A CASE STUDY OF THEORY-INFORMED TASK DESIGN: WHAT MIGHT WE, AS DESIGNERS, LEARN?

Wake, G.[a]

[a]University of Nottingham

Abstract

Task design plays a very important role in the day-to-day classroom experiences of both students and teacher: it comes to define their fundamental understanding not only of mathematical concepts but also their experience of mathematics as a practice. It is essential, therefore, that those of us who design tasks for others to use, often deliberately innovative in nature, consider carefully how our work might best be informed by underpinning theory. That is not to imply that there is one best approach to task design but rather here I provide insight into how theoretical considerations can provide insight into our activity as task designers and further to this how such considerations might, through our reflections, help inform our future work. This argument is illustrated with reference to a recent design research informed development of curriculum resources and an accompanying programme of teacher learning.

Keywords: *Design research, Communities of practice, Boundary objects, Dialogic learning, Professional learning.*

Resumen

El diseño de tareas juega un papel muy importante en las experiencias diarias en el aula, tanto para el alumnado como para el profesorado, llegando a definir la comprensión fundamental no sólo de los conceptos matemáticos, sino también de su experiencia de las matemáticas como práctica. Por consiguiente, es esencial que todos los que diseñamos tareas para que otros las usen, a menudo de naturaleza deliberadamente innovadora, consideremos cuidadosamente cómo nuestro trabajo podría estar informado por una teoría subyacente. Esto no implica que exista una mejor aproximación al diseño de tareas, sino más bien se pretende proporcionar un mejor conocimiento sobre cómo consideraciones de tipo teórico pueden ofrecer una mejor compresión de nuestra actividad como diseñadores y, más allá, entender cómo estas consideraciones podrían informar nuestro trabajo en el futuro, a partir de nuestra propia reflexión. Este argumento se ilustra con referencias a un reciente diseño de recursos curriculares y de un programa de aprendizaje para el profesorado que los acompaña, basados en la investigación.

Palabras clave: *investigación basada en el diseño, comunidades de práctica, objetos frontera, aprendizaje dialógico, aprendizaje profesional.*

INTRODUCTION

As designers of educational resources for mathematics I wish here to consider what we might learn from our consideration of theory.

First, I wish to situate our design work firmly in the paradigm of educational design research: that is, it is committed "to developing theoretical insights and practical solutions simultaneously, in real world (as opposed to laboratory) contexts, together with stakeholders" (McKenney y Reeves, 2012). That is, the focus of our work is on engineering research-informed solutions for practical use in education – primarily in the classroom. The work of our group in Nottingham, that is the work of the Shell Centre, situated in the Centre for Research in Mathematics Education has, for over fifty years sought to provide practical solutions for teachers in their day-to-day work in classrooms to support young people in learning mathematics (Burkhardt y Swan, 2017). Over this time, many

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resources have been produced all of which have been research-informed, drawing on prior underpinning knowledge drawn from the research literature appropriate to the context of the mathematics. In addition, our group has undertaken its own design research as indicated above, seeking to develop both theory and practical solutions. This primarily involves iterative cycles of inquiry led by the design team, involving multiple test sites in close collaboration with teachers providing rich data / feedback from their classroom trials.

This process has been important throughout our work and will to some extent be exemplified in the illustrative examples described here. I also want to draw attention to some theoretical considerations that I have found of increasing importance in informing the development of our work.

SITUATING DESIGN RESEARCH

Theories provide models or schema for understanding the nature and causes of (observable) phenomena: they help us make sense of the world. Importantly, they are developed over time, often being expanded, refined, modified as our understanding develops, is further informed by new insights, and so on. I would like to call upon a schema that will help us situate design research in education in the complex world that is that of research more generally. Figure 1 was first used by Stokes (Stokes, 1997) to consider the roles that different forms of research (primarily in the sciences) can be considered to play. This provides a two-dimensional matrix: the horizontal axis considers the use associated with the research (whether research has as its focus application as central or otherwise); the vertical axis considers how the research is situated with respect to its expectation of informing fundamental understanding.

		Consideration of use	
		No	Yes
Quest for fundamental understanding?	Yes	Pure basic research (Bohr)	Use-inspired basic research (Pasteur)
	No		Pure applied research (Edison)

Figure 1. Pasteur's quadrant as proposed by Stokes (1997)

Thus, as Stokes intimated:

- the research of Bohr that sought to understand atomic structure sits in the top left quadrant, i.e. it was pure basic research seeking to provide fundamental understanding by developing a model of atoms as the building blocks of matter;
- the research of Edison, that sought to inform how we might have generally available and commercially successful electric lighting, on the other hand, focused on applied goals without seeking to add to more general understanding of the phenomena of electricity as a scientific field, and consequently sits in the bottom right quadrant;
- the top right quadrant is where research sits that seeks to add to our fundamental understanding but also has as central considerations of applications. This quadrant has become known as Pasteur's quadrant after Pasteur who in his work both sought to understand the micro-biological processes he investigated as well as harness them to improve the health of humans and animals.

- The bottom left quadrant provides space for research that neither seeks to add to our general understanding nor has a focus on immediate application. Maybe a more recent type of activity that might be considered to sit within this quadrant is that of data mining, where data scientists explore large data sets in the hope of maybe being able to contribute to either or both of knowledge and/or application.

