Teaching mathematics through problem solving

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Abstract
The purpose and contribution of this paper is to explore and discuss how project-based learning (PBL), a general instructional strategy that can be used in numerous core content areas, can be integrated with teaching mathematics through problem solving (TMPS), a mathematics specific classroom approach. This instructional approach, TMPS, discusses two elements of a teacher's facilitation role, including the planning of valuable tasks and the classroom facilitation of this task. We describe and explore a case of a sixth (6th) grade mathematics teacher named Mrs. Miller who incorporated PBL in her teaching. We then address Mrs. Miller's concerns and perspectives through related observations and connections made by the authors, organized using the essential elements of a teacher's role in TMPS. We believe these essential elements of a teacher's role in TMPS should be maintained when a teacher integrates PBL in his/her practices in order to ensure a project effectively teaches students significant mathematical content.

Keywords
Project-based learning, teaching mathematics through problem solving, teacher's role.

La enseñanza de las matemáticas mediante la solución de problemas

Resumen
El propósito y la contribución de este trabajo son explorar y discutir cómo el aprendizaje basado en proyectos (PBL, por sus siglas en inglés), una estrategia de enseñanza general aplicable en numerosas áreas de contenido básico, puede integrarse a la enseñanza de las matemáticas mediante la solución de problemas, método didáctico específicamente diseñado para las matemáticas. Este método de enseñanza –conocido como TMPS, por sus siglas en inglés– analiza dos elementos de la función del profesor como facilitador, incluidas la planificación de tareas útiles y la facilitación de esta tarea en el salón de clases. Describimos y analizamos el caso de una profesora de matemáticas del sexto (6º) grado, la señora Miller, quien incorpora el PBL en su enseñanza. Enseguida, abordamos las preocupaciones y perspectivas de la señora Miller por medio de observaciones y conexiones que organizamos según los elementos esenciales de la función del profesor planteados por el método TMPS. Consideramos que estos elementos esenciales de la función del profesor en la enseñanza de

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las matemáticas mediante la solución de problemas deben mantenerse cuando un maestro integra el PBL en sus prácticas, con el fin de garantizar que un proyecto determinado logre enseñar a los estudiantes un contenido matemático significativo.

Introduction

In an era of educational reform movements promoting new strands of alternative instructional approaches, being able to adapt their current teaching is an essential yet challenging task for teachers. While new approaches are promising to enhance student learning, teachers still need to meaningfully connect their growing understanding of the new –often not content specific-strategies with their current content-specific knowledge about teaching and learning. Considering adopting an alternative instructional approach with the focus of particular content areas can be daunting for teachers. The purpose and contribution of this paper is to explore and discuss how *project-based learning* (PBL), an alternative instructional strategy that can be used in numerous core content areas, can be integrated with *teaching mathematics through problem solving* (TMPS), a mathematics specific classroom approach. In this paper, first we share existing literature for PBL. Included in the literature review is a distinction between problem based learning and PBL. Then, we describe and explore a case of a sixth (6th) grade mathematics teacher named Mrs. Miller who incorporated PBL in her teaching. Following, we share existing literature for TMPS. We then return to Mrs. Miller's PBL with a focus on the mathematics embedded in the project. We focus on identifying the issues the teacher encountered and offering suggestions about the effective planning and implementation of the two integrated instructional approaches.

Project Based Learning

PBL is an educational reform movement gaining a reputation for helping teachers to comprehensively meet higher level learning standards for students. There have been large scale PBL initiatives, notably in West Virginia (Williamson, 2008) and Indiana (Ravitz & Blazevski, 2010). A PBL approach centers on carefully orchestrated experiences for students investigating a driving question or problem. These experiences include student production of high quality products and/or presentations demonstrating their learning (Buck Institute for Education, 2011). Many of the principles of PBL are common to problem-based learning. For example, both have learning activities organized around achieving a shared goal (Savery, 2006). However, in problem-based learning the emphasis is rather on the self-directed acquisition of knowledge by the learner throughout the experience with no particular focus on the end product or presentation (Savery, 2006). In PBL, the central emphasis of instruction is on projects where students express what they learn through the end product and/or presentation (Helle, Tynjala, & Olkinuora, 2006).

Research has demonstrated that PBL can increase student engagement (Bernt, Turner, & Bernt, 2005), autonomy (Worthy, 2000), development of 21st century skills (Ravitz, Hixson, English, & Mergendoller, 2011), reflective experiences (Grant & Branch, 2005), and core curricular academic achievement on standardized tests (Geier, Blumenfeld, Marx, Krajcik, Fishman, Soloway, & Clay-Chambers, 2008). There has also been substantial interest in mathematics teaching reform involving PBL and a growing body of evidence indicating that under the right conditions, PBL can benefit both mathematics teaching and learning (Yetkiner, Anderoglu, & Capraro, 2008).

