TECHNOLOGY AND SEMIOTIC REPRESENTATIONS IN LEARNING MATHEMATICS

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Abstract

New technologies modify the socioculturals environments. The educational field changes too, but is necessary yet a deep discussion about how to do a correct implementation of this tools in the classroom, to transform them into cognitive instruments.
In this article, we present many reflections about the role that can develop the technologies in these processes, and its relation with representational systems and semiotic representations, that constitute the kernel for understand the students' mathematical knowledge building.

1. Introduction: mediation and representation

One of the central thesis in sociocultural psychologist studies supports that the *human* cognitive action is always an action mediated by some kind of tool. This tool can be symbolic (as the natural language) or material (as a telescope). Thus, we can deduce on learning an immediate consequence: the nature of the knowledge depends of the tool. Just a big process of instrumental “descontextualization” will can translate this piece of knowledge into anothers contexts later (Moreno, 1999).

We will talk of the calculator (TI-92), as a tool of mediation in the construction of students' mathematical knowledge.

The thought of that many representations of mathematical concepts are basics to understand them, has developed its study for some time now. So many researchers have investigated the concept of representation and the role that can carry out into students' reasoning (Duval, 1999).

We assume that, in the realm of mathematics, a *representation* is a symbolic, graphic or verbal notation to express concepts and procedures of the discipline, as well as their more relevant characteristics and properties. According to these properties, representations can be classified in *registers of representation* (Duval, 1999). For instance, with the concept of function, there are graphic, algebraic and tabular registers. Certainly, there are others, but these are the most used in teaching. Into each register it is possible to carry out *processings*, that is, transformations of the representations in the same register in which they were created. Also is possible to realize *conversions* between different registers of representation that are transformations of one representation made in a register into another representation in another register. In the functions instance, a conversion can be a translation of the function's tabular information into a graphic.

2. Mathematical Education and Technology

In spite of the increasing interest of studying how the computational tools' possibilities can be useful for the teaching and learning mathematics, we can find many arguments against the use of technology in the classroom. One of them explaints that students forget and give up what they do with paper and pencil, and this fact is considered as a damage for the quality of their education.

We believe that it is necessary understand the arrangement of computational technologies in teaching mathematics as a process of enrichment, and not as a substitution, trying to improve de cognitive capacities, and not to substitute them.

A deep reflection teaching us that these critics hide a poor understanding of technology. The first trouble is that when we speak about technology, the *last* technology is the only one recognized. Hardly is it mentioned that writing (above all writing!) is a form of technology.

The introduction of his book *Orality and Literacy*: *The technologizing of the Word*, Ong (1988) show us that the investigation has established a distinction between oral cultures and those that are affected by the use of writing. For instance, many of the characteristics accepted nowadays by the scientific thinking, were motivated by the resources that writing technology supplies to the human conscience. This is a very deep reflection that we mentioned before: tool mediation affects so much to cognition products and, in this case, the affectation follows from the writing.

Then, when a child carry out a paper and pencil' arithmetic calculus, the intellectual work that he realizes depends on as the writing system as the decimal notation that are mediating its actions. The technology is there, but we can't almost see it: it becomes invisible, and one of the most interesting technologies' features is when this invisibility becomes immersed in a sociocultural environment.

If we take into consideration most recent examples, such as simple calculator that only can work with the four basics arithmetic operations, we see arise critics about its use in primary school: using calculator, children will not understand to add…Is that true? We should say that this statement is only a half-true? An indiscriminate use of this tool can introduce mistakes in the learning process, but in the same way that the writing isn't an obstacle for doing mental calculus, neither plays the calculator this role.

The calculator doesn't disorganize the students' cognitive activity, but it offers them the opportunity of participating, in a cognitive way, in new fields. For instance, if the students use calculators with symbolic processing, they can focus on the interpretation of what they're doing, without wasting any time just in repetitive and boring syntactic calculus.

3. Mathematical Education and Representational Systems

The role played by the representations in the educational frame is especially important. The NCTM, for instance, suggests in its *Principles and Standards for School Mathematics* (NCTM, 2000), the study of representations as one of the most relevant. On the other hand, an increasing number of events are about this topic; for example, the 21st Annual Meeting of the PME-NA, celebrated in 1999 in Mexico, focused on visualization and representation in mathematics education.

Joined to the concept of *semiotic system of representation* also appears complex through its representations, so, how can we understand the relations between the representations and an object which doesn't exists before representing it? How can we know that a mathematical object isn’t only all its representations and, in addition, without using them?

To answer these questions, it is necessary to accept that the construction of a mathematical concept is a process in a continuous development, so this way, the objectivity’s level with which we understand that concept, is only temporary. We never hold the concept, and therefore it isn’t possible to think about platonic conceptions of the mathematical objects.

In this work, we will think about representations that the calculator supplies us. These representations have some very productive features for the mathematical learning. They are *executable representations*, that is, they can simulate cognitive actions, and these actions don’t depend on the calculator’s user. That happens, for instance, when calculator represents graphically a function.

From the teacher’s point of view, an appropriate strategy can be thought about calculator as an artificial cognitive system. The calculator manipulates internally the representations, and shows us these modifications on the screen, and this feedback helps the user to succeed in solving mathematical problems. Although all this is possible because the calculator has been programmed by expert humans, its relations with every particular user confers this instruments certain autonomy. Thus, we can regard the calculator as a new *cognitive partner*, which, interacting with the student, helps him to build new meanings.

It’s very important to understand that the objects which appear on screen and that the calculator manipulates, they are neither concrete objects nor objects from the formal mathematical world: they are virtual objects in the *interface* that separate the conceptual world of mathematics, from the concrete objects world. Therefore themselves are knowledge instruments, and not knowledge itself.

