AIEM - Avances de Investigación en Educación Matemática. - 2017, Nº 11, 47 - 68

Developmental stages and important periods of probability cognition in 6 to 14 year-old students

Zikun Gong, Hangzhou Normal University (China)

Shengqing He, Beijing Normal University (China)

Recibido el 6 de julio de 2016; aceptado el 26 de febrero de 2017

Etapas de desarrollo y períodos importantes de cognición probabilística en estudiantes de 6 a 14 años

Resumen

En este estudio se seleccionaron 906 estudiantes de 6 a 14 años de edad y se estudiaron las etapas de desarrollo y los períodos de cognición de la probabilidad. El estudio muestra que la cognición probabilística de los estudiantes de 6 a 14 años experimenta las siguientes 5 etapas: desarrollo lento I (6-7 años), desarrollo rápido I (8-9 años), desarrollo lento II (10 años), desarrollo rápido II (11-12 años) y fase consolidada (13-14 años). Además, hay dos períodos importantes en el desarrollo cognitivo de los estudiantes: el primero a lo 8-9 años de edad y el segundo a los 11-12. Incluso en la etapa de desarrollo más alta, los estudiantes pueden entender la representación numérica, la distribución de probabilidad y la representación fraccional mientras que no pueden alcanzar el nivel de maestría, lo que sugiere la limitación de la cognición probabilística de los estudiantes. En consecuencia, el plan de estudios debe tener en cuenta el nivel de desarrollo cognitivo de los estudiantes y establecer objetivos cognitivos razonables.

Palabras Claves. Estudiantes; probabilidad; probabilidad; cognición, aprendizaje y comprensión.

Estágio de desenvolvimento cognitivo de o conceito de probabilidade e importantes períodos de estudo em estudantes 6-14 anos de idade.

Resumo

906 estudantes com idades entre 6-14 foram testadas para investigar o conceito de estágio de desenvolvimento cognitivo da sua probabilidade e os períodos importantes. Os resultados mostraram que o desenvolvimento cognitivo tem experimentado um período de crescimento lento 1 (6-7 anos de idade),

Para citar: Gong, Z. & He, S. (2017). Developmental stages and important periods of probability cognition in 6 to 14 year-old students. *Avances de Investigación en Educación Matemática*, 11, 47 – 68.

um período de rápido desenvolvimento (8-9 anos), período de desenvolvimento lento 2 (10 anos), o período de desenvolvimento rápido (11-12 anos) e período de estagnação do desenvolvimento (13-14 anos). Ao desenvolvimento cognitivo tem dois períodos importantes: os primeiros 8-9 anos de idade é um período importante, os estudantes dominar esta fase da aleatoriedade e quantidade de crescimento rápido; 11-12 anos de idade, é o segundo período importante de desenvolvimento. Ao nível mais alto de conceito de probabilidade os estudantes tem período de rápido desenvolvimento, no entanto, para a quantidade de cognitiva e distribuição aleatória atingiu apenas o nível de compreensão. Isto sugere que a os estudantes têm limitada cogniçao em probabilidade, e, por conseguinte, o currículo padrão para um posicionamento preciso.

Palavras-Chave. Estudantes, probabilidade, cognição, aprendizagem e compreensão.

Developmental stages and important periods of probability cognition in 6 to 14 year-old students

Abstract

This study chose 906 students of 6 to 14 years of age and focused on the developmental stages and important periods of probability cognition. The study shows that probability cognition of students aged 6-14 experiences the following 5 stages: slow development stage I (6-7 years old), quick development stage I (8-9 years old), slow development stage II (10 years old), quick development stage II (11-12 years old) and stagnant stage (13-14 years old). Additionally, there are two important periods in students' cognitive development: 8-9 years old is the first period and 11-12 is the second. Even at the highest development stage, students can just understand the number representation, probability distribution and fraction representation while ca not reach the mastery level, which suggests the limitation of students' probability cognition. Accordingly, curriculum should take students' cognitive development level into account and set reasonable cognitive objectives.

Keywords. Students, probability, cognition, learning and understanding.

6-14 ans élèves connaissance de concept probabilité et périodes importante d'étude

Résumé

906 élèves âgés de 6-14 ont été testés pour étudier le stade de développement et la période importante sur leurs connaissances de probabilité. Les résultats ont montré que leur développement cognitif a connu cinq étages: première période de développement lent(6-7 ans), première période de développement rapide (8-9 ans), deuxième période de développement lent (10 ans), deuxième développement rapide (11-12 ans) et une période de stagnation du développement (13-14 ans). En plus, le développement cognitif a deux périodes importantes: les 8-9 premières années de l'âge est une période importante où les enfants maîtrisent le concept de distribution aléatoire et leur connaissance de quantité accroit rapidement; les années de 11-12 ans sont la deuxième période importante qui n'a pas beaucoup de progrès par rapport à la période précédente. La deuxième période de développement rapide a connu le plus haut niveau de leurs connaissances de probabilité et de concept. Cependant, leur connaissance pour la quantité et la distribution aléatoire a atteint seulement le niveau de compréhension. Ceci suggère que la connaissance de la probabilité des élèves a ses limites. Par conséquent, il faut repérer précisément *le standard du programme scolaire.*

Mots-clés. *élèves*, probabilité, cognition, apprentissage et compréhension.

1. Introduction

Probability is a tool for describing, quantifying and modelling randomness in order to help people making reasonable decisions under random conditions (Jones, Langrall & Mooney, 2007). Because probability literacy is one of the mathematics literacy goals for current citizens (Gal, 2005; Jones, et al., 2007; Nilsson & Li, 2015), since the 1980s international countries (e.g. Australian Education Council, 1991; Department of Education and Science and the Welsh Office, 1991; National Council of Teachers of Mathematics, 2000) started to introduce probability knowledge into primary and middle school curricula.

When compared with European and U.S. curricula, the Chinese curriculum reform in probability content was delayed. The Mathematics Curriculum Standards for Compulsory Education (Experimental Edition) (China Ministry of Education, 2001) published in 2001, organized probability related knowledge as a content branch in the compulsory curriculum and determined the cognitive objectives for the first time. However, some studies (Gao, 2011; Gong & Song, 2006; Wang, 2012; Yan, 2007) have shown that teachers cannot adapt their teaching to these requirements, and specifically, teachers lack the probability related knowledge (Gong & Song, 2006; Yan, 2007) and encounter various difficulties in teaching probability (Yan, 2007).