Larmer and Mergendollar (2010) identified eight essential elements that teachers should incorporate in meaningful PBL projects. These include: 1) significant content, 2) a need for students to know more information, 3) a powerful driving question, 4) student voice and choice, 5) 21st century skills, 6) student inquiry and innovation, 7) students receiving feedback and revising their work, and 8) a publicly presented final project. We now present the case of Mrs. Miller, a teacher new to PBL planning, with a focus on how she incorporated the first essential project element, significant content. In particular the focus is on the mathematics embedded in Mrs. Miller's PBL project. Mrs. Miller's project ideas and perspectives were shared through reflective interviews. The interviews used video captured during classroom observations, classroom planning documents, and reflective notes as a rich context for discussions. Our previous report extensively documented the research methodology and findings in this case (Kale, Selmer, & Ravitz, 2011), to which the readers can refer for further details. Our scope in this paper is rather practice-oriented, highlighting both Mrs. Miller's ideas and perspectives and related observations and connections we made as the authors.

The case of Mrs. Miller to exemplify Project Based Learning

Mrs. Miller, a teacher with 18 years of experience, currently works as a sixth grade multi-subject teacher at Wiley Middle and High School, which has around five hundred students in grades six to twelve. She was the driving force behind a PBL project that represented not only a change in the learning experience for the students but a change in the teaching experience for her. Mrs.

Miller's project was the culmination of a great deal of theory, training, and preparation that represented a significant change in the way she taught mathematics. There were a total of 21 students (4 boys, 17 girls) in Mrs. Miller's sixth grade mathematics classroom, which she classified as a high ability group. The students had no prior experiences in PBL though they had practiced mathematics content and skills, such as performing number and decimal computation and converting between fractions, decimals, and percents prior to the project. As they engaged in the PBL experience, the learning of mathematics content to solve real world problems was embedded within the project. Students employed real world 21st century skills (such as collaboration, communication and problem solving), and created presentations of their projects. The role of Mrs. Miller was as a coach or facilitator of the PBL experiences. This role was different from the more traditional teaching role of providing direct instruction to students. Prior to implementation, Mrs. Miller spent countless hours attending formal training sessions and working in smaller school based collaborative groups. She also spent time on her own to create and implement the mathematics PBL exercise.

Mrs. Miller's plan for her PBL is to begin with a visit from a parent planning a birthday party for her thirteen-year-old daughter. The students will be divided into small groups representing catering companies. She wants to choose a real-world context that would be both motivating and engaging for her students. The challenge for the catering companies is to prepare a bid for the birthday party to accommodate 30 guests with a budget limit of $250. The bid components are to include a party budget, menu, and description. The budget is to be represented as both a spreadsheet and a circle graph. The menu is to include food choices and recipes to be modified based on the number of guests. The description is to include decorations, entertainment, food and favors. Each catering company is to present a party proposal at the end of the project. The relationship within and between fractions, decimals, and percents represents the mathematics focus of the project. Table 1 presents the specific driving project question, overarching mathematics focus, and guiding mathematics-focused project activities and resulting artifacts that Mrs. Miller designs using a PBL approach.

Although such a framework is useful for planning the PBL activities, one common challenge for a novice PBL teacher is to focus on the embedded content within a project. In the following sections, we address Mrs. Miller's challenge specific to teaching mathematics by discussing TMPS, an analogous instructional approach specific to mathematics education. This allows us to offer suggestions about the effective planning and implementation of the two integrated instructional approaches. A first step is to clarify definitions and the facilitating teacher role for TMPS.

Table 1.
Components of Mathematics project-based learning Lesson Framework.