Research that informs the design of educational resources, often considered as design research (see for example, McKenney y Reeves, 2012), is ideally, focused on the interaction of students, teachers and designed resources. This can prove very effective in the development of a better product set for use in learning settings, often classrooms. Here I will illustrate a case of such detailed design research that we have undertaken as part of a project run by our research and design group in the UK. However, before doing so I wish to further situate this particular work referring to some theoretical ideas that we use to inform our design at tactical and in detail at technical levels. As an aside, at this point, I raise an issue that a colleague Burkhardt (2009) draws to our attention. He helpfully identifies three major levels of educational design—strategic, tactical and technical—with the first, strategical design, being "concerned with the overall structure of the product set and how it will relate to the user-system". Tactical design is at the level of the internal structure of the product: that is, it is focused on specifying the core design principles of the product, the different aspects and how these are structured in a way that will ensure they will bring about the desired change they are designed to effect. The technical level of design is that which focuses on the development of individual, often material resources of the product: for example, the classroom materials such as tasks to be used with students, elements of the professional development programme, lesson plans and so on. It is our work at this latter level with which we talk directly to teachers and students, clearly an important aspect of our work as designers.

THE ARCHITECTURE OF TACTICAL DESIGN

Here I wish to initially focus at the tactical design level and consider how theoretical considerations can inform such design. Fundamental to such work, in our experience, is having an underpinning theory of learning that helps us define our whole approach as to the experiences of learning we would hope result from our carefully engineered products.

Our recent work, draws on Wenger's theory of learning that considers learning to be fundamentally experienced socially and involves the learner being engaged in practice, identity development, making meaning, and becoming a participant in a community of practice. Figure 2, provides a schematic overview of this and is reproduced from Wenger's seminal work on *communities of practice* (Wenger, 1988) in which he considers human behaviour as being social with individuals as members of multiple communities of practice. Foremost in *our* work as designers for teachers working in classrooms and in their interactions with colleagues is to design artefacts that support both student and teacher learning. Fundamental in this regard is our understanding of "teachers as learners", that is, in Wenger's terms developing their community of practice with individuals and the community as a whole involved in developing their practice in ways that support development of each of practice, identity, meaning and community.

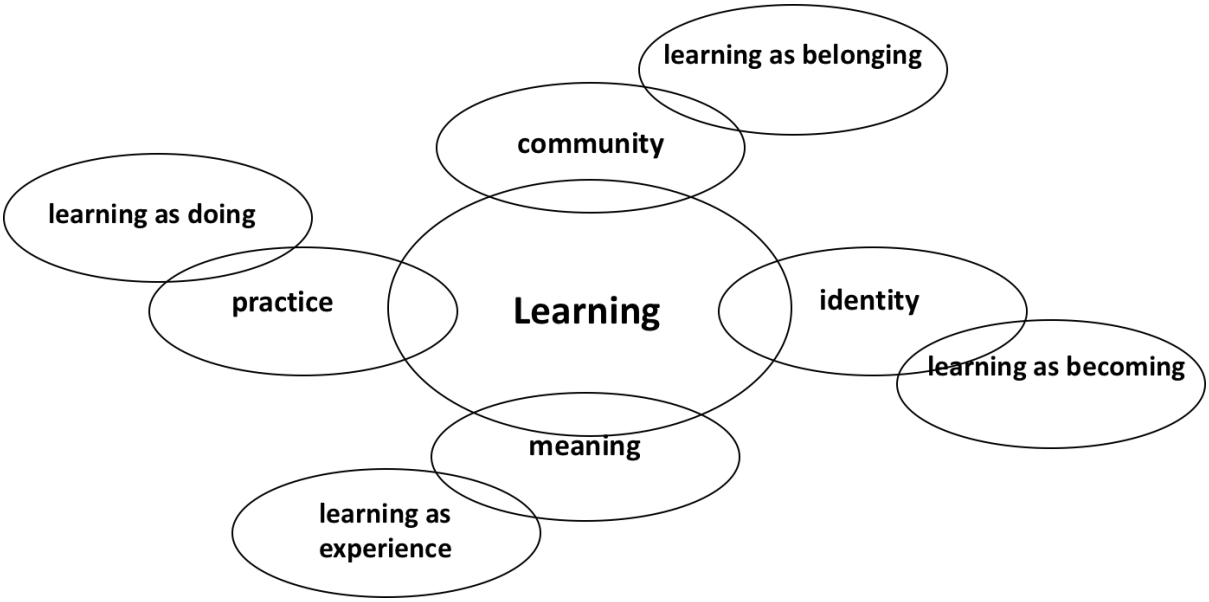

Figure 2. Wenger's components of a social theory of learning (Wenger, 1998. p.5).

What are the implications for us as designers? This is a question that Wenger raises and addresses by pointing to how architects and computer software developers, amongst others as designers, frame their designs by reference to conceptual architectures that capture relevant and appropriate aspects of human behaviours that are mediated by physiological and cultural factors. He goes on to argue that in design for learning we need to consider four dimensions and that these present four dualities that challenge the efficacy and effectiveness of our designs.