In view of the above issues, the *Mathematics Curriculum Standard Revision Panel* was established by the Ministry of Education in 2005 and the revision work by this panel was started since then. The Mathematics Curriculum Standards for Compulsory Education (China Ministry of Education, 2011) (hereinafter Curriculum Standards, 2011 Edition) started some adjustments and revisions of the probability content to reduce this content and decrease the difficulty of the topic. The theoretical and practical basis of this revision involves a key question: what are the development stages and levels of students' probability cognition? A widely shared principle in educational psychology was stated by Ausubel and colleagues (Ausubel, Novak & Hanesian, 1983): the most important factor to influence learning is the student's previous knowledge, and therefore, we ought to discover it and to teach consequently. Only based on the data collected from psychological research that define students' cognitive development stages on the concept of probability can we solve the curricular problems.

The aim of this study is to provide some initial response to the above question. We present a study that was intended to explore two issues:

1. The cognitive development stages and the relevant levels of students' probability cognition.

2. The important period(s) in the cognitive development of probability.

2. Theoretical framework and background

Piaget and Inhelder (1975) defined the following three developmental stages of probability cognition: In stage 1 (before 7 to 8 years), children have not acquired the concept of probability and cannot distinguish between random and casual events; in stage 2 (8 to 12 years), children start to differentiating between certainty and uncertainty, and qualify probability in simple situations; in stage 3 (around 12 years), children can make connections between deductive logic and probability and make accurate calculations in probability. In addition, the more profound theoretical contribution of Piaget's study was analysing the cognitive development from the developmental perspective (age); this research opened a window for later research.

However, some subsequent studies pointed out that Piaget's stages and level descriptions were not widely applicable. For instance, Fischbein and Gazit (1984) pointed out that Piaget's research underestimated students' cognitive ability, while Carpenter (1981) suggested that even in developed countries, students' cognition of probability failed to get the levels that Piaget described.

Another perspective that focused on the hierarchical features was defining the reasoning levels according to different psychological theories. Biggs and Collis (1982) proposed a SOLO model (Structure of the Observed Learning Outcome) to describe cognition levels of probability, which divided students' probability cognition into five levels, namely pre-structural, uni-structural, multi-structural, relational and extended abstract levels. Li (2002, 2003) proposed a framework for describing probability conceptual development based on SOLO model, which rendered the aforementioned levels more specific.

Furthermore, other authors studied knowledge structure and conceptual development in primary and middle curricula, in specific sub-concepts, such as randomness (Batanero & Serrano, 1999; Engel & Sedlmeier, 2005), chance (Nikiforidou & Pange, 2010; Watson & Moritz, 2003b), probability comparison (Jones, Langrall, Thornton & Mogill, 1999), probability measurement (Chernoff, 2008), and probability distribution or sample space (Chernoff, 2009; Chernoff & Zazkis, 2011; Jones, Langrall, Thornton & Mogill, 1997, 1999). Accordingly, we should also take the different sub-concepts into account since the developmental stages of different sub-concepts maybe different. And consequently, the above researchers were interested in investigating cognitive development research in probability with specific cognitive tasks.

Jones and colleagues (Jones, et al., 1997, 1999) proposed a framework for describing students' probability thinking and its development, which characterized students' developmental level of probability by four different sub-concepts (sample space, probability of events, probability comparison and conditional probability) and four hierarchies (subjective level, transitional level, nonstandard quantitative level and numerical level). However, the subjects in this research were limited to elementary *3th* graders, many of which failed to achieve the different developmental features.

In the 1980th, Zhang and colleagues (Zhang, Liu, Zhao, Sun, & Zuo, 1985) initiated a series of experiments to investigate 5-15 years old students' mastery of probability in China, in which they described *the first key period* in students' probability cognitive development; but a systematic research of a second key period is still lacking. In the process of developing the concept of probability, that is, from a preliminary perception of possible outcomes for an experiment to some probability concepts, the first key period appears around 9 years of age and there may be a second key period later (Zhang, Liu & Qiu, 1983).

In the present research, we basically focus on two issues: describing the cognitive developmental stages and identifying the important periods of students' probability cognition. By analysing these issues, we expect to explore the progression in probability cognition from the perspective of age. More specifically, we expect to classify the developmental stages by differentiating the speed and degree by which this concept develops, and, in addition, we also intend to find the dominant sub-concepts that develop at different stages. By analysing the latter issue, we try to verify the key period suggested by Zhang' team (Zhang, et al., 1985) and to explore other important periods that these authors hypothesized. By studying the above issues, the developmental features can be uncovered and specific conclusions for curriculum and teaching design can be achieved.

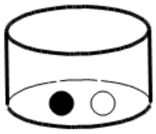
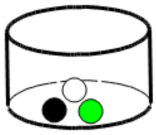
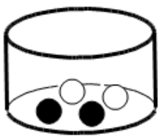
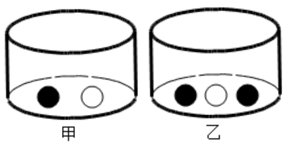
3. Research design

3.1. Questionnaire

The questionnaire involved four sets of questions and each question included five cognitive tasks, related to understanding randomness (hereinafter randomness), understanding probability distribution (probability distribution in the following), qualitative comparison of probability (hereinafter qualitative comparison), number representations of probability (hereinafter number representation), and fraction representation of probability (fraction representation in the following). The first three questions were adapted from previous research (Zhang, et al., 1985) while question four was designed originally. The contexts of the four questions were basically the same, while the tasks were hierarchically designed according to variables such as the total numbers of balls, the colours of the balls, the number of balls taken out, or the number of boxes (see Table 1).

By controlling the above variables, the questions can be classified into two basic types; the first is "selecting a ball" (simple experiment) and the second "selecting two balls" (compound experiment). In the latter situations, questions were comparatively difficult, since students should apply combinatorics; in addition, the difficulty of combination can be different by controlling the colours of the balls, and the numbers of boxes.

