a speed of 0.889 m/s (or 3.2 km/h) on a treadmill for 30 minutes helped add 7.876 (KJ/kg) /day (or 1.8824 (Cal/kg) /day) to the average daily expended energy, and therefore, to the exponential component. This emphasises the importance of walking.

The following steps, from 8 to 10, did not involve a mathematical manipulation, but a critical interpretation and discussion of the model, its adequacy to reality and possible social implications.

Step 8 was just for the students to analyse what was produced and give a brief explanation of the modelled phenomenon — which was done to everyone's satisfaction. A few groups (2 in total) had mathematical difficulties, solving the ODE involved in the wrong way. Despite this, the graphics expressed an adequate description of the phenomenon, as illustrated in Figure 2. Thus, the interpretations of these groups were consistent with reality. Possibly facilitated by the intuitive factor of the problem in question, these students said that a person who is on a diet to lose weight will reduce body weight, and this will be accelerated if he or she does some physical activity.

Figure 2

Graphics created by a group



Continuing with Step 9, the groups agreed that the model fits reality. It is intuitive to expect a person who does some type of physical activity to lose weight faster. Despite this, many students recognised several factors that could be improved in the model. Pedro, for example, mentioned that “the model does not fully fit reality as it disregards factors such as the individual’s body fat percentage and metabolism”. In one of the meetings to produce the model, student Tatiane (names were changed) mentioned:

> I believe the model is a valid approximation, but it can be improved because this issue of diet has many variables in addition to several personal aspects. Other aspects to consider are: eating habits, types of exercise, taking into account duration and frequency. (Tatiane)

Age also influences, as burning off energy becomes harder with age (McCargar, 1996; Newby, 2019). Other factors that could be considered are the person’s gender and height.

In addition, the pattern of energy intake, or eating habits, has an influence: those who eat breakfast tend to expend more energy than those who fast in the morning, as the body starts to function and expend energy earlier (Cho et al., 2003; Wyatt et al., 2001). On the other hand, adding such modifications to the model make the new ODE harder to solve. We will illustrate this with a case. When the ambient temperature is quite different from the body temperature, the body needs to regulate its temperature, in a process known as thermoregulation. Thus, if we include the effect of ambient temperature in the model, we will have the following result:

$$B'(t) = A(C - (b + F(T))B)$$

where *F(T)* is the function dependent on the ambient temperature and the other letters, as described above. Thus, the new ODE is non-linear, which makes resolution difficult. This was recognised by many groups, i.e., a new model closer to reality may be more difficult to resolve.

The mathematical model developed by the groups was considered realistic, as it takes into account factors such as diet and exercise. The weight loss that we can see in the graph (Figure 1) is also within the normal range, as it is something doable, being a useful prediction for those who want to lose weight. Furthermore, the model showed that simple and easy periodic exercise could have a big effect on body weight loss.

Ending the activity with Step 10, the groups held critical discussions about the model. They reflected on questions such as: What are the difficulties in following a diet and exercise program?

We realized that the discussions were facilitated by the fact that the model produced was close to reality and described a topic that was familiar to all students in some way. Student Fabiana said that:

> It would be ideal to include a constant that represents a margin of error, since most people aren't always able to follow a diet that strictly every day. In addition, it would be good to consider a possible consequence of biological factors in the body, such as the response time of metabolism. (Fabiana)

Pedro and Samantha recognised that energy expenditure can be different in each organism and that the quality of food ingested is a factor that can cause changes. Samantha even added:

> For a more complete modelling, we can include: amount of each macronutrient ingested associated with a constant that

> represents its metabolisation time in the organism, as well as its acceleration or delay impacts on it; type of body morphology – endomorph, mesomorph or ectomorph – linked to a constant that represents their specific metabolic response to physical activities. Therefore, it will be possible to obtain a more reliable result for the individual's data, giving the equation a greater degree of precision and, consequently, a greater possibility of bringing results that describe reality. (Samantha)

CONTRIBUTIONS FROM MODELLING ACTIVITIES

From the development of mathematical modelling activities, the records in the field diary during remote classes, the answers given to the questionnaire applied after carrying out the activities and, in view of our theoretical framework, we were able to establish two central categories of analysis that we will describe below.