4. Executable Representations

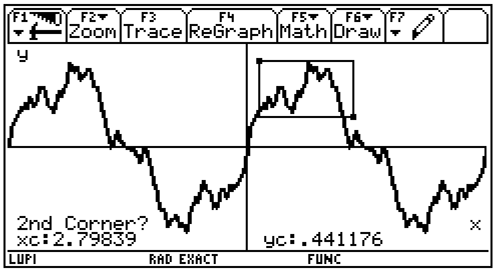
The graphic calculators (specially the TI-89 and the TI-92 Plus calculators) support an array set of mathematical objects and relations in different registers. And even more important, they permit go from one of them to another, that is, they permit the conversions of registers, which is supposed to be a powerful work tool in mathematics education.

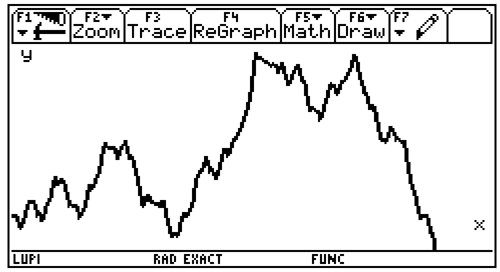
In the expression environment that calculator support, we can obtain mathematical properties and relations very different from there that we can observe only whit paper and pencil. An example: to represent functions which are almost impossible of being drawing by hand, they could conjecture properties and *prove it visually* (and there are activities very interesting from an educational point of view), some complex facts for an algebraic analysis. To illustrate this, we will consider a very important function in the historic search of continuous functions but not differentiable at any point.

Before surprising presentation that Karl Weierstrass did in Academy of Berlin in 1872 of trigonometric series $f(x)=\sum_{n=0}^{\infty} b^n\, Cos(a^n \pi\, x)$ (with a an odd integer, and b a real number between 0 and 1, such that $ab > 1 + 3\pi/2$), as continuous function at every value of x and differentiable at none, Riemann introduced in 1861 an example of a continuous function and not differentiable for infinitely many values of x:

$$f(x)=\sum_{n=1}^{\infty} \frac{Sin\left(n^2\, x\right)}{n^2}$$

The convergence of this series is uniform, so it is continuous at every x, but nondifferentiabiliy is harder to prove. It was not until 1916 that G. H. Hardy proved that in any finite interval there will be infinitely many values of x for which the derivate does not exist, but in 1970 it was demonstrated that there are also infinitely many values at which the derivate does exist (Bressoud, 1994). If we represent some partial sums of this series whit the calculator, we can observe how much complicated its graph is. On the left figure appears the sum of its 15 first terms, and the box marked there is expanded on right figure:

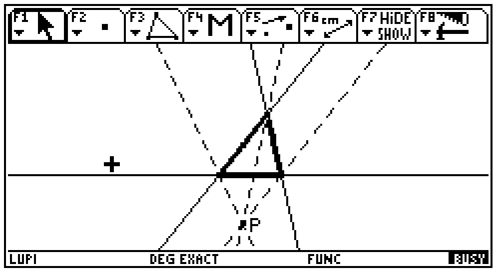


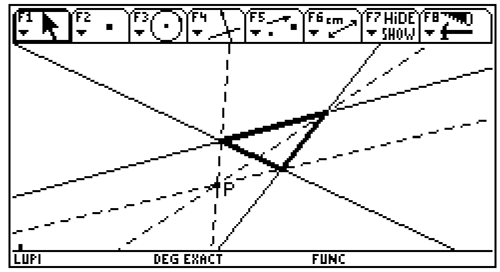


Into calculator environment, a function is differentiable at a point if, when doing successively zooms of its graph around this point, the graph looks like a line. If we do it in the example below, we will approach to an argumentation form of existence of many infinitely values for which the derivate does not exist, although the study of points in which function has derivate requires a very different treatment. Anyway, a functions' study under this perspective must be always carefully planned in order to don't lead students to misconceptions or mistaken interpretations. Besides, it is very important to conjugate the graphical and analytic possibilities of these technologies, because from these relations arise an useful and a consistent knowledge.

The technology' power is, above all, epistemological. Its impact is based on a reification of mathematical objects and relations that students can use to act more directly on these objects and relations than ever before (Balacheff y Kaput, 1996). The calculator permits us that mathematical objects can be manipulated, we can *act about there*, and therefore, the technology's force is based on that reification of mathematical objects and relations.

The traditional analytic representations have been complemented and enriched by these new technologies. The static nature of the traditional representational systems disappears with the *executable representations*, which can be manipulated, which can act directly against them. We can see a good example of this in dynamic geometry environments as Cabri, from which TI-92 introduces a version. If we represent a triangle in this environment, and we trace the three angle bisectors to its external angles, we can observe that these three lines on intersect among themselves, and that if we change the position, form or measurements of the triangle using the dragging, this property follows been true.





All this emphasizes the executable representations' notion that offers new technologies, in comparison to traditional static representations, with which can be almost impossible to visualize some mathematical objects' properties. The abstract concepts of mathematics become real with the use of a calculator, in the way that they can be manipulated, transformed.

The reflections we mentioned in this work allow us to get an idea of capability that these instruments have to modify the students' mathematical instruction, since they supply representations and relation between mathematical objects with which they can interact, giving a new dimension to mathematical knowledge construction.

We want to emphasize the fact that taking to classroom a work of this kind, it is necessary to think about curricula and, principally, about teachers' training. We aren't supposed to do only the things that we did without technologies, but we need to reorganize goals, activities, and assessment in mathematics, and all this lies in a precise teachers' training.

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