Table1. *Questionnaire for probability cognition*

Contexts	Tasks
1 black ball and 1 white ball are put in 1 box; you select 1 ball 	Consider the ball that was selected, Task 1: is it certainly a black ball? Task 2: how many possibilities are there in total? Please list them. Task 3: please compare the probability of 1 black and 1 white. Task 4: if we put back the ball in the box and then select 1 ball repeatedly for 10 times, please estimate the times you will get a white ball. Task 5: please describe the probability of white ball with a fraction.
1 black ball, 1 white ball and 1 green ball are put in 1 box, then you select 2 balls 	Consider the 2 balls that were selected, Task 1: is it certainly one of the balls a black one? Task 2: how many possibilities there are? Please list them. Task 3: please compare the probability of 1 black and 1 green and 1 white and 1 green. Task 4: if we replace the 2 balls in the box and then take out 2 balls repeatedly for 10 times, please estimate the times you will get 1 white ball and 1 green ball. Task 5: please describe the probability of 1 white and 1 green using a fraction.
2 black balls and 2 white balls are put in 1 box, you select 2 balls 	Consider the 2 balls that were selected, Task 1: is it certainly one of the balls a black one? Task 2: how many possibilities there are? Please list them. Task 3: please compare the probability of 1 black and 1 white and 2 whites. Task 4: if we put back the 2 balls in the box and then take out 2 balls repeatedly for 10 times, please estimate the times you will get 1 white and 1 black. Task 5: please describe the probability of 1 white and 1 black using a fraction.
There are 1 black ball and 1 white ball in the left box, and 1 white ball and 2 black balls in the right box, you select 1 ball from each box. 	Consider the 2 balls that selected, Task 1: is it certainly one of the balls a black one? Task 2: how many possibilities there are? Please list them. Task 3: please compare the probability of 1 black and 1 white and 2 whites. Task 4: if we put back the 2 balls in the box and then take out 1 ball repeatedly for 10 times, please estimate the times you will get 1 white and 1 black. Task 5: please describe the probability of 1 white and 1 black using a fraction.

We admit that questions related to compound experiment are to some extent difficult for lower graders, but we aimed to control the questions consistency in the different groups in order to explore the potential abilities in lower groups. Furthermore, combinatorial issues are included in Chinese mathematics textbooks even as early as grade 2 (Che, 2014), and investigation also indicated that students in grade 2 can deal with some simple combinations such as matching all the possibilities of 3 coats and 5 skirts (Che, 2014).

We define *probability cognition* to describe students' knowledge, recognition and understanding of probability and of the relevant sub-concepts. Specifically, cognition of randomness refers to students' understanding of random events and of the meaning of certain, possible and impossible; Cognition of probability distribution includes students' estimation of all the possibilities of random events; Qualitative comparison of probability refers to students' qualitative judgment for the probability of two or more events; Number representation of probability consist in describing the probability of an event by numbers (repeating the experiment 10 times); Fraction representation of probability is describing the probability of an event using fractions.

Taking into account the above definitions, we used students' response (choice of correct answers) to describe their cognition and the percentages of correct responses to describe the cognitive levels. The question context was the following:

> An opaque box contains x white balls, y black balls (and z green balls), which are identical except for the colour. Close your eyes and shake the box; then select two balls simultaneously from the box. Please answer the following questions.

3.2. Subjects

The *experimental group* was made of 906 students in grades K-8 (6-14 years old, and about 95 students in each age) from 27 groups; the sample was stratified to select a representative sample from urban, suburban and rural areas (Table 2).

Table 2. *Ages of the experimental subjects*

Age	N	Min	Max	M	SD
6	95	5.8	7.2	6.45	.30
7	90	6.8	8.3	7.38	.36
8	88	7.8	9.3	8.43	.35
9	106	8.8	10.4	9.43	.40
10	108	9.7	11.4	10.48	.34
11	107	10.8	12.2	11.38	.39
12	102	11.8	13.3	12.48	.43
13	99	12.5	14.3	13.47	.40
14	111	13.6	15.3	14.43	.41

Students in the *experimental group* use the textbooks published by People's Education Press (PEP) (Lu & Yang, 2009; 2010a; 2010b) in primary grades and those published by Zhejiang Education Press (ZEP) (Fan, 2006; 2010) in middle grades. The students involved in the study acquired probability related knowledge as early as in grade 3, and, from the perspective of conceptual development in the long run, these subjects studied all the sub-concepts mentioned in the present study. Specifically, they preliminary learned randomness and qualitative comparison in the first semester of grade 3 (Lu & Yang, 2009), and learned equal probability, quantitative comparison, fraction representation and probability distribution in the first semester of grade 5 (Lu & Yang, 2010a). The middle grades subjects further learned probability distribution and fraction representation in the second semester of grade 7 (Fan, 2006).

Control group. To eliminate the impact of learning in the evaluation of students' probability cognitive development, we additionally chose students which have not learned probability as from other schools as the control group, which consisted in two age groups: 9 years-olds (67 students) and 11 year-olds (66 students) (Table 3).

Table 3. *Ages of the control subjects*

Age	N	Min	Max	M	SD
9	67	8.8	10.5	9.63	.39
11	66	10.2	12.7	11.5	.39

Students in the control group used the ZEP textbooks in primary grades. The experiment of the control group was not originally planned and only age 9 and 11 were involved, since we found two periods (that is, 7-9 and 10-12) in which students developed quickly in probability cognition. To eliminate the impact of learning, we conducted this additional experiment to confirm whether students' quick development in these periods was the result of learning or not. Students of 9 year-olds in this group had not learnt any probability related knowledge, and the 11 year-olds just had learnt randomness and preliminary probability distribution at the age of 10.

It is worth noting that the decision of selecting the editions of textbooks was made by local authorities, namely, the Bureau of Education of certain regions. In Hangzhou city in which the present subjects selected, only the Shangcheng region adopted the ZEP textbooks in primary grades.

