In this paper, we propose the didactical analysis as a tool to assist future secondary mathematics teachers in their analysis of mathematics content in order to plan, implement, and assess teaching and learning tasks in class. We propose a conceptual structure that supports the curriculum design of a methods course and present the elements composing the didactical knowledge that we expect preservice teachers to develop in such course.

Introduction
The methods course we are in charge of is taken by last year mathematics students. It is a theoretically oriented course that is complemented with a second course in which future teachers perform some practical activities in a high school. One of the main goals of our course is to develop preservice teachers’ didactical knowledge as the basis for curriculum design at a local level (didactical units). The content of the course is therefore related to the disciplinary knowledge that serves as reference to didactical knowledge: the concept of curriculum, school mathematics, their teaching and learning, and the curriculum organizers. Preservice teachers work in groups of four to six people. Each group chooses a specific mathematical subject on which it develops its curriculum design. This work is periodically and systemically presented, shared and discussed in class at different levels of development, until the final document is produced. This allows everybody to participate, individually and socially, in the construction of the technical meanings they are using in their own work. In what follows, we introduce the idea of didactical analysis and show its relationship with the notion of didactical knowledge. We show how the content component of the curriculum design of the course is based on these two notions.

Didactical analysis as a tool for local curriculum design
The notion of pedagogical content knowledge (Shulman, 1987) highlights that knowledge of mathematics and pedagogy are not sufficient in order to become a mathematics teacher. Teachers need a specific knowledge for understanding “how particular topics, problems or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction” (p. 8). Despite of its importance, this notion has not been treated with precision in the research on teachers’ knowledge (Bullough, 2001, p. 657). The issue of which types of knowledge are needed by mathematics teachers in order to be efficient and how this knowledge can be developed remains open and is central in teacher training (Cooney, 1994, p. 608, Richardson, 2002).

The design of a mathematics methods course should be based on a conceptualization of the activities that the teacher has to do in order to promote students’ learning and of the knowledge that is necessary to perform those activities. We call the structuring of a cycle of these activities a didactical analysis. It is organized around four analyses: content, cognitive, instruction, and performance. Didactical analysis allows the teacher to examine and describe the complexity and multiple meanings of the subject matter, and to design, implement, and assess teaching/learning activities.
Any cycle of the didactical analysis begins with the identification of the student’s knowledge for the subject matter at hand (see Figure 1). We expect the teacher to use his knowledge and previous experience for establishing the tasks that the students can and cannot solve, the mistakes they can make, and the difficulties underlying those mistakes. With this information, and taking into account the global planning of her course, we expect the teacher to determine the goals she wants to achieve and the mathematics content she wants to work on (box 1).

The next step of the cycle involves the description of the mathematical content from the viewpoint of its teaching and learning in school (box 2). The content analysis stresses the relationship among concepts, highlights its multiple representations, and distinguishes the connections between the elements of the conceptual structure and between those elements and the phenomena from which they emerge. This information is used in the cognitive analysis, in which the teacher describes hypothesis about how students construct their knowledge when they face the learning activities that are proposed to them. The cognitive analysis involves the identification of the skills, reasoning, and strategies necessary to solve the tasks, of the mistakes students can make when they are solving them, and of the difficulties and obstacles they might face. The information from the content and cognitive analysis allows the teacher to carry out an instruction analysis: the identification and description of the tasks that can be used in the design of the teaching and learning activities that will compose the instruction in class (box 3). These tasks should mobilize students’ knowledge in order to generate cognitive conflicts and promote the construction of meaning using the materials and resources available. In the performance analysis the teacher observes, describes, and analyzes students’ performance in order to produce better descriptions of their current knowledge and review the planning in order to start a new cycle (box 5).

The didactical knowledge is the knowledge that the teacher enacts when she performs the didactical analysis (box 6). In other words, it is the knowledge needed for organizing teaching and learning activities in didactical units. Expert teachers perform didactical analysis based on their experience and the materials they have available. However, preservice teachers need guidelines and criteria with which to organize their activities, produce their work, and structure their future experience. The didactical analysis provides guidelines for preservice teachers to 1) explore and recognize the richness and variety of meanings of the mathematics subject matter, 2) collect, organize and select information concerning these multiple meanings, and 3) use this information to design materials and activities to promote students’ mathematical learning. The knowledge underlying these guidelines can be organized in three categories: 1) the concept of curriculum as a tool for planning and global structuring, 2) the foundations of school mathematics (mathematics, learning, teaching, and assessment), and 3) mathematics education notions that can be used, as conceptual and methodological tools, for local planning, as suggested in the cycle of didactical analysis. We call these notions “curriculum organizers” (Rico et. al, 1997): conceptual structure, representation systems, didactical phenomenology, modeling, errors and difficulties, materials and resources, and problem solving. Didactical knowledge is the integration of these three types of knowledge.
References


