Theory and practice in pre-service mathematics teacher education from a social perspective

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Contributing to the growing literature on the conceptualisation of mathematics teacher knowledge and mathematics teacher education, I put forward a model of how mathematics teaching can be, the activities that ought to be involved in it and their relationship with teacher knowledge. On the basis of this model I show that it is possible to support the design and development of pre-service teacher education programs on conceptual arguments that highlight a connection between mathematics education theory and teachers' practice. Those programs should be designed from a social perspective emphasizing local curriculum design.

Introduction

It is possible to establish a conceptual relationship between some concepts of mathematics education theory and mathematics teachers' practice, in the context of pre-service methods courses. In order to depict that connection, it is necessary to assume a stance about mathematics learning and about how teaching ought to be carried out. I describe the activities that I think the teacher should fulfil in order to produce a curriculum design at the local level. On the basis of that description, I identify the teacher knowledge necessary to perform those activities and show that this knowledge is related to some of the technical knowledge of mathematics education theory. On the other hand, I show how I use this relationship to design a pre-service methods course based on a sociocultural perspective.

The paper starts by reviewing some of the literature on teacher knowledge and teacher education. In particular, I discuss Shulman's notion of pedagogical content knowledge and relate it to some of the issues that Cooney (1994) put forward some time later. I use Simon's (1995) mathematics teaching cycle model as the starting point for my model. Having described that model, I identify its relationship with the teacher's didactical knowledge and establish the connection between some concepts of mathematics education theory and teacher's practice. Finally, I show how a pre-service methods course can generate settings for developing this knowledge from a social perspective.

Mathematics teachers education

While reviewing the literature on mathematics teacher education and teacher knowledge, Cooney (1994) poses some questions concerning these issues: "What kinds of knowledge do teachers need to become effective teachers of mathematics? What sorts of experiences are needed for teachers to acquire this knowledge" (p. 608), and "What theoretical perspectives can better enable us to understand what teachers are experiencing? What theoretical perspectives can enable us to develop a research and development program that can propel our efforts forward?" (pp. 627-628).

Most of Cooney's questions are related to the problem of designing appropriate mathematics teachers education programs. One may think that in 1994, as it might still be the case today, many of these programs were based on the mixture of mathematics and general pedagogy that Shulman (1986) fought against. I interpret Cooney's questions as a request for systematic and justifiable curriculum designs in mathematics teachers education courses. Before deciding the contents of those courses, we have to assume a stance concerning mathematics learning, supporting the way we think mathematics instruction should be done. On that basis, we can decide on what kinds of teacher's knowledge are necessary, justify the goals we have to look for, and produce the curriculum design of the course, so that the teachers can "[live the] experiences needed for [them] to acquire this knowledge" (Cooney, 1994, p. 608).

Eight years before, Shulman (1987) introduced the notion of *pedagogical content knowledge*. He argued that this notion involved a dramatic shift in teacher understanding "from being able to comprehend subject matter for themselves, to becoming able to elucidate subject matter in new ways, reorganize and partition it, clothe it in activities and emotions, in metaphors and exercises, and in examples and demonstrations, so that it can be grasped by students" (p.13). As powerful as this notion was at that time, since it showed new ways of looking at teacher knowledge and teacher education, the notion itself did not evolve in the education research literature (Bullough, 2001). However, in 1995, Simon makes some contributions on this issue. On the one hand, he takes an explicit stance concerning mathematics learning (from the social constructivist perspective) and tries to look at teaching on the basis of those constructivist principles. On the other hand, he sees the teacher as a cognising and reflecting agent. In his words, "starting from a social constructivist perspective on knowledge development, my paper continues the discussion of pedagogical deliberations that lead to the determination of problem contexts for student involvement. In particular, the paper extends the notion of teaching as inquiry, examines the role of different aspects of teachers' knowledge, and explores the ongoing and inherent challenge to *integrate the teacher's goal and direction for learning with the trajectory of students' mathematical thinking and learning*" (p. 121). The mathematics teaching cycle he proposes (Figure 1[1]) is a "schematic model of the cyclical interrelationship of aspects of teacher knowledge, thinking, decision making, and activity that seems to be demonstrated by the data" (p. 135), having three properties: it is a *local* model, it is cyclic and it is dynamic. The model is local in the sense that it is focused on the teaching of a specific topic in a given lesson. It is cyclic and dynamic in the sense that the results of a cycle are used

1.We have modified the orientation of Simon's figure for economy of space.

by and affect the starting point of a new cycle.