Contributions to learning differential equations

To analyse aspects related to DE learning, we will present some positive and some negative aspects, from our point of view.

Unanimity among students was that mathematical modelling activities provided a greater interest in DE. We realised that this may be because students are not used to different methodologies, especially in mathematics classes in higher education.

Motivation led students to do research and be more dedicated to their studies. One student said that "she tried to study and research the subject, seeing applications in practice." Doing research is already expected in a modelling activity. However, we note that some groups still did not do so. We noted that some groups had a solution strategy that demonstrated research being done. In addition, all groups used software to present the graphical part of the model. This certainly contributed not only to an understanding of the model, but also to learning ODE.

The applications of mathematics were highlighted in the following comments, which also show some contributions of modelling from the students' perspective:

> I think these methodologies are very important, professor, mainly because we are studying Engineering. We will probably apply this knowledge in practice, hardly in theoretical studies as in pure mathematics. It was good to be able to add on the practical part that we will have in our daily life on top of the theoretical knowledge on ODE. (Enzo)

Another student, Hannah, commented: "I thought that the modelling activities, even though it was a lot of work, were good for us to be able to see the application of what we learned. We hadn't seen any courses like that. and I found it very interesting".

In relation to mathematical modelling, as it is new to some, there was an initial estrangement. Student Guilherme said that "I had never done anything related to modelling; it was even a concept that scared me at first". But Guilherme and several other students (the majority) liked the new methodology adopted. For example, student Amanda stated: "I think it is super viable to use mathematical modelling as a methodological alternative". This feeling was also expressed by several other students.

Mathematical modelling as a methodological alternative was widely accepted by students, as noted by the comments of some who made clear the importance of knowledge built through the activities developed. In general, the students could explain, in their words, contributions from the use of modelling, as summarised in the words of student Lucas: "I found the visualisation and application of the subject to be very positive; it ends up that, with this methodology, we apprehend more things and learn better".

Thus, we note that "the teaching practice based on the precepts of mathematical modelling in education, highlighting the mediating character of the teacher and making the student more autonomous in relation to their learning" (Scheller et al., 2015, p. 17) allows us to reconcile theory and practice. It unites the world of academic mathematics with the mathematics present in everyday life (Lopes, 2020). In this sense, student Alex stated: "I liked it, I found it interesting how to deal with everyday problems with mathematics".

However, some students feel that it is necessary to have a broad and vast knowledge in mathematics to be able to carry out a modelling activity. For example, a student, Sara, said that she did not think mathematical modelling was viable because she “didn't have enough knowledge of mathematics to do it”.

The application of modelling in the Differential Equations discipline was considered positive by all students. In this way, the students “connected” the theoretical content with the reality that surrounds them, as some explained:

> Carrying out the modelling activities contributed to give new meaning to my mathematical knowledge in relation to differential equations. It contributed by making me study and go deeper into ODE, but the most significant part was opening my mind to see how modelling is used in everyday life. (Amanda)

This comment is in line with what Biembengut (2016, p. 178) defends, when stating that modelling “also allows the student to enjoy and be interested by some area of knowledge, by realising that these topics being learned serve them as important fundamentals or even ‘means’”.

For some students, a contribution to learning was the possibility of observing the connection that mathematics and modelling can have with other disciplines. For others, the contribution of the use of modelling in the discipline extended to a better understanding of previous topics:

> During the course, I needed to review several topics, and I had clearer reinterpretations of Calculus 1, which really helped me to understand differential equations. (Arthur)

Another very positive aspect for learning was related to the question of learning style. Each student has a certain learning style (Barros, 2008). In general, the acceptance of the completion of the work was positive and generated motivation, as exemplified by a student's comment:

> Much more than just knowing how to do the math, studying ODE to carry out the activity was super valid, it gave me a lot more energy to study. The involvement with the work, the desire to make it work, and especially the desire to understand everything that was happening and how the model would

> behave, what would result, made the study much more meaningful. (Pedro)

The question of learning style also involves the study environment. Some students did not have a suitable environment to study. Student Diego said: “the study environment at home itself does not provide me with a positive experience”.