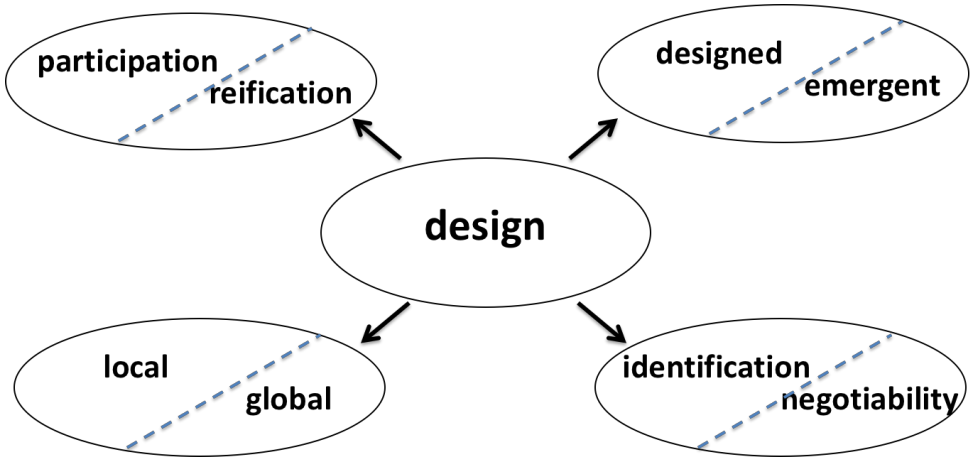

Figure 3. Wenger's four dimensions of design for learning (Wenger, 1988. p. 232)

First, he considers that in developing their *practice* teachers, at various stages of their induction into, and participation in, teaching negotiate their *meaning* of this professional practice. Wenger unpacks his use of the term *meaning* at some length considering its fundamental importance in his view to our participation in communities of practice. In essence, he sees this constantly ongoing and evolving process as involving the duality of participation and reification, with the latter term being used to capture how aspects of our activity are captured in the artefacts that embody the history and culture of the practices at the heart of the community of practice. Classroom tasks are examples of just such artefacts and help come to define, for users, day-to-day teaching and learning practices and importantly how members of the community of practice, including students, make meaning of the very nature of what it means to learn and do mathematics. Consequently, it behoves us as designers of tasks to consider what we reify in our tasks and what our designs for participation reveal in relation to their negotiation of *meaning* of what they engage with.

The second duality we need to consider is that of what is designed and what is left to be emergent: more practically in our design we need to consider what is fundamental to our design and that we desire to be central to the practice we wish to support and how we ensure that the emergent practice is 'designed for' in relation to such essential features. As Wenger, succinctly puts it: "practice is not the result of design but rather a response to it." (Wenger, 1988. p.233).

The third duality is that of the local and the global. This highlights how in our design we need to be aware that each community of practice will need to make the intended practice its own, that is, in adopting the *global* aspects of the design they will *localise* aspects that ensure their practice develops in ways that are compatible with their usual social practice whilst having fidelity to the main design intentions. Again, careful design will be sensitive to such issues.

Finally, Wenger draws attention to the duality of identification and negotiability: how our design can prompt/facilitate identification or non-identification with the proposed participatory activity. This draws our attention to how as individuals, and a community, we have the agency to negotiate and shape the meaning associated with the activity of the group's social enterprise.

Perhaps we might summarise, even if perhaps a little simplistically, as educational designers we need to develop a tactical design that is constructed around our core principles but also serves to facilitate much scope for adaptability to the different ecologies of learning in which our products will be used.

EXAMPLES OF AN ARCHITECTURE OF TACTICAL DESIGN

Change in teacher practice is often fundamental to our design of new products for classroom use. This we know, from experience, is unlikely to be facilitated by the production of classroom materials on their own and we have, particularly more recently, developed professional development (Swan et al., 2013) programmes alongside our materials (see for example, http://www.bowlandmaths.org.uk/pd/). This professional development is ideally informed by the design research programme that accompanies the development of the classroom materials.

We know from a synthesis of research that addresses issues of teacher professional development that it is effective when

- Experiential: stimulating and drawing on teachers' experiences.
- Sustained: involving cycles of planning, predicting, enactment & reflection.
- Grounded: involving practical, and well-resourced, experiences; related to context & culture.
- Safe: ensuring teachers are able to speak their minds, permission to take risks.
- Collaborative: involving networks of teachers & administrators.
- Informed: by outside expertise and research.
- Provocative: involving both pressure and support.
- Focused: attentive to the development of the mathematics itself.

(for example, Guskey, 2002; Joubert and Sutherland, 2009; Villegas-Reimers, 2003; and many others…)

It is our further experience that professional development that is designed in line with principles of lesson study as practised in Japan meets all of the above criteria and we have increasingly used such programmes as part of our work; most significantly in the funded Lessons for Mathematical Problem Solving (LeMaPS: https://www.nottingham.ac.uk/research/groups/crme/projects/lemaps/index.aspx) project funded by

the Nuffield Foundation in the UK. Our analysis of such programmes points to boundary objects as being most significant in helping to support learning that transcends the different communities of classroom and teacher research group in which teachers operate. Lesson Study as a boundary crossing experience (Wake et al., 2016) is central to our design at a tactical level. This conceptualises the learning architecture associated with our learning materials design as supporting developing expertise in the two settings of classroom and teacher research group. It is this architecture that is at the focus of our design.

I now illustrate this in the particular case of the project Maths-for-Life which has just completed a development phase in the UK in which design research has informed both the tactical aspects of the programme and the technical level design of boundary objects that facilitate practice across the communities involved. There are two aspects of the architecture of tactical design that here I characterise as being (i) *structural* (concerned with the pragmatics of time, place, organisational structures and dynamics of social interactions) and (ii) *conceptual* (the frameworks, schemas and theories that inform intellectual engagement).

Central to the structural aspects of our design of Maths-for -Life is the development of teacher inquiry communities/groups led by a "Lead Teacher" who had been part of the design research phase. These Lead Teachers had experienced working with drafts of materials as well as participating in exploratory inquiry research groups working with two or three others from a total cadre of 20 lead teachers.