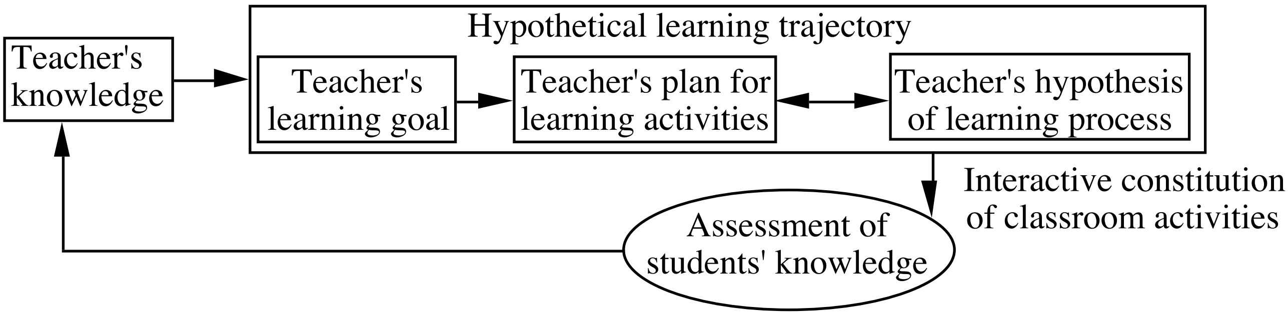

Figure 1. The mathematics teaching cycle

Didactical analysis and curriculum design in mathematics teachers education

I think that Simon proposal shows that it is possible to conceive mathematics teacher education from a perspective that goes beyond the traditional mixture of mathematics and general pedagogy. Following the ideas of sociocultural theory, I assume that "some of the responsibilities of the old-timer [the teacher] involve the communication of the norms, values, and discourse practices of the community to the newcomers [the students] and the design of meaningful mathematical activity settings" (Forman, 1996, p. 118). As teacher educators[2], we were interested in producing a model similar to the mathematics teaching cycle, based on this view of the teacher's responsibilities, in order to set up and justify the curriculum design of the methods course we are in charge of. This is a course for last year mathematics students who want to become secondary mathematics teachers. The main purpose of this course is to train these prospective teachers in one aspect of their future teaching practice: the planning of didactical units[3]. Therefore, the main concern of the course is curriculum design. Curriculum development and class management are dealt by a parallel (practice) course, which is only partially related to ours. In this and the next two sections, I present my approach to the teacher's activities and knowledge. After that, I lay out my view of prospective teachers' learning and the way I use some ideas of sociocultural theory for the methodological design of the course.

We work then within a *local* conception of the curriculum design of didactical units in which: a) there is a global design that frames the local problem; and b) the object of the curriculum design is a specific mathematics structure (e.g., the sphere or the quadratic function). At the end of the course, the prospective

2. The methods course we are referring to had two teacher educators. The author was one of them.

3. The term "didactical unit" is used in Spain with no clear-cut meaning. On the one hand, a didactical unit is composed of more than one and less than eight lessons. Its "unity" is given by the identification of a specific mathematics structure to be taught.

teachers are supposed to have the knowledge and abilities that will allow them to carry out efficiently the planning activities. Therefore, in order set up the curriculum design of the course, it is necessary to determine that knowledge and those abilities. For this purpose, we have to provide a description of the way we think the planning process should be realized. We see this planning as the sequential arrangement of activity settings that would drive students' participation in the mathematics practice community of the classroom.

Simon's mathematics teaching cycle was the starting point for the outlining of a model (that I call *didactical analysis*[4]). This model represents my view of how I think the mathematics teacher can produce the planning activities. In this model I try to: a) expose the relationship between the global and the local design; b) identify and describe the activities the teacher is supposed to carry out in order to produce the planning of a didactical unit; and c) establish the teacher knowledge that is necessary in order to fulfil these activities and the role this knowledge plays in the process.

Didactical analysis involves four types of analysis that the mathematics teacher is supposed to perform for the design, implementation and assessment of activity settings in the classroom (Figure 2).

The content analysis, as the structured description of the subject matter that is involved in the activities. This description includes the conceptual structure of the subject matter, its representation systems, its phenomenological analysis and its modelling possibilities.

The cognitive analysis, as the identification and description of the difficulties that students might face and the errors that they might make while accomplishing the tasks that constitute the activities.