On the other hand, learning difficulties add up with the difficulties caused by remote teaching. For some students, the difficulties involved studying alone, being away from their peers and not adapting (up to the moment of modelling) to the remote education system. This was made clear in the responses of some students to the questionnaire. For example, student Ênio said: “The biggest difficulty was because the semester was online; I find it easier to learn in the classroom, where my mind encompasses more”.

Also probably related to learning style is the problem of working in groups. Some students said they benefited from the group activity. For example, a student said:

> In addition, this type of activity promotes interaction among the students themselves, who exchange knowledge and arouse our interest in these types of more practical methodologies. (Glauber)

However, a difficulty with peers in the group was expressed by a few students. Some groups adopted the following strategy accepted by three groups: about half of the group members took the 1st mathematical modelling activity and the others took the 2nd. This strategy, which may have been adopted by more than the three groups that admitted this, was not positive for Pamela’s group, as she stated: “I found it selfish that the colleagues who solved the first problem did not help those involved in the second”.

Five other students also expressed this lack of cooperation and collaboration. Glauber said that some colleagues only participated in the recording of the presentation and then “disappeared”. Jeane said that colleagues who “mastered the subject the most” were overwhelmed and this “was crucial to an erroneous execution to some degree”. Luís Edson said that one of the biggest difficulties of the modelling activities was “managing a group during the RTE”. These and other difficulties with peers in the groups may be related to the fact that everything was conducted remotely, i.e., a difficulty intrinsic to the RTE. However, our teaching experience shows that, even in the classroom, not all students collaborate with their colleagues to carry out a group activity.

Other learning difficulties that we noticed are not (necessarily) linked to the way the discipline was conducted in the semester through remote teaching. Such difficulties would likely arise, even if the activities had taken place in face-to-face classes. Some of them were related to the modelling itself. Few students said they had used modelling in any discipline before, in the answers to the questionnaire. The positive answers from the previous use of modelling referred mostly to the discipline of Linear Algebra. Despite this, student Ramon said that “it was not modelling in itself, but I have already participated in a similar work, in Linear Algebra, on the topic of the Kalman Filter”.

Additionally, few students said in the questionnaire that they think they will no longer apply modelling in any other courses. One student said that he does not see himself using modelling in other courses that were taking place. Another student, Henri, said that “modelling is very difficult to apply”. Others said that they would use modelling, but with “reservations”, and in “simpler situations”.

Other difficulties reported by students are related to difficulties in mathematical topics. The difficulties presented by these students are not necessarily related to the modelling activity. Surely, such difficulties would appear in some other assessments, such as a test, an exercise, etc.

The mathematical difficulties started with the interpretation of the modelling to be done. These difficulties appeared in greater quantity in the 2nd block of activities (involving 2nd order ODE). Except for one group, all groups in the Model 2B activity (the spread of an epidemic) left essential items blank (in Steps 7 and 8). Student Priscila’s comment sums up the feeling of most groups during the development of this activity: “This task demanded work”.

Some of these interpretation difficulties can be seen clearly in some of the responses to the questionnaire. Several students said that the epidemic modelling was inadequate for a real situation because it did not include the possibility of a vaccine. However, one of the items (unsolved by most groups) of the 2B Modelling activity was precisely to consider the “vaccinated case” and compare the possible improvement in that case.

We saw that interpretation difficulties also occurred at the time of conclusions and reflections on the model found. Some conclusions did not make much sense and seemed more an expression of intuition about the phenomenon than an analysis of what had been done, as the model obtained did not match the conclusions. For example, in the considerations of a group regarding the Modelling 1A activity (risk of accidents when combining alcohol and driving) data from the Ministry of Health were presented, but there was no connection with the model constructed by the group.

With the 2nd block of Modelling activities, mainly activity 2B, some specific difficulties surfaced. As pointed out by the students, the greatest difficulty was to obtain the numerical parameters to find a specific solution.

Even explaining to the students which letters were constant and which were variable, there were several misunderstandings in the questionnaire, as stated by Daiane: "In the second activity, when I started researching the variables, we were almost lost when realising the number of variables". Related to this difficulty, many students were not able to determine the necessary parameters for the ODE boundary conditions or the PVI (Initial Value Problem).

Another great difficulty presented, both in remote classes and in the questionnaire, was the sketch of graphics. Only one group mentioned difficulties in graphing the activities of the 1st block. But all groups had difficulties in drawing up the graphs of the 2nd block activities. Students did not detail their difficulties, which are possibly related to the interpretation of the phenomenon, the model and obtaining the parameters.