3.3. Procedure

All the subjects answered the same four sets of questions. Considering that the reading and writing literacy were limited for students aged 6-7, we conducted interviews with them and showed them the questions in visual way by presenting the students real balls and boxes and helped them to imagine the process described in the different questions. For students aged 8, we asked their mathematics teachers to project the questions on the blackboard and explain these questions to the children in front the classroom; we also organized a procedure for the children answer independently the questions. Finally, for the students aged 9-14, we gave them paper and pencil tests. And considering that students fewer than 9 had not acquired any knowledge related to fraction, we did not gave task 5 to these groups.

All the testing was conducted in mathematics classes with permission of schools principals and relevant teachers; the testing times were different among different graders. Generally, students in grade 3 and above answered the tests within 40 minutes or less, and the students in grade 2 (aged 8) usually finished in a whole class, while the children in preschool grade (aged 6) and grade 1 (aged 7) who answered our interviews, needed longer time. Specifically, the project members (2 professors, 1 middle school teacher and 4 master students) did this part of the interviews and generally we spent 30 to 50 minutes to talk with a single subject. For each task, we coded 1 for the correct responses and 0 for wrong responses.

4. Results

4.1. Reliability and Validity

We computed the *Crombach α coefficient* for the 5 cognitive tasks, which varied in the interval 0.70-0.870, which shows that the questionnaire had comparatively high homogeneity reliability.

The *correlation coefficients* among the four sets of questions were all statistically significant (0.28-0.716), which shows that the questionnaire had comparatively high construct validity, since responses to the different items were related.

4.2. Developmental Stages of Probability Cognition

In Table 4 and Figure 1 we present the summary statistics for the total scores for the four sets of questions. From these results we can conclude that students' cognition of probability (as measured by our questionnaires) generally increased along age. In addition, two separate periods where students developed comparatively faster than in other ages were 7-9 and 10-12 years, while along 6-7 and 9-10 years students developed slowly and 12-14 years old was a stage where no progress was observed.

Table 4. *Summary statistics of the scores*

Age	Number of students	Average score	SD	Max	Min
6	95	6.73	3.23	13	0
7	90	6.91	2.80	12	0
8	88	9.11	1.96	13	4

9	106	11.66	2.74	17	3
10	108	12.23	2.65	17	0
11	107	13.55	2.16	18	8
12	102	14.84	3.18	20	6
13	99	14.73	3.09	20	5
14	111	14.02	2.57	20	4

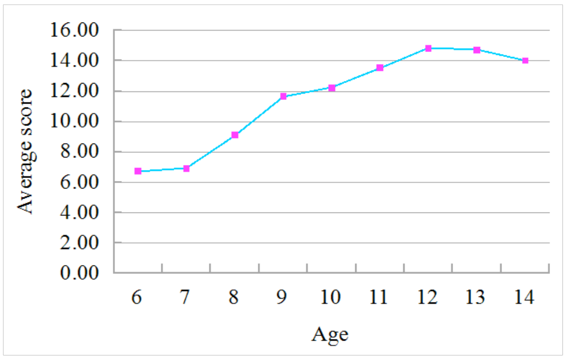


Figure1. Average score in the probability cognition test for students aged 6-14

Table 5. *Multiple-comparison between probability cognitive levels of 6 to 14 years*

Age	6	7	8	9	10	11	12	13	14
6									
7	-.19								
8	-2.39***	-2.20***							
9	-4.93***	-4.75***	-2.55***						
10	-5.51***	-5.32***	-3.12***	-.57					
11	-6.83***	-6.64***	-4.44***	-1.89***	-1.32**				
12	-8.11***	-7.92***	-5.72***	-3.18***	-2.60***	-1.28*			
13	-8.00***	-7.82***	-5.61***	-3.07***	-2.50***	-1.18	.11		
14	-7.29***	-7.11***	-4.90***	-2.36***	-1.79***	-.47	.82	-.71	

**p*<0.05, ***p*<0.01, ****p*<0.000

We conducted a one-way ANOVA with age as an independent variable (factor) and probability cognition as dependent variable. The homogeneity tests (W= 5.76, 8,897 d.f., p<.001) showed that variance was not homogeneous, which is not a serious in ANOVA. In this case it is suggested to resort to Welch's variance-weighted ANOVA as an alternative method (Li, Zhang & Shoo, 2014, p.129). The results were t=124.739, 8,371 d.g., p<.001, which suggests there are significant differences between different age groups in terms of probability cognition, as measured by our test. Specifically, we performed the multiple comparison test (Table 5), which shows there are significant differences between most age groups ($p < .05$).

Accordingly, we can conclude that children's probability cognition experience 5 developmental stages:

Stage 1 (6≤Age≤7): Slow development stage I. Students at this stage have not received any formal instruction on probability, so that their probability knowledge emerge from their daily life experience. The correct percentage of responses in the test was 42.6% (more specifically, 63.8% in cognition of randomness, 37% in qualitative comparison number representation, 37% and probability distribution 32.5%. Considering the standard of 20%, 50% and 80% respectively as goals for preliminary understanding, understanding and mastery (Shen & Liu, 1984), students aged 6-7 in our sample understood randomness and acquired preliminary knowledge of qualitative comparison, number representation and probability distribution in the questions proposed to them. We used the terms *preliminary understanding*, *understanding* and *mastery* as reference in the present study to describing and classifying students' level more clearly.

Stage 2 (7＜Age≤9): Quick development stage I. Students at this stage quickly develop probability concepts (the rate of development can be calculated by the slope of the line graph in Figure 1 (11.66-6.91)/2 = 2.375). In addition, there is significant difference between 7, 8 and 9 years of age. Specifically, students achieve high development in every task; the correct percentage for 8 year-olds was 56.9% and they even reached 60.3% in number representation; the correct percentage for 9 year-olds was 58.3%, 52% in number representation and 47.8% in fraction representation (considering that 9 years had learned preliminary fraction, we designed the fraction representation tasks for students older than 9 years). In conclusion, students in this stage achieved a sound development in probability quantification and preliminarily developed fraction representation. They mastered randomness and understood probability distribution, qualitative comparison and number representation in our questionnaire.