The instruction analysis, as the description of the activities that are to be proposed to the students, taking into account the types of tasks that emerge from the content analysis, the needs of students (as a consequence of the cognitive analysis), and the materials and resources available.

The performance analysis, as the description of the students' cognitive status as a result of the activities. This information feedbacks a new cycle of the didactical analysis.

At the end of one cycle (which can be a lesson or a portion of a lesson), the information produced in the performance analysis is used in the formulation of new goals, contents and cognitive status. Additionally, this information may also have an effect on the teacher's knowledge. It is in this sense that I see the teacher as a cognising and reflecting agent, taking into account the role of the context in which learning takes place.

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4. We use the expression didactical analysis generically. We do not pretend to introduce a new term but to use it to encapsulate a set of activities that will be described below. Even though Simon's model is based on a cognitive view of students' learning, our proposal, grounded on a sociocultural view of learning, looks at teachers' activities in the design of meaningful mathematical activity settings.

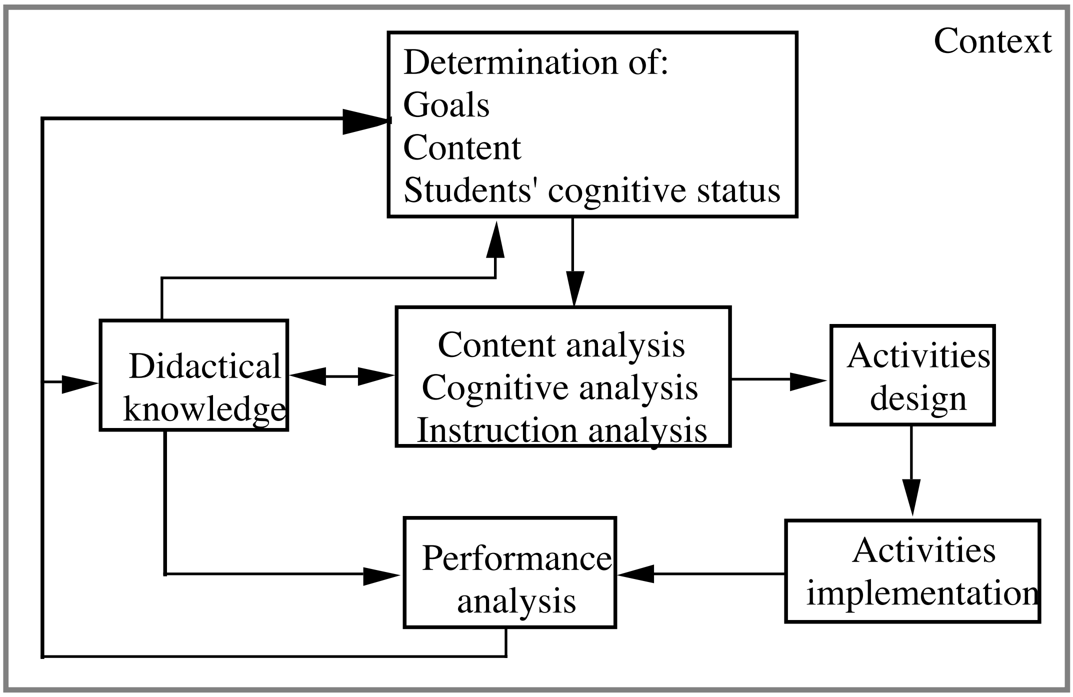

Figure 2. Didactical analysis

Didactical knowledge

Having described the activities that I think the teacher should perform in order to produce the curriculum design at the local level, I can identify the knowledge that can be necessary for that purpose. I can organize this knowledge in three categories: a) about the concept of curriculum; b) about some mathematics education concepts that I consider relevant; and c) about the implementation of a) and b) on a specific mathematical structure in order to carry out the didactical analysis. In the methods course we are concerned about we expect prospective teachers to construct a) and b) and develop capabilities for realizing c). The knowledge I am interested in is the assembling of a), b) and c).

Didactical knowledge has then a disciplinary knowledge that serves as reference. These are some concepts from mathematics education (which will be enumerated later). It has a practical purpose, as the knowledge that is used in order to design, implement and assess didactical units. Furthermore, its implementation is framed within an analytical structure, the didactical analysis.