The Maths-for-Life project professional development programme is designed to improve student examination re-sit grades in mathematics. The students involved follow a "resit" course for one year, post-16, in attempt to improve outcomes in the mathematics examination taken at the end of compulsory schooling (GCSE). The majority of these students attend post-16 "Further Education" colleges and their course in mathematics is taken alongside a range of other academic, and often vocational, studies. The context is such that this is an intensive experience often running for only seven or eight months from September/October through to April/May in the following year. Consequently, and pragmatically, the Maths-for-Life programme focuses on just five lessons spread throughout this period: teacher's collaboratively work with the Lead Teacher on considering the "research lesson" in some considerable detail before teaching it themselves, often to multiple classes of students, and collaborating in small inquiry groups, to observe one of them teach the lesson and in a post-lesson discussion focus on a carefully framed "research question". The five lessons are timetabled throughout the period so that the professional development has opportunities for cycles of planning, predicting, enactment & reflection.

The conceptual framework is summarised by the pentagons of Figure 3. These illustrate (a) the five principles of dialogic learning that we are seeking to develop in classrooms defining the behaviours that we expect teachers, teaching assistants and students to develop over the course of the programme (b) the five key aspects of pedagogy that we consider underpin such learning, and (c) mathematical content associated with the five lessons.

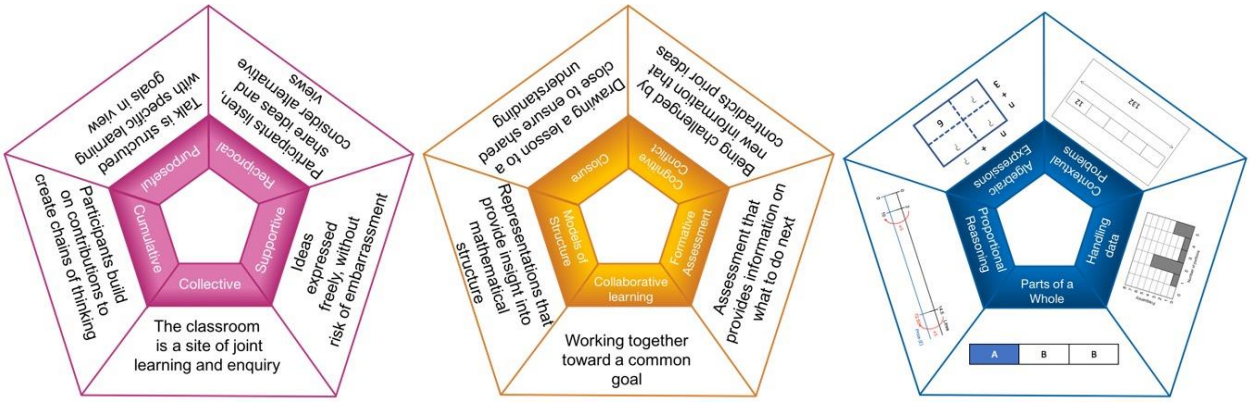

Figure 3. The conceptualisation of dialogic learning, pedagogies and mathematical content central to the Maths-for-Life programme.

The project builds on the earlier work of Swan who, working at a smaller scale with students in a similar context, designed classroom materials that facilitated dialogic talk in their classrooms (Swan, 2006).

The dialogic classroom encourages talk that Alexander (2006) and Mercer (1995, 2000) identify as being:

- Collective - teachers and children address learning tasks together, as a group or as a class, rather than in isolation
- Reciprocal - teachers and children listen to each other, share ideas and consider alternative viewpoints
- Cumulative - teachers and children build on their own and each others' ideas and chain them into coherent lines of thinking and enquiry
- Supportive - children articulate their ideas freely, without fear of embarrassment over 'wrong' answers and they help each other to reach common understandings
- Purposeful - teachers plan and facilitate dialogic teaching with particular educational goals in view

Such practices, as suggested earlier, and will be illustrated later, have to be designed for at a technical level: likewise, the associated pedagogies:

- Collaborative learning: where teachers and students work jointly towards a common goal;
- Models of structure are central: with representations that provide insight into these models;
- Cognitive conflict is prompted: materials provide challenge to students' thinking if they, as many do, hold conceptual understanding at odds with 'scientific understanding' (Vygotsky, 1986);
- Formative assessment practices (Black y Wiliam, 2009) are used to provide teachers with insight into students' thinking throughout lessons
- Closure: lessons are carefully brought to a close in ways that understanding is shared and further opportunities provided for understandings to be clarified.

These five key aspects of dialogic classrooms and five signature pedagogies are designed to be part of all five lessons but the professional development programme has been designed to focus on pairs of these in turn in one of five individual lessons focused on topics that have significant importance in the examinations. These are:

- Parts of a whole (ratios and fractions);
- Proportional reasoning;
- Algebraic expressions;
- Contextual problems;
- Handling data.

We bring together these three key features of our conceptual design of the professional development in ways that permeate the two different communities that teachers engage in as part of the programme: the teacher inquiry group meetings and the "lesson study" process. Because of pressures of time we allocate only half a day to the introduction of each lesson led by a Lead Teacher and in which the five maths topics, pedagogies and aspects of dialogic learning are discussed facilitated by a number of materials both written and video (developed during the design research process of the first year of the project). A further half day is dedicated to the "lesson study" which is facilitated by a "research question" designed to focus observations of the lesson which is taught by one of the teachers, and reflected upon in a discussion immediately following the lesson. Table 1 summarises key aspects of the conceptual design across the five lessons at central to the structural design of the Maths-for-Life project together with the research questions associated with each.