Stage 3 (9＜Age≤10): Slow development stage II. Students aged 10 scored 0.57 higher than 9 year-olds, while there was no significant difference between both groups. These students mastered randomness, understand qualitative comparison, number representation, probability distribution and fraction representation. Specifically the correct percentage of the whole test for 10 year-olds was 61.2% (while for 8 to 9 years-olds was around 60%); randomness was 89% (84% for 8 to 9 year-olds), probability distribution was 52% (45.8% for 8 to 9 year-olds), qualitative comparison was 58.5% (51.3% for 8 to 9 yea-olds), number representation was 55.3% (55.8% for 8 to 9 year-olds) and fraction representation was 50.8%.

Stage 4 (10＜Age≤12): Quick development stage II. This was another quick developmental period, although the development rate (1.3) was lower than that in Stage 2 (2.375). Compared to students aged 10, 11-12 in the responses to the questionnaire these students mastered randomness and qualitative comparison, understood number representation, probability distribution and fraction representation. There was no further development in the cognition of probability distribution and fraction representation. Compared to age 10, these students achieved further development in every tasks: 70.9%

correct percentage of the whole test was (in 10 year-olds was 61.2%) 95.8% in randomness(in 10 year-olds was 89%), 60.8% in probability distribution (52% for 10 year-olds), 73.5% in qualitative comparison (58.5% for 10 year-olds), 65.8% in number representation (55.5% for 10 year-olds) and 58.8% in fraction representation (50.8% for 10 year-olds).

It is worth noting that there were some differences in the tasks solutions between the two quick development stages. In the quick development stage I, students achieved significant development in every task, while in the quick development stage II, number representation and fraction representation failed to develop further, as we expected. In fact, students had learned fraction representation at grade 5 (11 years old) (Lu & Yang, 2010a), and systematically reviewed probability related knowledge at grade 6 (12 years old) (Lu & Yang, 2010b). Theoretically, it was expected that students at this stage further develop in every task while this did not happen. Instead they stopped to improving their responses after 12 years of age; we tentatively interpret this fact in the sense that it was not learning but students' cognition what affected their further development in these sub-concepts.

*Stage 5 (12＜Age≤14): Stagnant stage***.** The average score difference between 13 years old and 12 years old was -0.11, and between 14 years old and 13 years old -0.71. That is to say, students experienced lack of progress and even negative development from 13 to 14 years in their responses. There was no significant difference in the cognitive level between 13-14 and 10-11 years old students, and from a larger extent, the cognition of probability for students aged 11-14 were roughly in the same level.

The percentage of correct responses for 13 to 14 year-olds in the whole test was 71.8% (70.9% in 11 to 12 year-olds); for randomness it was 95% (95.8% for 11 to 12 year-olds), 58.8% for probability distribution(60.8% for 11 to 12 year-olds), 78.3% for qualitative comparison (73.5% for 11 to 12 year-olds), 65% for number representation (65.8% for 10 year-olds) and 62% for fraction representation (58.8% for 11 to 12 year-olds). Apart from qualitative comparison (that improved in 4.8%) and fraction representation (improving 3.2%), 13 to 14 years old students scored lower than 11 to 12 year-olds in other tasks.

A possible question is whether the same or similar questions to those proposed in our tests were taught in grade 7 to 8 textbooks. Actually, in the ZEP textbooks that the subjects in the experimental group studied, the related or similar knowledge, that is, compound experiment, is included (Fan, 2006; Fan, 2010). An example is the following problem: "if we throw a dice for two times, could you calculate the probability of the sum of the two independent outcomes". We admit that this example is different from the questions involved in the present study, because the former focus on 'two steps' experiment while the latter emphasizes 'taking out two balls simultaneously'; however both problems require the operation of combination. Apparently, students had been taught probability knowledge at this stage, but students failed to get a further development; we interpret this fact as a sign that that neither their cognition have reached the necessary maturity nor their related knowledge changed.

4.3. Important Periods of Probability Cognition

As mentioned above, 7-9 and 10-12 years were two quick development stages so that were interested in analysing if these stages are important spontaneous developmental periods To answer this question, we should exclude the influence of teaching.

Students aged 8 in the experimental group had not been taught probability, while 9 year-olds learnt some preliminary probability (randomness, probability distribution and qualitative comparison); therefore, a possibility was that the quick development in 9 year-olds was due to learning. Similarly may have happened to students aged 11 years who were taught some simple probability (number representation and fraction representation). The 9 year-olds students in the control group have not learnt probability, and the 11 years old students, just learned preliminary probability (randomness and probability distribution) at the age of 10. By comparing the group that had learnt probability (experimental group) and the group that did not learn (control group) we analysed the impact of learning in probability cognition.

Firstly we study these results in grade 9. The summary statistics in the experimental and control group are reported in Table 6. We conducted a one-way ANOVA with group (experimental and control group) as independent variable and probability cognition as dependent variable and obtained $F(1, 141) = 0.200$ (p=.656), which suggests there was no significant difference between different both groups in terms of probability cognition.

Table 6. *Summary statistics in experimental and control group for students aged 9*

	N	Mean	SD
Experimental group	76	11.39	2.866
Control group	67	11.18	2.892
Sum	143	11.29	2.870

Table 7. *Summary statistics for the different tasks in experimental and control group(9 year-olds)*

		N	M	SD	SE	Min	Max
Randomness	Experimental group	76	3.57	.94	.11	0	4
	Control group	67	3.40	.94	.12	1	4
	Total	143	3.49	.94	.08	0	4
Probability distribution	Experimental group	76	1.82	.63	.07	0	3
	Control group	67	1.96	.71	.09	0	4
	Total	143	1.88	.67	.06	0	4

Qualitative comparison	Experimental group	76	1.97	1.32	.15	0	4
	Control group	67	2.04	1.26	.15	0	4
	Total	143	2.01	1.29	.11	0	4
Number representation	Experimental group	76	2.09	.85	.10	0	4
	Control group	67	2.10	.99	.12	0	4
	Total	143	2.10	.91	.08	0	4
Fraction representation	Experimental group	76	1.95	.89	.10	0	4
	Control group	67	1.67	1.21	.15	0	4
	Total	143	1.82	1.059	.09	0	4

Moreover, we analysed the difference in scores in all the tasks in both groups (see Table 7). Apart from cognition of randomness and fraction representation, the control group scored higher than the experimental group in all tasks. One-way ANOVA with group as independent variable and each task as dependent variables produced the following results with 1,141 d.g.: $F_{\text{randomness}} = 1.07$; p=0.304; $F_{\text{probability distribution}} = 1.57$, p=0.213; $F_{\text{qualitative comparison}}= 0.11$, p=0.743; $F_{\text{number representation}}= 0.01$, p=0.936; $F_{\text{fraction representation}}= 2.44$, p=0.121, which suggested there was no significant differences between the two groups in terms of probability cognition.