Didactical knowledge is a psychological construct. It attempts to give a more specific meaning to Shulman's pedagogical content knowledge notion. But, having described how curriculum design at the local level should take place on the basis of didactical analysis, and having identified the relationship between didactical analysis and didactical knowledge, there is a question that remains unanswered: Which are the mathematics education concepts that I consider relevant in order to carry out the different analysis that make up didactical analysis? I think this is a matter of choice. Nevertheless, the selection of those concepts should be such that each of the analysis involved in the didactical analysis can be carried out.

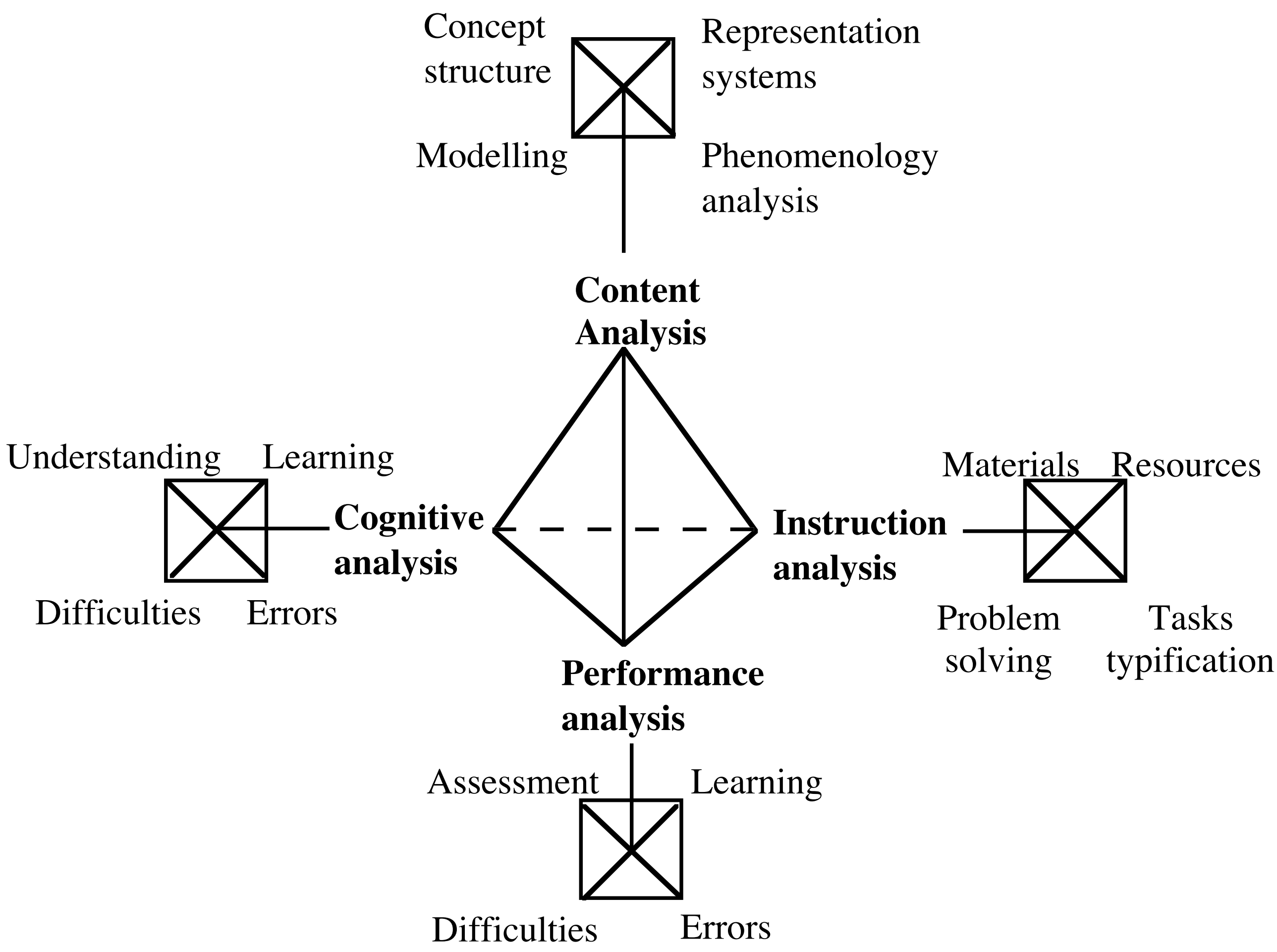

Figure 3. Mathematics education concepts involved in didactical knowledge

Figure 3 shows the choices we have made in our methods course. Some of these concepts have been integrated in what Rico et al. (1997) call the "curriculum organizers". The general concept of curriculum is on the background as the basis for the local curriculum design. As a matter of fact, the didactical analysis can be seen as the local expression of the global curriculum design. It is the way in which teachers can go from the usual planning as a sequencing of contents to a broader and more profound view of local curriculum design in which learning activities are the result of the analysis of the subject matter, the cognitive issues involved in it, its teaching possibilities and the performance assessments of students.