Analysing the groups' graphs, we see that some of them do not match the presented model. Despite having a correct intuitive interpretation (ascending line: increase in cases; descending line: decreasing cases, in the case of activity 2B), many of the graphs were inadequate or incomplete for the model found.

Another factor raised by some groups was time. Some students said, both in remote classes and in the questionnaire, that it would be better to have more time for the 2nd block Modelling activities.

Thus, we believe that, despite the difficulties discussed here, the contributions of mathematical modelling to learning differential equations and giving meaning to the study of the discipline were recognised by almost all students.

Contributions to the development of criticality

The mathematical modelling activities carried out with the students were conducted in such a way that the critical discussion fell on the model itself. The themes were chosen to arouse students' interest and, in this way, bring a critical discussion to the classroom. But the focus on the careful choice of themes ended up leading the activity to a critical discussion directed at the model produced and not about social, political, and/or economic issues.

It should be noted that the critical discussion of a model must be included in any modelling concept, whether in the context of applied mathematics or mathematics education (Rosa et al., 2012). Thus, we will present an analysis of the moments of criticism of the built models, pointing out how the criticality of a future professional in Engineering was present.

Analysing the data, we look for signs of a critical attitude of a future professional in Engineering. Effectively, the students perceived the relationship between the modelling activities and a critical posture, according to the students' comments below:

> I believe that mathematics is paramount in the critical formation, logical thinking, and reasoning of an individual, being an important part in the use of multi-knowledge, be it for carrying out simple tasks as well as in solving a more complex problem, allowing for more reliable decision making. (Talita)
>
> After this modelling work, I have a more critical view of some problems faced in the world, better understanding of how to solve them through what we are seeing throughout higher education. In this way, the use of both basic and higher mathematics, when applied to life, unravels, guides, helps, and solves different situations, opening up a mix of possibilities within reality. (Felipe)

This critical stance observed was initially based on mathematical procedures and certainties to reach conclusions:

> The risk of a person being involved in a car accident after drinking alcohol is HIGH. Based on the work, we verified that

> the risk is proportional to the rate of alcohol in the individual's body, that is, the more you drink and the higher the alcohol content of the beverage, the greater the chances of a traffic accident occurring. [...] Another approach that we can take between the graph and the 1st order ODE refers to the number of factors that are related, since the factors that comprise the relationship between accidents and alcohol refer to more variables, therefore, the equation modulates the interpretation of graphics in a specific way. For this issue to be circumvented, a more in-depth study and work on the topic would be necessary. (Group 1 (T1) – Modelling 2A)

Like Araújo (2012), we realise that the criticisms made by students in a modelling activity are based on mathematical certainties and on the reality that surrounds them. Some groups have outlined a stance in this regard:

> The risk of someone being involved in a car accident after drinking is high, and it is expressly necessary to educate the population not to drive after drinking, as we can use the graphs and formulas of the work as a basis to corroborate the exponential growth of risk x ingestion of alcohol. (Group 9 (T2) – Modelling 1A)

Most groups had a good understanding of the phenomenon involved. When making a critical analysis of the model, students realised that adding some other variables can help to obtain a more complete model. Among the numerous groups that realized this, we selected the following:

> The model presented fits the reality in parts, however it can be improved if other variables are added. Its strengths are in the analysis by gender and in making the relationship between the individual's data (height, weight, and blood volume in the body) with the volume of drink ingested, its alcohol content. Negative points include the fact that the consumption of other types of beverages is not analysed simultaneously. The model could be improved by adding the variables of different types of beverages, their consumption time and perhaps the time it takes to be eliminated from the body, which could lead to other results. (Group 3 (T2) – Modelling 1A)

The comment of this group reveals a questioning of mathematical certainties, of the ideology of certainty (Borba & Skovsmose 2001). The ideology of certainty is the current structure around absolutist conceptions of mathematics. The problems dealt with in mathematics classes are artificial. When they are problems taken from the real world, simplifications are made to fit them into the world of mathematics. Mathematics applications seek an ideal, optimal solution, but disregard factors from reality. Borba and Skovsmose (2001) bring as a solution the ideology of certainty, a different course of action in the traditional classroom, showing that mathematics can be questioned and that it does not always provide a final answer.