The above analysis suggests that, neither for the total score nor for the score in the different probability tasks, there was significant difference between both groups. That is to say, the teaching of probability did not affect these students' probability cognition significantly as regards the tasks proposed in the questionnaire. Our interpretation is that, the development of probability cognition at 9 year of age in these tasks was led by students' general cognitive development. In other words, 9 years old is an important period for probability cognition, since students at this age achieved further development in their general cognition development. It was worth noting that, our findings on the first important period agree with those of Zhang's research (1983). Secondly we study results in grade 11 (Table 8). The subjects in the control group had only learned some preliminary probability knowledge (randomness and probability distribution) at age 10.

Table 8. *Summary statistics in experimental and control group for students aged 11*

	N	Mean	SD
Experimental group	76	13.44	2.079
Control group	66	13.38	1.951
Sum	142	13.41	2.014

One-way ANOVA with group (experimental and control) as independent variable and probability cognition as dependent variable produced the following results with

1.140 d.g.: F=0.035 (p=.853), which suggests there was no significant difference between different groups Moreover, we analysed the scores in the different tasks in the two groups (see Table 9). One-way ANOVA with group (experimental and control) as independent variable and each task as dependent variable produced the following results with 1,140 d.g.; $F_{randomness}$ = 0.29, p=0.594; $F_{probability\ distribution}$= 0.31, p=0.577; $F_{qualitative\ comparison}$=0.80, p=0.372; $F_{number\ representation}$ = 0.20, p=0.656; $F_{fraction\ representation}$ = 0.01, p=0.928, which suggested there was no significant differences between the two groups in any of the tasks.

Table 9. Summary *statistics of different tasks in experimental and control group for 11 year-olds*

		N	M	SD	SE	Min	Max
Randomness	Experimental group	76	3.80	.65	.08	0	4
	Control group	66	3.74	.69	.08	0	4
	Total	142	3.77	.67	.06	0	4
Probability distribution	Experimental group	76	2.05	.51	.06	1	4
	Control group	66	2.00	.61	.08	0	4
	Total	142	2.03	.56	.05	0	4
Qualitative comparison	Experimental group	76	2.78	.78	.09	2	4
	Control group	66	2.89	.79	.10	1	4
	Total	142	2.83	.78	.07	1	4
Number representation	Experimental group	76	2.53	.76	.09	1	4
	Control group	66	2.47	.75	.09	1	4
	Total	142	2.50	.75	.06	1	4
Fraction representation	Experimental group	76	2.28	.66	.08	1	4
	Control group	66	2.27	.85	.11	0	4
	Total	142	2.28	.75	.06	0	4

The above analyses show that, neither for probability cognition nor for different probability tasks, there was significant difference between the two groups. That is to say, having learned the preliminary knowledge did not affect students' probability cognition significantly. We can conclude that the development of probability cognition at 11 to 12 years old was led by students' general cognition development. In other words, 11 to 12 years of age was another important period for probability cognition, since these students at this time achieved further development in their general cognition development. It was worth noting that, our findings on the second important period confirmed Zhang's (1983) prediction that there may be another key period of probability cognition after 9 years of age.

5. Discussions and suggestions

5.1. Students' Cognitive Stages of Probability

The following five developmental stages were identified in our research in relation to the tasks proposed:

1. *Slow development stage I* (6-7 years): students at this stage understand randomness and acquire preliminary knowledge of qualitative comparison, number representation and probability distribution;

2. *Quick development stage I* (7-9 years): these students quickly develop probability concepts and have obtained high development in probability quantification; they also preliminarily develop fraction representations;

3. *Slow development stage II* (9-10 years): students at this age master randomness, understand qualitative comparison, number representation, probability distribution and fraction representations;

4. *Quick development stage II* (10-12 years): these students master randomness and qualitative comparison, understand number representation, probability distribution and fraction representation. Specifically, there is no further development in the cognition of probability distribution and fraction representation;

5. *Stagnant stage* (12-14 years): students do not growth in the probabilistic reasoning and even experience negative development from 13 to 14 years. There is no significant difference in cognitive level between 13-14 and 10-11 years old students as regards the proposed tasks, and from a larger extent, the cognition of probability for students aged 11-14 were roughly in the same level.

Compared to previous studies (Zhang, Liu, & Qiu, 1983; Zhang, Liu, Zhao, Sun, & Zuo, 1985) that described either the developments with age or the progressions among different sub-concepts, our work does not only elaborated students' developmental stages in relation to the proposed tasks, but also clarified the cognitive levels reached for five specific sub-concepts used in our research. In addition, we explored the other important period as Zhang et al. (1985) hypothesized. But still, in the present study inevitably exist some limitations; for example the testing questions in the questionnaire could be refined or varied, and higher order sub-concepts (e.g. conditional probability, etc.) in each question could be considered. Even so, our results help the curriculum reform in analysing the reasonability of the selected content and in providing a hierarchy of cognitive requirements. This will serve to formulate the curriculum guidelines and select materials according to the students' actual cognitive level.

We also determined some *important periods of development of probability cognition:* 8 to 9 years of age was the first important period of development in the students' probability cognition, since these students mastered randomness, and number representation in the questions included in the questionnaire; 11 to 12 years of age was the second important period of development in probability cognition, although in this period students failed to further develop number and fraction representations.

