Conjecturing and proving in AlNuSet

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This report proposes an approach to algebraic proof, based on the use of the AlNuSet system, a dynamic, interactive system to enhance the teaching and learning of algebra. The research hypothesis is that educational activities performed in AlNuSet favours the "transparency" of algebraic proof.

In the secondary school curricula of many countries, the approach to proof is still taught in the context of traditional geometry (Hanna & Jahnke, 1993). Nevertheless, rigorous proof is generally considered as a sequence of formulae within a given system, each formula being either an axiom or derivable from an earlier formula by a rule of the system. This kind of proof clearly reveals the influence of algebra (Hanna & Jahnke, 1993). In school practice, however, algebra is usually not considered as a way of seeing and expressing relationships but as a body of rules and procedures for manipulating symbols. Thus, algebra is taught and learned as a language and emphasis is given to its syntactical aspects.

This report proposes an approach to algebraic proof. It is based on the use of the AlNuSet[1], a system which can be used to propose specific tasks requiring the construction of a conjecture and the production of an algebraic proof (Pedemonte, 2011). AlNuSet is a dynamic, interactive system for enhancing the teaching and learning of algebra for lower and upper secondary schools (Pedemonte, & Chiappini, 2008). It is constituted by three integrated environments: the *Algebraic Line*, the *Algebraic Manipulator*, and the *Functions*. In this report we consider two of them: the Algebraic Line (AL) and the Algebraic Manipulator (AM). The AL is an explorative environment to construct conjectures through a motor perceptive approach; the AM is a symbolic calculation environment to produce algebraic proof. The aim of this report is to show

[1] www.alnuset.com

Pedemonte, B. (2013). Conjecturing and proving in AlNuSet. En P. Perry (Ed.), *Memorias del 21º Encuentro de Geometría y sus Aplicaciones* (pp. 71-74). Bogotá, Colombia: Universidad Pedagógica Nacional.

how this system can be used to support the teaching and learning of algebraic proof, making proof "transparent" (Hemmi, 2008). The concept of transparency combines two characteristics: visibility and invisibility. Visibility concerns the ways that focus on the significance of proof (construction of the proof, logical structure of proof, its function, etc.). Invisibility concerns the proof as a justification of the solution of a problem without thinking it as a proof. It has been underlined that "*Proof as an artifact needs to be both seen (to be visible) and used and seen through (to be invisible) in order to provide access to mathematical learning*" (Hemmi, p. 425).

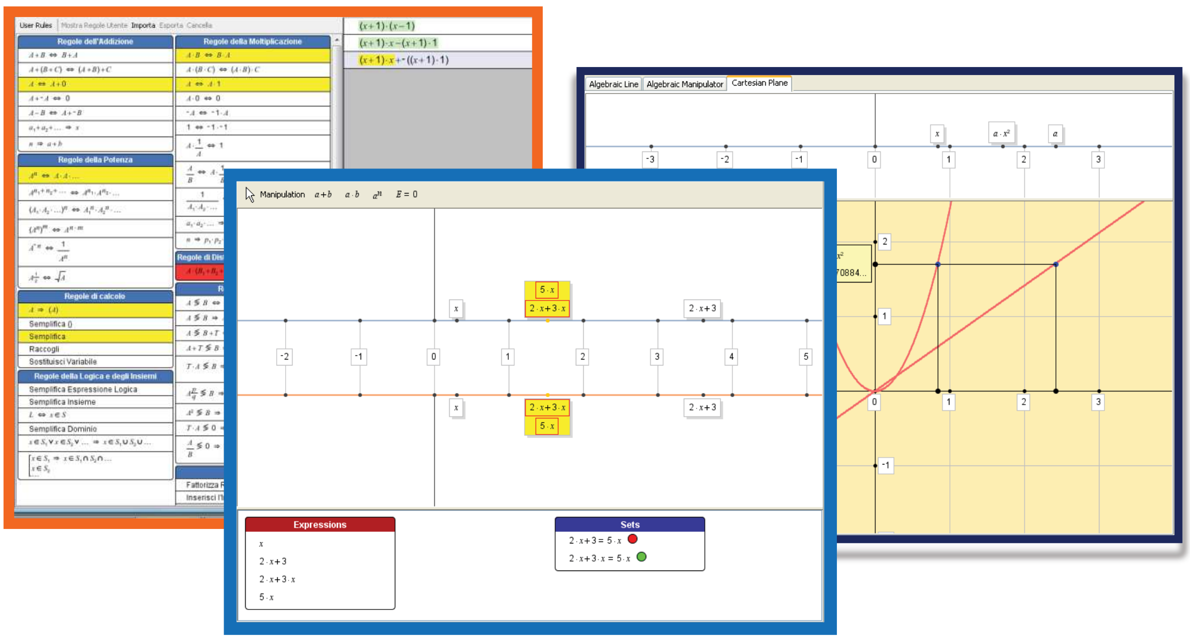

Figure 1: The three environments of AlNuSet: the Algebraic Line, the Algebraic Manipulator, the Function environment