On the other hand, some of the conclusions reached by the groups were confused or not based on the obtained model:

> However, the SIR model is not able to explain the persistence or eradication of infectious diseases, the main reason for this is that the SIR model considers the distribution of individuals spatially and temporally homogeneous, based on the premise that the population size is so large to the point of allowing the approximation by continuous variables of the different states. Therefore, the number of infected people in the country may be six times greater than the number released by the government. (Group 4 (T2) – Modelling 2B)

> The disease will interfere with the curves of the graphs and their behaviour, and may be more incisive (exponential) or less incisive ("linear"). In this way, it is possible to specify specific graphs for each disease, given their peculiarities. (Group 9 (T2) – Modelling 2B)

In the previous comments, we can see the use of "6x greater" and "exponentials". Students' use of such terms (and others) can represent a media influence and society's tendency to make use of mathematical terms and estimates, without having a clear basis. Since the beginning of the pandemic, numbers, graphs, estimates, predictions, and mathematical terms became commonplace.

We note, therefore, that several students, even when faced with a model, made conclusions based on intuition or through other sources. Certainly, doing research and relating problems to the surrounding reality is an important aspect of modelling, however, doing this without looking at the built model itself leaves a feeling of a poor understanding or distrust in the model obtained. In addition, we see here another sign of the ideology of certainty, through unquestioning reliance on an already established model.

A characteristic of the criticisms of some groups and some students was evident in the reading of the data. The criticism made to the model produced and the way in which the student faces and uses mathematics in everyday life was presented in a polarised way. On the one hand, some negative conceptions were presented:

> As for the negative points, it would be the fact that the model cannot represent reality. It also does not cover variables that can interfere with the expected result. Another fact is that the model does not consider the biological factors of the body, and that a person does not always follow the proposed diet and exercises very strictly, affecting the real result when compared to the theoretical one. (Group 7 (T2) – Modelling 1B)

> Mathematics taught in schools is very limited, does not represent reality, it only gives an idea, but it is not very applicable. (Gabriel)

In the first comment above, we note that the group challenged the ideology of certainty, questioning the model produced. On the other hand, other students had a positive view of the relationship of models obtained with reality, as illustrated below:

> So, yes, we can consider that the model obtained fits very well with reality. [...] we built a model that compares to one of the most used today, which has extensive application and visibility by health professionals, which gives us the conclusion that our model could be applied in the real world. (Group 5 (T1) – Modelling 2B)

> I think the main difficulty was: Ok, we reached this result, but is that really true in practice?". When looking at the graphs, we were making assumptions, guessing values, to see if this was really the way it would proceed over time, and sometimes finding graphs that didn't fit with that was quite frustrating. (Lucas)

In the first comment mentioned above, we realised that the group confirms the validity of the model by comparing it to another one that is widely used today, i.e., something mathematically guaranteed. This similarity leads to the assurance that it can be used in the real world. However, this reasoning is a manifestation of the certainty ideology (Borba & Skovsmose, 2001).

Despite noticing polarised critical discussions, influenced by the media and with the bias of the certainty ideology, we can say that the groups held critical discussions that "made sense", given the context of modelling and application to reality. Therefore, we realise that the discussions held with the students, the discussions held between the members of the same group, and the discussions between the groups allowed for incipient critical reflection on mathematics that, to a certain extent, fosters the formation of criticality that we believe is essential for the future Engineering professionals.

FINAL CONSIDERATIONS

In conclusion, we recall that the research detailed here aimed to identify and analyse the possible contributions of mathematical modelling activities, which were carried out remotely, to aspects related to learning and the development of criticality in Engineering students.

As in a face-to-face classroom, in remote learning, students need clear guidance and close monitoring during modelling activities. As in-person contact is not possible in remote education, then it is important that other types of meetings take place constantly and, if possible, in real time, for greater and better interaction with students.

Regarding DE teaching, the use of mathematical modelling can be very fruitful, as students show a greater interest in the discipline when they get involved with some project. Despite this, the themes worked with students in modelling activities must be linked to their daily lives or have some relationship with the students' future area of professional activity. In the construction of the model, the production of graphics can present many difficulties for the students, however, the visual aspects of the model provide a good basis for the conclusions they will reach.