The modified lesson study structure follows, to as great an extent as possible, the key principles of lesson study (Wake et al, 2014) distilled from the work of our LeMaPS project (da Ponte and Wake, forthcoming). Namely the teacher inquiry group incorporated:

1. A **research focus**, that informs the 'bigger picture', that is the overall context of the endeavour. In the Maths-for-Life project this research focus was explicit for each lesson (see table 1) but also primarily the overall focus for all lessons is dialogic learning.
2. A **detailed lesson plan** that anticipates how students will respond to the task and how the teacher might respond.
3. The **research lesson** being taught by one of the team with the lesson being observed carefully by all members of the lesson study group.
4. The **post-lesson discussion** involving the teacher and all observers in analysis of the lesson with an outside expert (the Lead Teacher).

The two aspects of the architectural design of the Maths-for-Life programme, the structural and the conceptual, are made material in the artefacts that we design as *boundary objects*, designed and produced to stimulate both student and teacher learning in the sense of Wenger (1988) as explicated above.

THE DESIGN OF BOUNDARY OBJECTS

Fundamental to the design of our professional development programme, as I signal above, are the two communities of the teacher inquiry group and the classroom. The deliberate design of these two communities results in a distinct a boundary. This we consider is important in teacher learning and requires facilitating by carefully designed boundary objects, or what we might consider as boundary artefacts. A boundary object in the terms used by Star and Griesemer (1989) is made 'material' as a single device that has different meanings in two or more different communities, while retaining a common essence in each. In their consideration of the importance of boundaries in learning Akkerman and Bakker (2011) note that Star and Griesemer point to how "such artefacts are seen as potential 'bridges' or 'anchors' across different 'intersecting social worlds as they allow

cooperation and communication across sites." (Star y Griesemer, 1989). It is important to note that such devices may include easily discernible material artifacts such as classroom materials through to what might seem less tangible or more abstract devices such as the five signature pedagogies.

Important in the design of such artefacts is that they are effective in facilitating the desired boundary work that is envisaged as part of the learning process: and to ensure that this is feasible the design must be cognisant of the detailed activity that will occur in each community. Drijvers and Trouche (2008) in their contribution to the development of the theory of instrumental orchestration as elaborated by Drijvers et al. (2010) make the distinction between an artefact as having only the potential to support actions, whereas it becoming instrumental in its use when the user has a mental scheme that supports both technical and conceptual abilities to realise this potential in a specific situation. An example I often use to exemplify this idea is that of a graphing calculator. As an artefact it can be considered as a material object incorporating graphing facilities that has the potential to be used, for example, to plot a function and give the area enclosed by this, certain limits, and the x-axis, or its gradient at a specific point. For this to become instrumental in use in a particular situation, the user must understand both the potential and the appropriateness of doing so and have the technical expertise and conceptual understanding to do this.

The designer's role, then, is to design such artefacts, perhaps not as complex in potential and technical sophistication as graphing calculators, but the designer does require a detailed understanding of the contexts in which the artefact is required to be instrumentally used. The artefacts are designed so that, in Drijver et al.'s terms it becomes an instrument that both shapes the thinking of the user, in this case the teacher, (the instrumentation process) and is in itself shaped by the user (the instrumentalisation process), see Figure 4, below. The dual participatory communities in which we involve our teachers, classroom and teacher inquiry group, provide multiple opportunities for them to engage in such practices in relation to the artefacts we organise in line with our conceptual design.

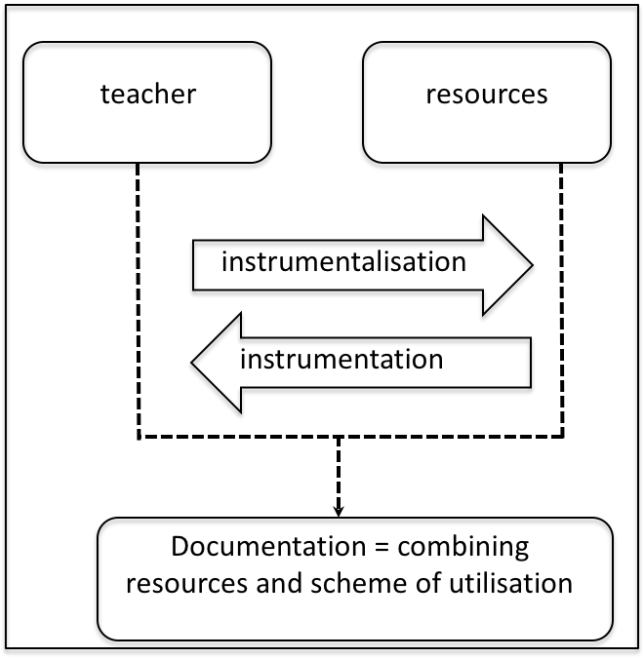

Figure 4. The documentation process (Drijver's et al., 201).

CLASSROOM TASKS AS BOUNDARY OBJECTS IN TEACHING LEARNING

It is this particular approach that I now wish to elaborate in the design of our classroom materials, with additional reference as to how these particular artefacts articulate with our work with teachers in the professional development programme. Here I draw on just one artefact central to the first lesson in our programme. This lesson focuses on the mathematical topic of "parts of a whole". Figure 5, brings together in a single schema, the important features of our conceptual design elements: maths content, pedagogy and feature of dialogic learning, as addressed by this particular lesson.