It is not my purpose in this paper to describe in detail each one of the mathematics education concepts portrayed in Figure 3, nor to establish the way those concepts should be put in practice in order to carry out the didactical analysis. Furthermore, I do not have space here to describe the systemic, dynamic and cyclic characteristics of these models. However, I expect that, as consequence of implementing didactical analysis on the basis of didactical knowledge, the mathematics teacher can produce at least three results: a) a rational and systematic justification of the activities chosen; b) a set of rationally grounded conjectures about the possible ways in which students will react to the tasks comprised in the activities; c) ideas for class management and for his/her performance in the classroom according the students' work and actions.

Theory in mathematics teachers education

In this paper I am interested in the role that theory can play into two practices in a mathematics methods course: the mathematics teacher educator practice and the prospective mathematics teacher practice. We have seen that the most common answer to these questions is to base these practices on theory on two different subject matters: mathematics and general pedagogy. Even though Shulman, through the notion of pedagogical content knowledge, gave some ideas of what we should expect the teacher to do (in his practice), he did not identify what knowledge was necessary for that purpose, nor did he propose the theory, if any, this individual knowledge should refer to. One possible reason for this is the fact that Shulman's group was working with a mixed group of teachers from several different subject matters. Therefore, they could not get into the details of a specific didactical theory.

I have tried to show that in order to produce mathematics methods courses curriculum designs that go farther than the traditional mixture of mathematics and pedagogy, and in order to give a more specific meaning to the idea of pedagogical content knowledge, we have to take into account several issues. First, we have to recognize the specificity of our subject matter. Mathematics teaching and learning have their own specific characteristics and problems, and that is also the case in mathematics teacher education. Second, we have to assume an explicit stance concerning mathematics learning in the classroom. This position will affect our view of how teaching should be. The curriculum design of mathematics teacher methods courses should have that view as a starting point. Third, we have to make a clear difference between the problems of global curriculum design (for a whole mathematics course in school, for example) from the problems of local curriculum design (for a didactical unit on a specific mathematical structure). If we only consider the problems of global curriculum design, teachers will tend to see planning as the sequencing of mathematics contents and teaching as the "covering" of those contents. By considering the problems of local curriculum design and concentrating on a specific mathematical structure, it is possible to break away from this view of planning and teaching and open a new working space. Notions like representation systems or phenomenological analysis have full meaning when they are used to describe a specific mathematical structure, whereas they tend to be less useful for analysing broader mathematical structures. It is within the context of carrying out the didactical analysis of a specific mathematical structure that many concepts of mathematics education acquire a practical meaning in the practices of the teacher educator and the prospective mathematics teacher. In other words, there is a theory, specific to mathematics education that is relevant to the mathematics teacher's practice. The arguments of the previous pages were intended to support this thesis.

Practice from a social perspective

In the previous sections my concern was the teachers' activities and knowledge necessary for the design of activity settings. I look now at teachers' learning from the viewpoint of the design of our methods course. This course is concerned with the planning dimension of the prospective teachers future practice. I see lesson planning as a social activity in school. This should be a social practice carried out by the mathematics teachers as a community in which there is a common goal and in which a social knowledge is constructed on the basis of individual knowledge through the discussion of possible solutions to the individual and social problems that teachers have to face in planning and implementing their teaching. This is a central work practice of the teaching community in the sense that, in order to produce lesson planning, "individuals come together with a shared goal and work toward a joint product that is meaningful to all participants" (Stein & Brown, 1997, p. 162). On the other hand, I see prospective teachers' learning as "changes over time in social participation patterns in the work practice of [a] community" (p. 162).

Our challenge was to design a methods course which: a) takes into account and takes advantage of the diverse identities of our prospective teachers; b) offers activity settings which involve prospective teachers in the participation in communities that mirror the practice communities in school; and c) takes into account our views on teachers' activities and knowledge suggested previously. In Lerman's words, we wanted to "encourage the expression of difference; teach methods of critique of orthodoxies concerning mathematics and mathematics education; and encourage theorising about teaching and learning mathematics" (2001, p. 48).