Finally, but not the last aspect to be considered, during the entire modelling process, students need to be stimulated towards critical discussion.

This discussion can be outlined, at first, regarding the model under construction and the variables involved. In addition, the discussion of the model can and should be directed to questions about the reality that surrounds us, accompanied by relevant critical reflections, which can encourage students to take a critical stance in their life in society.

Thus, we conclude by emphasising the importance of DE teaching that breaks with the model that prioritises only the algebraic part, forms, and resolution methods, and is outlined from the contextualisation/modelling of problem-situations and/or natural phenomena for the formative character of students, which can boost rich research in mathematics education in higher education.

AUTHORS' CONTRIBUTIONS STATEMENTS

A.P.C.L. contributed substantially to the conception and design of the study, including data collection, analysis and interpretation, manuscript preparation, as well as a critical review. F.S.R. contributed substantially to the conception and design of the study, including data analysis and interpretation, manuscript writing, and critical review.

DATA AVAILABILITY STATEMENT

Data supporting the results of this study will be made available by the corresponding author, A.P.C.L., upon reasonable request.

REFERENCES

Abdel-Hamid, T. K. (2003). Exercise and Diet in Obesity Treatment: An Integrative System Dynamics Perspective. *Medicine & Science in Sports & Exercise*, *35*(3), 400–413. https://doi.org/10.1249/01.MSS.0000053659.32126.2D

Araújo, J. L. (2002). *Cálculo, Tecnologias e Modelagem Matemática*: as discussões dos alunos. (173 f.). Tese de Doutorado em Educação Matemática, Universidade Estadual Paulista, Rio Claro.

Araújo, J. L. (2012). Ser crítico em projetos de modelagem em uma perspectiva crítica de educação matemática. *Bolema: Boletim de Educação Matemática*, *26*(43), 839–859. https://doi.org/10.1590/S0103-636X2012000300005

Bassanezi, R. C. (2002). *Ensino-aprendizagem com Modelagem Matemática*: uma nova estratégia. Contexto.

Barros, D. M. V. (2008). A teoria dos estilos de aprendizagem: convergência com as tecnologias digitais. *Revista SER: Saber, Educação e Reflexão*, *1*(2), 14–28. http://www.revistafaag.br-web.com/revistas/index.php/ser/article/viewFile/70/pdf_45

Biembengut, M. S. (2016). *Modelagem na Educação Matemática e na Ciência*. Livraria da Física.

Borba, M. C. & Skovsmose, O. (2001). A ideologia da certeza em educação matemática. In: O. Skovsmose. *Educação Matemática Crítica*: a questão da democracia (pp. 127-148). Papirus.

Buéri, J. W. S. (2019). *Análise de fenômenos físicos no ensino de Equações Diferenciais Ordinárias de primeira ordem em cursos de Engenharia*. (118 f.). Dissertação de Mestrado em Ensino de Ciências e Matemática, Pontifícia Universidade Católica de Minas Gerais, Belo Horizonte.

Burghes, D. N. & Borrie, M. S. (1981). *Modelling with Differential Equations*. E. Horwood.

Charalampos, T. (2004). A Mathematical Diet Model. *Teaching Mathematics and Its Applications*, *23*(4), 165–171. https://doi.org/10.1093/teamat/23.4.165

Chin, M. K., Lo, A. Y. S., Li, X. H., Sham, M. Y. M. & Yuan, Y. W. Y. (1992). Obesity, Diet, Exercise and Weight Control – A Current Review, *182 J Hong Kong Med Assoc*, 44(3), 181–187. https://www.hkmj.org/sites/default/files/mjimage/sample.pdf

Cho, S., Dietrich, M., Coralie J. P., Brown, C. A. C. & Block, G. (2003). The Effect of Breakfast Type on Total Daily Energy Intake and Body Mass Index: Results from the Third National Health and Nutrition Examination Survey (NHANES III). *Journal of the American College* *of Nutrition*, *22*(4), 296–302. https://doi.org/10.1080/07315724.2003.10719307

Dullius, M. M. (2009). *Enseñanza y Aprendizaje en Ecuaciones Diferenciales con Abordaje Gráfico, Numérico y Analítico*. (514 f.) Tese de Doutorado em Ensino de Ciências, Universidade de Burgos, Burgos.