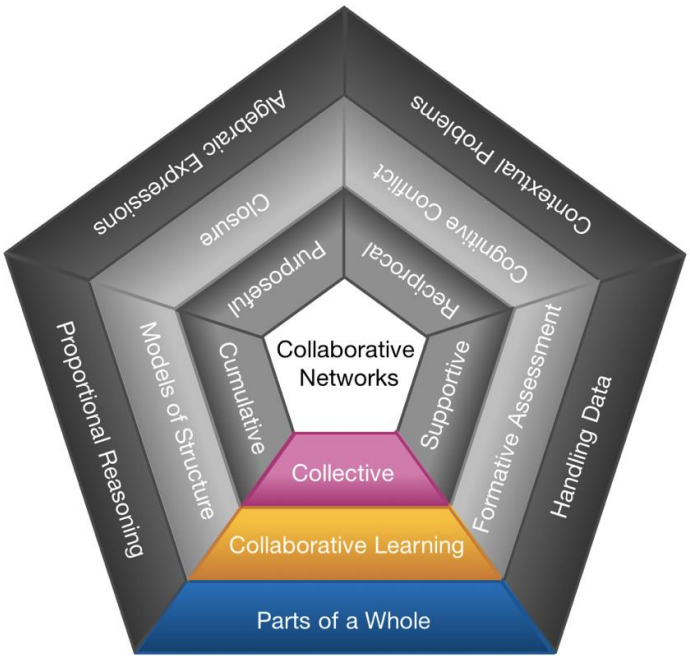

Figure 5. The conceptualisation of dialogic learning, pedagogies and mathematical content central to the Maths-for-Life programme.

As elaborated below (Table 1), the lesson focuses on *collaborative learning* with students working together in both pairs or small groups, and as a whole class, toward a common goal in a *collective classroom* in which both the students and the teacher see their lessons as being based around joint learning and enquiry. To facilitate reflection and boundary work the research question for this lesson is: "*How does collaborative learning (through the design of resources and the actions of the teacher) promote collective endeavour?"*

Table 1. The five Maths-for-Life lessons

	Topic	**Pedagogy**	**Dialogic learning**	**Research question**
1	Parts of a whole	Collaborative learning	Collective	How does collaborative learning (through the design of resources and the actions of the teacher) promote collective endeavour?"
2	Proportional reason	Models of structure	Cumulative	How do models of structure help to facilitate cumulative dialogue and insight into mathematical structure?
3	Algebraic expressions	Closure	Purposeful	How does purposeful dialogue contribute to student understanding during the

				closure phase of the lesson?
4	Contextual problems	Cognitive conflict	Reciprocal	How can cognitive conflict provide the opportunity to develop reciprocal dialogue?
5	Handling data	Formative assessment	Supportive	How does the use of formative assessment help to develop an environment of **supportive** dialogic learning?

The main task of the lesson was very much informed by the design research phase of the project. Initially it was a minor redesign of resources available as part of resulted in the 'Standards Unit Box' (DfES, 2005) resources. In its initial design it consisted of a card matching activity which provides two sets of cards that indicate how money earned by two students, Ali and Blair, might be divided expressed as either a fraction or a ratio. As Figure 6a illustrates most students (working in pairs) have the misconception that a ratio of 1:2 corresponds to the fraction 1/2, when faced with the initial design of the task. Having allowed students' understanding to have been expressed in this initial card matching activity the teacher is then asked to introduce the third set of cards which they are asked to match to the cards already paired up so that all three cards are consistent in being representative of the same splitting of the money. Figure 6b shows a typical pair of students' work at this point.

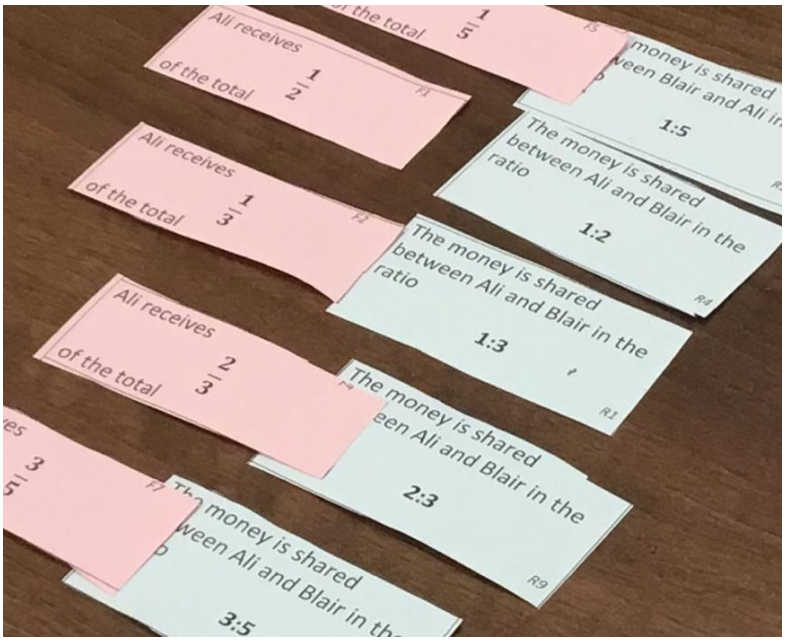

Figure 6a. Materials for lesson 1 of the Maths-for-Life programme.