Limitations of students' probability cognition in compound experiments. Even at the highest development stage (namely, quick development stage II), students in our sample could just understand the number representation, random distribution and fraction representation, while they did not reach the mastery level, which suggest the limitation of these students' probability cognition in relation to the test provided to them. We recognise that, the questions related to compound experiments in the present study were inevitably difficult for the students and even more for those in lower primary levels; therefore we carefully conclude that students' cognition on these questions was limited. In addition, these students had learnt number and fraction representations at 11 years and furthermore reviewed these contents systematically at 12 years. So, theoretically, these students should perform better at 11 or 12 than at 10 years of age, but they did not succeed. Specifically, having learnt all the contents assessed in our test, students aged 13 and 14 were still at a stagnant stage, which indicated the intrinsic difficulty of probability reasoning in these questions.

5.2. Suggestions for teaching

Taking into account our results we recommend proposing *limited objectives for each of the different stages.* Cognitive developmental stages provide a direct reference to organise the content in different grades (Lagnado & Shanks, 2003). According to our results, Chinese students can understand simple questions about randomness even at 6-7 years, and accordingly, we should basically focus on this topic in the first grades. Students achieve a great development in number representation at 8 - 9 years, so since these ages we should help them avoid their incorrect intuitions and develop an understanding that probability is measurable; they also achieve great development in simple questions about qualitative comparison and probability distribution. Considering that students' cognition of probability develop comparatively slower than, their cognition in other areas of mathematics, the curricular guidelines should take into account the students' cognitive development level and set reasonable cognitive objectives for each grade.

Teaching Interventions. Several studies (e.g. Fischbein & Gazit, 1984; Sharma, 2012) shown that, students actually, to some extent, acquire some probability intuitions and informal knowledge with careful instruction based on manipulative materials and experience with randomness. But still, there are potential conflicts or interactions between informally acquired probability knowledge and formally presented concept in school (Amir & Williams, 1999). Consequently, in the first important period of development of probability cognition (8-9 years old), we should avoid the negative influence from determinism and causal thinking on probability thinking, and focus on cultivating probability intuition. In the second important period of probability cognition, we can select some combinatorial situations (e.g. making combinations of 2 shirts and 3 trousers), as a prerequisite to develop students' ability of constructing sample spaces, and help them to make connections between sample space and probability calculation.

References

Amir, G. S., & Williams, J. S. (1999). Cultural influences on children's probabilistic thinking. *Journal of Mathematical Behavior*, *18* (1), 85-107.

Australian Education Council. (1991). *A national statement on mathematics for Australian schools*. Carlton, VIC: Curriculum Corporation, Department of Education and Science and the Welsh Office.

Ausubel, D. I., Novak, J. D., & Hanesian, H. (1983). *Psicología educativa*. Un punto de vista cognoscitivo, Mexico: Trillas.

Batanero, C., & Serrano, L. (1999). The meaning of randomness for secondary students. *Journal for Research in Mathematics Education*, *30*, 558-567.

Biggs J. B., & Collis K. F. (1982). *Evaluating the quality of learning: The SOLO taxonomy* (pp. 17-35). New York: Academic Press.

Carpenter T P. (1981). What are the chances of your students knowing probability? *The Mathematics Teacher*, *74*, 342-343.

Che, L. (2014). *Research on learning process and characteristic representations about permutation and combination in the second grade primary school.* Master's thesis of Hangzhou Normal University, Hangzhou.[In Chinese].

Chernoff, E. J. (2008). The state of probability measurement in mathematics education: a first approximation. *Philosophy of Mathematics Education Journal*, *23*, 1-23.

Chernoff, E. J. (2009). Sample space partitions: an investigative lens. *Journal of Mathematical Behavior, 28*(1), 19-29.

Chernoff, E. J., & Zazkis, R. (2011). From personal to conventional probabilities: from sample set to sample space. *Educational Studies in Mathematics*, *77,* 15-33.

China Ministry of Education. (2001). *Mathematics Curriculum Standards for Compulsory Education (Experimental Edition)*. Beijing Normal University Press. [In Chinese].

China Ministry of Education. (2011). *Mathematics Curriculum Standards for Compulsory Education (2011 Edition)*. Beijing Normal University Press. [In Chinese].

Copeland, R. W. Li, Q., & Kang, Q. (1985). *How do children learn mathematics?*. Shanghai: Shanghai Educational Publishing House.

Department of Education and Science and the Welsh Office. (1991). *National curriculum: Mathematics for ages 5 to 16*. York, UK: Central Office of Information,

Engel, J., & Sedlmeier, P. (2005). On middle-school students' comprehension of randomness and chance variability in data. *Zentralblatt für Didaktik der Mathematik (ZDM)*, *37(*3), 168-177.

Fan, L. (2006). *Mathematics (7th (semester 2))*.. Zhejiang: Zhejiang Education Publishing House.[In Chinese].

Fan, L. (2010). *Mathematics (9th (semester 2))*. Zhejiang: Zhejiang Education Publishing House.[In Chinese].

Fischbein, E., & Gazit, A. (1984). Does the teaching of probability improve probabilistic intuitions? *Educational Studies in Mathematics*, *7*, 15-19.

Gal, I. (2005). Towards probability literacy for all citizens: Building blocks and instructional dilemmas. In G. A. Jones (Ed.), *Exploring probability in school: Challenges for teaching and learning* (pp. 39-63). New York: Springer.

Gao, H. (2011). *The understanding of probability concepts in 6-12 year old pupils*. Master's thesis.f Hangzhou Normal University, Hangzhou.[In Chinese].

Gong, Z. & Song, N. (2006). Teaching and learning of statistics and probability: reflections and suggestions. *People's Education*, *10*, 1-6. [In Chinese].

Jones, G. A., Langrall, C. W., Thornton, C. A., & Mogill, A. T. (1999). Students' probabilistic thinking in instruction. *Journal for Research in Mathematics Education*, *30*, 487-519.

Jones, G. A., Langrall, C. W., Thornton, C. A., & Mogill, A. T. (1997). A framework for assessing and nurturing young children's thinking in probability. *Educational Studies in Mathematics, 32*, 101-125.

Jones, G. A., Langrall, C. W., & Mooney, E. S. (2007). Research in probability: Responding to classroom realities. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 909-955). Reston: The National Council of Teachers of Mathematics.

Lagnado D. A. & Shanks D. R. (2003). The influence of hierarchy on probability judgment. *Cognition*, *89,*157-178.