Fecchio, R. A. (2011). *Modelagem Matemática e a Interdisciplinaridade na introdução do conceito de Equação Diferencial no Ensino de Engenharia*. (208 f.). Tese de Doutorado em Educação Matemática, Pontifícia Universidade Católica de São Paulo, São Paulo.

Fitrakis, P. (1985). *Modern Dietetic*. Fitrakis.

Klüber, T. E. (2012). *Uma metacompreensão da modelagem matemática na Educação Matemática*. (396 f.). Tese de Doutorado em Educação Científica e Tecnológica, Universidade Federal de Santa Catarina, Florianópolis.

Lopes, A. P. C. & Reis, F. S. (2019). Vamos viajar? – uma abordagem da Aprendizagem Baseada em Problemas no Cálculo Diferencial e Integral com alunos de Engenharia. *Remat: Revista de Educação Matemática*, *16*(23), 449–469. http://dx.doi.org/10.25090/remat25269062v16n232019p449a469

Lopes, A. (2020). Formação crítica dos professores de Matemática articulada às questões contemporâneas. *REnCiMa: Revista de Ensino de Ciências e Matemática*, *11*(6), 809–817. https://doi.org/10.26843/rencima.v11i6.1901

Lopes, A. (2021). Modelagem Matemática e Equações Diferenciais: um mapeamento das pesquisas em Educação Matemática. *REnCiMa: Revista de Ensino de Ciências e Matemática*, *12*(4), 16–31. https://doi.org/10.26843/rencima.v12n4a16

Mackarness, R. (1988). *How to Weaken Eating*. Fitrakis.

McCargar, L. J., Sale, J. & Crawford, S. M. (1996). Chronic Dieting Does Not Result in A Sustained Reduction in Resting Metabolic Rate in Overweight Women. *Journal of the American Dietetic Association, 14*(2), 142–146. https://doi.org/10.1016/S0002-8223(96)00301-X

Newby, P. K. (2019) *Food & Nutrition*: what everyone needs to know. Oxford University Press.

Oliveira, E. A. & Igliori, S. B. C. (2013). Ensino e aprendizagem de equações diferenciais. *Em Teia: Revista de Educação Matemática e Tecnológica Iberoamericana*, *4*(2), 1–24. https://periodicos.ufpe.br/revistas/emteia/article/view/2231

Laudares, J. B., Miranda, D. F., Reis, J. P. C. & Furletti, S. (2017). *Equações Diferenciais Ordinárias e Transformadas de Laplace*: Análise gráfica de fenômenos com resolução de problemas – Atividades com softwares livres. Artesã.

Rosa, M., Reis, F. S. & Orey, D. A. (2012). Modelagem Matemática Crítica nos Cursos de Formação de Professores de Matemática. *Revista Acta Scientiae*, *14*(2), 159–184. http://www.periodicos.ulbra.br/index.php/acta/article/view/227

Reis, F. S., Cometti, M. A. & Santos, E. C. (2019). Contribuições do GeoGebra 3D para a aprendizagem de integrais múltiplas no cálculo de várias variáveis. *REnCiMa: Revista de Ensino de Ciências e Matemática*, *10*(2), 15–29. http://revistapos.cruzeirodosul.edu.br/index.php/rencima/article/view/2328

Santos, R. J. (2010). *Introdução às Equações Diferenciais Ordinárias*. Imprensa Universitária da UFMG.

Scheller, M., Bonotto, D. L. & Biembengut, M. S. (2015). Formação Continuada e Modelagem Matemática: Percepções de Professores. *Educação Matemática em Revista*, *46*, 16–24. http://sbemrevista.kinghost.net/revista/index.php/emr/article/view/499

Wyatt, H. R., Grunwald, G. K., Mosca, C. L., Klem, M. L., Wing, R. R., & Hill, J. O. (2001). Long-Term Weight Loss and Breakfast in Subjects in the National Weight Control Registry. *Obesity Research*, *10*, 78–82. https://doi.org/10.1038/oby.2002.13

Zill, D. G. & Cullen, M. R. (2001). *Equações Diferenciais*. Pearson Makron.