THE ALGEBRAIC LINE OF ALNUSET

The Algebraic Line (AL) of AlNuSet is constituted by two lines[2] where it is possible to insert letters and mathematical expressions involving numbers and letters. These expressions can be inserted (or constructed) and represented as points on the line depending on the mobile point of the variable contained in such expressions. Once an expression has been inserted, dragging the x mobile

[2] The two lines are used to construct expressions through a geometrical model. It is not possible here to explain in which way they work. In this report we use these lines as a unique line.

point, the expression(s) that depend on it move accordingly. This dynamic characteristic is very important to allow students experience important algebraic concepts –the dependence of the expression from a variable, the meaning of denotation for an expression, the equivalence among expressions, etc.

THE ALGEBRAIC MANIPULATOR OF ALNUSET

The Algbraic Manipulator of AlNuSet is a structured symbolic calculation environment for the manipulation of algebraic expressions and for the solution of equations and inequalities. Its operative features are based on pattern matching techniques. In the Algebraic Manipulator pattern matching is based on a structured set of basic rules that correspond to the basic properties of operations, to the equality and inequality properties between algebraic expressions, to basic operations among propositions and sets. These rules are explicit for students. They appear as commands on the interface and made active only if they can be applied to the part of expression previously selected. An expression is transformed into another through this set of commands that corresponds to axioms and rules. Students can see the transformation of an expression as the result of the application of a rule on it.

EXEMPLE OF USE OF ALNUSET

Consider the following task:

> Let x be an integer number. Write an expression for the triple of x. Represent this expression on the AL. Write an expression for the consecutive of the triple of x. Represent it on the AL and verify your answer. Consider the expression $x + 2x + 1$. Compare it with the previous one. Check your answer using the AL and AM of AlNuSet.

This task requires to construct the expression $3x$ in the AL and verify that this expression represents the triple of x. Moving x on the line the expression $3x$ moves accordingly. Through a perceptive approach students can see that the point associated to the expression $3x$ assumes values that are multiples of 3. In this way, what the expression $3x$ denotes is made more explicit. Furthermore, only moving the variable x it is possible to move $3x$ allowing students to experience the expression's dependence from the variable x.

The second part of the task requires to construct the consecutive of the triple of x and to compare it with the expression $x + 2x + 1$. The aim of this second question is to point out the equivalence between the two expressions from a perceptive point of view and not from a formal one. In the AL the equivalence among expressions is represented by a post-it (see Figure 2). The two expressions $3x + 1$ and $x + 2x + 1$ belong to a same post-it for each value the variable x assumes on the line.

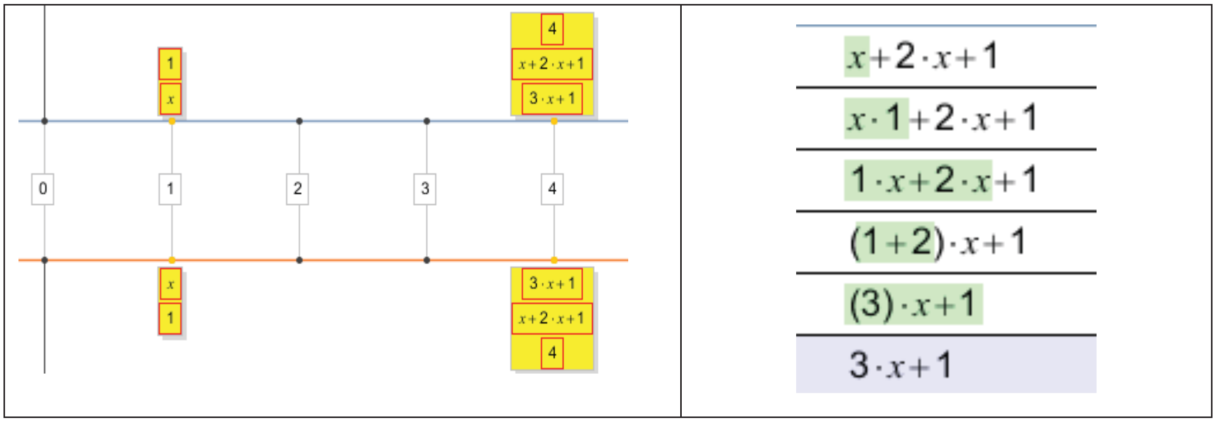

Figure 2: The equivalence between two expressions in the AL and in the AM

Students can "experience" the equivalence of the two expressions and then they can prove the equivalence in the AM. In the AL, students make visible the equivalence between the two expressions. The focus here is not to prove the equivalence but to experience it. In the AM proof is made explicit – students are obliged to explicit the rules necessary to transform the first expression into the other. This is not obvious because this transformation in a paper and pen environment is usually not treated as a proof; here proof is in general invisible to students.

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