Li, J. (2002). Developmental structure in understanding of probability, *Journal of Mathematics Education*, *4*, 1-5.[In Chinese].

Li, J. (2003). *Learning and teaching of probability in primary and secondary schools*. Shanghai: East China Normal University Press.[In Chinese].

Li, W., Zhang, H., & Shoo, H. (2014). *Quantitative research methods and statistic analysis in education and psychology*. Beijing, Beijing Normal University Press. [In Chinese].

Lu, J., & Yang, G. (2009). *Mathematics (3th (semester 1))*.. Beijing, People's Education Press. [In Chinese].

Lu, J., & Yang, G. (2010a). *Mathematics (5th (semester 1))*. Beijing, People's Education Press. [In Chinese].

Lu, J., & Yang, G. (2010b). *Mathematics (6th (semester 2))*. Beijing, People's Education Press. [In Chinese].

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM.

Nikiforidou, Z. & Pange, J. (2010). The notions of chance and probabilities in preschoolers. *Early Childhood Education Journal*, *38,* 305-311.

Nilsson, P., & Li, J. (2015). Teaching and learning of probability. In. S. J. Cho (Ed.). *The Proceedings of the 12th International Congress on Mathematical Education: Intellectual and Attitudinal Challenges* (pp. 437-442). New York: Springer.

Piaget, J. & Inhelder, B. (1975). *The origin of the idea of chance in children*. New York: Norton.

Sharma, S. (2012). Cultural influences in probabilistic thinking. *Journal of Mathematics Research, 4* (5), 63-77.

Shen, J., & Liu, F. (1984). Study on the cognition of volume for students aged 5-17. *Acta Psychologica Sinica*, 155-164. [In Chinese].

Wang, H. (2012). *Study on elementary teachers' probability knowledge*. Master's thesis Hangzhou Normal University, Hangzhou.[In Chinese].

Watson, J. D. & Moritz, J. B. (2003a). Fairness of dice: A longitudinal study of students' beliefs and strategies for making judgments. *Journal for Research in Mathematics Education*, *34*, 270-304.

Watson, J. D. & Moritz, J. B. (2003b). The development of comprehension of chance language: evaluation and interpretation. *School Science and Mathematics*, *103,* 65-80.

Watson, J. M., Collis, K. F., & Moritz, J. B. (1997). The development of chance measurement. *Mathematics Education Research Journal*, *9,* 60-82.

Yan, B. (2007). *A study of questions of statistics and probability in teaching in elementary mathematics*. Master's Thesis.Southwest University, Chongqing. [In Chinese].

Zhang, Z., Liu, F., Zhao, S., Sun, C., & Zuo, M. (1985). A study of the development of 5 to 15-year-old' concept of probability. *Acta Psychologica Sinica*, 1-6. [In Chinese].

Zhang, Z., Liu, Z., & Qiu, M. (1983). On the budding and developing of the cognitive structure for the notion of probability in children of 5-11. *Journal of the southwest teachers college*, *2,* 29-42. [In Chinese].

Referencias a los autores

Zikun Gong, Hangzhou Normal University (China), zkgong@163.com

Shengqing He, Beijing Normal University (China), heshengqing99@163.com

Developmental stages and important periods of probability cognition in 6 to 14 year –old students

Zikun Gong, Hangzhou Normal University (China)

Shengqing He, Beijing Normal University (China)

Probability literacy is one of the core mathematics literacy goals for current citizens. To cultivating probability literacy, the Chinese Mathematics Curriculum started, developed and adjusted probability related content in the past years. However, curriculum reforms in the field of probability faced the dilemmas that the students failed to adapt to probability thinking and teachers encountered various difficulties in teaching. These facts led mathematics education researchers to consider the theoretical and practical basis of curriculum revision, since curriculum design should be based on students' cognition.

In the present research, we basically focused on two issues: cognitive development stages and important periods of students' probability cognition. Specifically, we proposed the following questions: what are the cutting stages and the according levels? What concepts students develop in every stage? What are the important periods of development of probability cognition?

We designed a questionnaire which involves four sets of questions, each of them containing 5 hierarchical cognitive tasks which refer to randomness, probability distribution, qualitative comparison, number representation and fraction representation. 906 6 to 14 year-old students from 27 groups were chosen as the study subjects, and the sample was stratified sample from urban, suburban and rural areas. To eliminate the impact of learning on students' probability cognitive development, 67 9 years old and 66 11 years old students which have not learnt probability related knowledge were chosen from other schools as the control groups.

The results show that there were significant differences between different age groups in terms of probability cognition. Accordingly, we concluded 5 developmental stages in students' probability cognition. *Stage 1 (6≤Age≤7): Slow development stage I.* Students in this stage understood basic ideas of randomness and acquired preliminary knowledge of qualitative comparison, number representation and probability distribution. *Stage 2 (7<Age≤9): Quick development stage I.* Students in this stage obtained great development in probability quantification and preliminarily developed fraction representation. They mastered basic ideas of randomness and understood probability distribution, qualitative comparison and number representation. *Stage 3 (9<Age≤10): Slow development stage II.* Students in this stage mastered randomness, understand qualitative comparison, number representation, probability distribution and fraction representation. *Stage 4 (10<Age≤12): Quick development stage II.* Students in this stage mastered randomness and qualitative comparison, and understood number representation, probability distribution and fraction representation. Specifically, there

was no further development in the cognition of probability distribution and fraction representation. *Stage 5 (12 < Age ≤ 14): Stagnant stage.* Students in this stage experienced stagnant and even negative development.

The results also reported two important periods of probability cognition. We chose the control groups (9 and 11 years old students from S region of Hangzhou city) to analysing the impact of learning to probability cognition. For 9 years old students, there was no significant difference between experimental group and control group in terms of probability cognition. Apart from randomness and fraction representation, the control group scored even higher than the experimental group in other tasks. For 11 years old students, there was also no significant difference between experimental group and control group in terms of probability cognition. We concluded that the preliminary knowledge did not affect students' probability cognition significantly. That is to say, 9 and 11 were two important developmental periods in which the development of probability cognition was led by students' general cognition development.

We suggested that, reasonable teaching objectives should be organized by stage, and cognitive oriented teaching interventions should be implemented.