With these conditions in mind, I have set up a design with the following characteristics. Prospective teachers organize themselves in groups of five people. Each group chooses a mathematics topic on which they will carry out the didactical analysis and produce a local curriculum design. The didactical analysis is produced in sequential steps. For each step, each group puts in play a mathematics education concept for analysing its mathematics topic, using the information produced in previous analysis and improving those analyses with the new information. For each one of these tasks, each group has to share the results of their work to the whole class. Once this is done, teacher educators and prospective teachers discuss on the difficulties they have faced and try to set up a social meaning for the concepts involved. For example, once a group has tried to construct the concept structure of its mathematical subject, teacher educators introduce informally the notion of representation systems through an example. The example is discussed and each group is asked to improve its previous work by using the notion of representation systems to analyse its corresponding mathematical structure. At the following session, each group presents its proposal to the whole class. Each proposal is criticised by the class. Once all proposals are presented a discussion naturally develops concerning the meaning of the mathematics education notion at hand. Teachers educators, as "old-timers", guide this discussion in order to support the "newcomers" participation and consolidate norms, values and discourse practices.

We work then in two practice communities. The working group practice community in which a small group of prospective teachers has to face and solve a didactical problem (the use of a given mathematics education concept in one of the analysis composing the didactical analysis). In this practice community each individual brings her/his knowledge and puts into practice her/his individual meanings, interests and motivations. This setting generates a great deal of discussion within the group until an agreement is reached. The result of this agreement is presented to the whole class. We have seen how the differences in experience and expertise drive individuals to coparticipate with different roles in the group (Gómez, 2001).

The discussion in the classroom of each group's proposals sets up the classroom practice community. In this community, each group shares its meanings, interests and goals, and, with the participation of the teacher educators, they are compared, analysed and discussed in order to construct a classroom agreement increasingly closer to the technical meanings, norms, values and discourse practices of the mathematics education community. It is at this point in which some social participation patterns are made explicit and new forms of meaning and understanding are created. Our experience shows that this is a very complex task. The mathematics education concepts we use on the course are difficult ones. The construction of those technical meanings by the prospective teachers cannot be achieved by "transferring" that meaning to them. They have to construct them by themselves through a social process in which they put into play those concepts while solving the problems involved in the didactical analysis of a given topic (Gómez, 2001).

Discussion

In this paper, I have suggested that it is possible to approach some of Cooney's questions, especially those concerning pre-service teacher education, from a sociocultural perspective. The models I have presented pretend to shed some light on the "kinds of knowledge […] teachers need to become effective teachers of mathematics [and the] sorts of experiences [that] are needed for teachers to acquire this knowledge" (p. 608). I have tried to show that it is possible to support the design and development of pre-service teacher education programs on conceptual arguments that establish a connection between mathematics education theory and teachers' practice. With these arguments, I have tried to give a mathematics education specific meaning to Shulman's notion of pedagogical content knowledge. I have argued that in order for these mathematics education concepts to have a practical meaning it is necessary to go from the global curriculum design to the problems of local curriculum design. On the other hand, I have also argued that the pedagogical development of prospective teachers cannot be achieved by the transfer and reception of information in a methods course. A social perspective is necessary in which individual meanings and interests evolve in a process of putting them in practice for the construction of social meanings close to the technical meanings of mathematics education theory. These social meanings affect in turn the construction of the individual meanings.

A sociocultural view of learning is central to the design of the methods course. On the one hand, the teacher's activities and knowledge necessary for lesson planning are based on a sociocultural view of her/his students' learning. On the other hand, the actual design of our methods course is based on a similar view of prospective teachers' learning.

This conceptual framework is part of an on-going research project in which I am exploring the development of prospective teachers' didactical knowledge. Current partial results show that this development is a slow and difficult process in which it is possible to characterize prospective teachers' partial meanings and identify moments in which difficulties arise or conceptual reorganizations take place. I have found that the paths of meaning construction of prospective teachers are substantially different from those of in-service teachers. For the latter, many mathematics education concepts, previously unknown to them, acquire quickly a practical meaning that they can implement in their local curriculum designs (Gómez & Carulla, 2001). These results corroborate the relationship between theory and practice in mathematics teacher education I have proposed here.

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