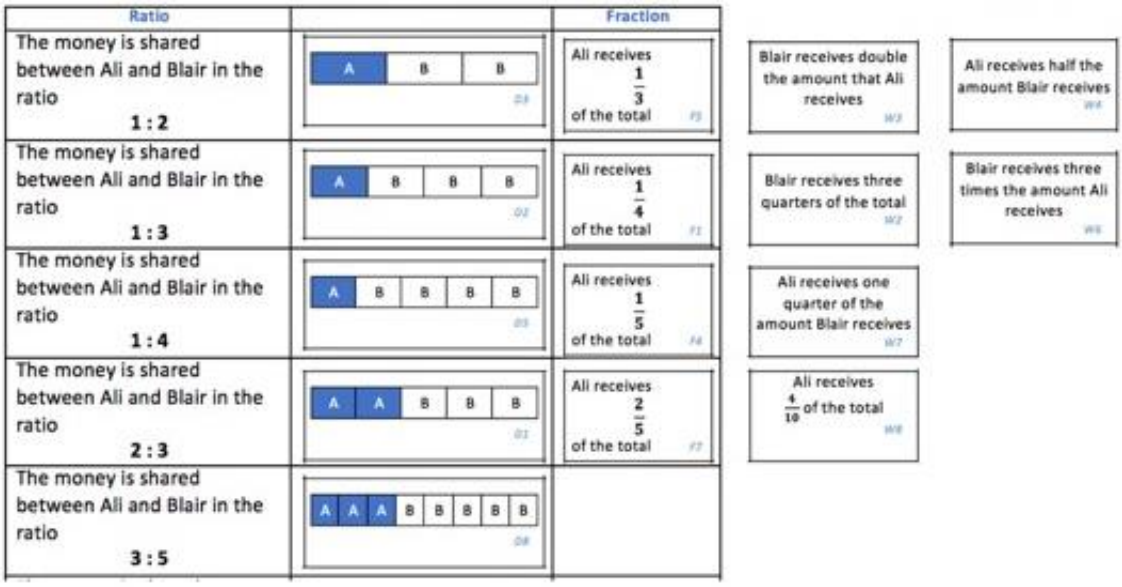

Figure 6b. Redesign of the materials for lesson 1 of the Maths-for-Life programme.

The design research process identified that as it stood students often worked individually in matching the cards with little effort to come to collective/shared decisions. Bringing about a change in the socio-mathematical norms (Yackel and Cobb, 1996) or the didactique contract (Brousseau, 1997) of the classroom is something that needs a more significant design input than this card-matching activity as it stood was able to bring about. The lesson re-design is apparent in Figure 5b. Important changes include:

i. Students are provided with a template structure that can be used in their student-to-student discussion which the teacher initiates around expected behaviours, for example, to signal when justification of thinking might be expected.

ii. The use of a template helps the teacher more quickly identify student thinking, thus supporting formative assessment.

iii. The positioning of representations on the template encourages resolution of the task in its totality.

iv. The planned gradual hand out of cards ensures that the task isn't overwhelming in the first instant.

The design of the task for the 'parts of a whole' lesson in its current form, consequently, embodies and encapsulates the key aspects of the conceptual design of the programme. In a little detail:

Maths topic. The topic that is central to the task here is that of 'parts of a whole', perhaps more often referred to as fractions. The term 'parts of a whole' is used here so that teacher discussion can focus around conceptual understanding of notation/representation of part to part and part to whole. As illustrated above this is central to the first task of the lesson.

Pedagogy. This being the first lesson of the programme the pedagogy chosen for discussion is that of collaborative learning, in the sense of encouraging joint endeavour with a shared understanding of what individuals, pairs/groups and the class as a whole aim to achieve.

Aspect of dialogic learning. The aim is to encourage collective work with students and teacher focusing their collaborative enquiry/talk around their shared and collaborative work.

The task as a boundary artefact has been designed to best facilitate these particular important features of the conceptual design whilst also paying attention to all other important aspects of pedagogy and aspects of dialogic learning.

For example, the 'bar model' representation that provides insight into issues surrounding 'part-to-part' and 'part-to-whole' provides a representation of mathematical structure that has potential uses beyond that illustrated here. This and similar representations are explored in further lessons in the programme (for example, in the lesson focussed on contextual problems). Not only does it aim to provide such insight, but also in its use here it is designed to provoke cognitive conflict or dissonance (Limon, 2001). That is, the representation provides a new/alternative insight which cannot be matched successfully to ensure consistency across the row when the students have already aligned representations that are not compatible in terms of their representation of both 'part-to-part' and part-to-whole. Discussion of how students resolved the discrepancy in their initial matching can form part of meaningful discussion of the whole group at closure of the lesson.

DISCUSSION AND CONCLUSION

As the single case above illustrates, design of innovative classroom materials that seek to support and develop teachers' practices, in ways that might improve student learning, is a complex endeavour. Perhaps, much of what has been achieved in the Maths-for-Life development so far might have been achieved without reference to any theoretical underpinning. Maybe such considerations could perhaps have been left almost entirely unspoken, and with potentially little loss to the eventual outcomes in terms of its "products". What, then, might we as educational designers learn from this theory-informed approach? The schematic overview in Figure 7 attempts to help clarify / organise the theoretical underpinnings referred to throughout the article as they apply to the Maths-for-Life project in particular, but also to much of our work in general.

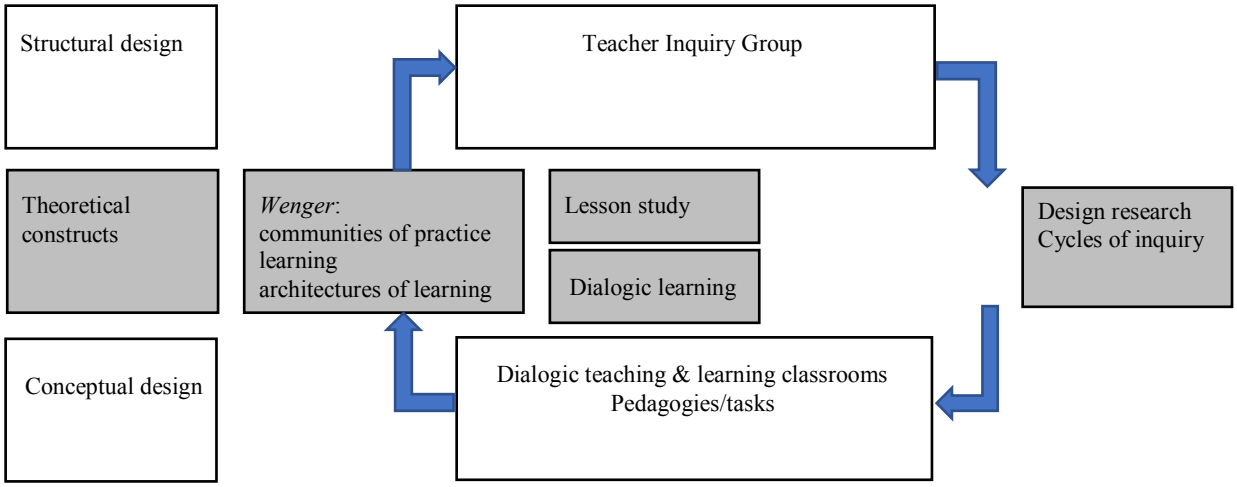

Figure 7. Schematic overview showing the articulation of theory with practical design considerations in the maths-for-Life programme.

Figure 7 highlights the thread of theoretical constructs that are brought together to inform design of the Maths-for-Life intervention that seeks to improve student learning in mathematics for students who need to improve their grade in the end-of-compulsory school examinations in England.

Fundamental to our model of change is recognition of the need to involve the teachers of these students in a process of teacher learning that will support them with the immediacy of day-to-day lessons and also provide them with new modes of working that might support the expected changes to become much more deeply embedded in their practice. Consequently, we draw on Wenger's theoretical stance in relation to *communities of practice* and in particular draw on how such communities support learning, that in taking a social view, considers how this involves much more

than "knowing that". Wenger's view fundamentally considers how learning engages the individual and the community of which they become part in a symbiotic relationship in which they develop together with each adapting in their accommodation of the other. Helpfully, Wenger points to how as designers our designs might be informed by the metaphor of 'architectures of learning' which are sensitive to the aspects of learning that he identifies as involving practice, identity development, making meaning and community participation. This expanded notion of learning provides constructs that we find helpful in considering both structural and conceptual aspects of our design of a new community of practice, that of the teacher inquiry group, and informing how this needs to articulate with the day-to-day, bread and butter, work of the teacher that focuses on day-to-day student learning.

These may appear at first as rather abstract constructs but in the project they can be seen made real in practical aspects of design of the structure such as in how we ensure cycles of enquiry that are spread throughout the time allotted to the project and how these are scheduled so that a sense of community is engendered from the outset and supported by a lead teacher who has access to a set of materials designed to facilitate meaningful professional discussions. Further, our support materials include video sequences, developed during the first 'pilot' year in which the materials underwent a design research cycle of improvement. These can be used to signal expected behaviours of the teacher inquiry groups (for example, in their new practice of post-lesson discussion).

As indicated above the new communities of teacher inquiry groups have been informed by Japanese Lesson Study practices and what we have learned through our prior work in this field about how this approach might be effectively adopted in the UK context.

Our model of classroom teaching and learning is informed by the work of Mercer (1995, 2000) and Alexander (2006) who researched student-student and teacher-student talk in classrooms and in line with our philosophy of developing learning as a social activity provide insights into how we might design for classrooms that support their dialogic approach. In seeking to develop and support the different aspects of such classroom talk we consider carefully how our design of tasks might facilitate such outcomes whilst also supporting the aspects of teacher learning and the new communities of teacher inquiry groups we wish to establish and have just outlined. It is the design of these tasks that are central to the whole dialogic approach as they embody the very essence of what we advocate. However, as designers, who have been involved in task design for very many years, we recognise that conveying the essentials of how a task might be effectively made instrumental in line with our design intentions in a classroom requires considerable expertise. Hence, our understanding of tasks and their accompanying material support as boundary objects that need to facilitate both classroom activity and community involvement in professional reflection and learning is important. Fundamental then, are these professional questions that as boundary artefacts support professional collaborative growth. Such aspects of our design are perhaps best exemplified in the video sequences that we have been designing that draw on classroom use of the tasks by Lead Teachers and their students' responses in relation to the key aspects of our conceptual design. These video sequences are framed in terms of the research questions (Table. 1), that we have crafted in light of our design research experiences during the first year of the project, to support teacher discussions.

We find that the theoretical constructs, which I highlight as a central thread to our design process in Figure 7, provide us as a research team, with insights to the important and multiple aspects of our design work to which we need to be sensitive. In many ways, this not only points to the complexity of our undertaking in such projects, but also provides us with a language and emerging discourse around which we ourselves might effectively become a community of practice of designers of tasks. It sits comfortably with our efforts to make meaning of our own work in ways in which support communication both within our own group and in the wider design community. It allows us to probe / critique each other's work in language that allows for some neutrality, rather than perhaps

appearing rather personal. As designers we can use such theoretical constructs to help us develop our sense of identity, come to an understanding of design as a practice, assist us in making meaning of the work in which we engage and fundamentally develop a community of practice of designers. For these reasons I recommend designers/ design communities explore such theoretical considerations as they seek to firmly establish their work as a valued activity in Pasteur's Quadrant.

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