Component	Description
Driving Project Question	Applying mathematical standards, what is the best way to plan a birthday party, while working within a fixed budget?
Mathematics-focused	Development of number sense by understanding relationships within and among fractions, decimals, and percents and by computing and making reasonable estimates as part of real-world problem solving.
Primary Project Activities and Resulting Artifacts with a Mathematics-focused	
• Party budget spreadsheet	The students are to create a party budget spreadsheet that includes an expense sheet and a summary sheet to display percentage representation for each budget category.
• Recipes	The students are given a recipe for chocolate chip cookies. Using the given quantities for each ingredient, students are to answer questions such as, "If you want to make two batches, how much flour would you need?" Students do similar modifications for their own chosen party recipes. For example, if a group chooses a chocolate cake that serves 12 people they are asked to modify the recipe to serve 30 guests.
• Budget represented in a circle graph	Using the party budget spreadsheet the students are asked to express line items in dollars, fractional equivalent, decimal form, and as a percentage. For example, if the budget for party favors is $40, the figure is to be represented as a fraction of the total budget (40/250). The fraction is converted to a decimal using a calculator and then multiplied by 100 to provide a percentage of the total budget. The percentages are used to construct a circle graph representing the contribution of the line items to the total party budget.
• Mathematics journal prompts	Students are to answer open-ended journal prompts. The following examples are from classroom artifacts: "How can you determine the percent of the budget that each category represents?" "If you plan to spend 40% of your budget on food expenses, how much would you spend on food if your total budget is $300?" "Briefly describe steps to plan a birthday party within a budget." "Explain all mathematical skills/concepts your team used to complete this project."

Teaching Mathematics through Problem Solving

During the last two decades TMPS has offered educational promise in improving mathematics education (Lester & Charles, 2003). TMPS embraces the idea that aspects of problem solving and learning substantive mathematical concepts are recognized as interdependent (Lesh, & Zawojewski, 2007). In other words, you do not learn how to problem solve separate from mathematics content. The mathematics learning is dependent on the problem solving and the problem solving is reliant on the mathematics. For example, consider upper elementary students working on a task involving the fair sharing of a number of brownies. The students are asked to illustrate how they would represent scenarios such as: 5 brownies shared with 2 children, 2 brownies shared with 4 children, 3 brownies shared with 8 children, and 5 brownies shared with 6 children. The task requires that all children in the scenarios must get an equal part of the item being shared and that the entire amount must be used. In this example the students are solving the problem of equally sharing the brownie while concurrently learning mathematics central to solving these tasks. The mathematics central to these tasks include the importance of the unit, varying interpretations of fractions as part/whole ratios and quotients (Lamon, 2005), issues of equivalency and congruency, and the meaning of the numerator and denominator. As students engage with these tasks the mathematics learning is dependent on the students solving the problem and the problem solving is the conduit for students to engage with the mathematics.

The role a teacher plays in a classroom based on TMPS entails the planning of valuable tasks and the classroom facilitation of these tasks (Cai, 2003).When choosing a task the teacher first needs to consider the mathematics they would like their students to learn (Van de Walle, 2003). Often teachers emphasize the importance of providing real-world mathematical problems for students, but more important for TMPS is that a task focus students' attention on the embedded mathematical concepts (Van de Walle, 2003). The teacher must question if a task they chose requires students to engage with substantial mathematics in order to find a solution. In other words, the mathematics cannot be peripheral to a solution. If the mathematics is not central to a solution then the students might be problem solving in the context of the real world, but they are not learning mathematics through problem solving (Van de Walle, 2003).

Once a teacher has chosen a valuable task they must facilitate the task with students. Typical teacher facilitation in TMPS involves first letting students work on a solution and then facilitating the use of student solutions as fodder for a classroom discussion. As students work on a solution (either individually or in small groups) the teacher must allow the students to struggle with the mathematics. The facilitation of students working on their own solutions seems simple enough; however, this can be difficult for teachers who often step in and help students (Hiebert, 2003). For example, if a student is struggling with the previously mentioned brownie problem, a teacher sensing a student's frustration might ask guiding questions that actually provide the student with the solution. This is actually very common in the United States where teachers often guide students to solutions because they feel bad that their students are frustrated (Hiebert, 2003). It is therefore essential that a teacher allow the mathematics to be problematic for the students—because otherwise students are not learning the mathematics—even if this involves struggling to find a solution.

The role of a teacher also includes facilitating the use of various student solutions to maximize the mathematical learning opportunities for students through classroom discourse (Cai, 2003). To drive classroom discourse, Hiebert (2003) suggests asking students to share their solution methods and procedures they used to solve a problem. Then, students are encouraged to elaborate their own thinking and to engage in the thinking of others through these shared solutions. During this process, the teacher must be an active guide, choosing which solutions to share as a group and asking mathematically challenging questions that do not remove the problematic nature of the task (Hiebert, 2003).

Returning to the brownie example, as students work on the problem, the teacher strategically facilitates sharing and exploring different strategies among the students, asks students to justify their responses, and asks questions to probe the conceptual understandings of important fraction concepts. This process allows the students to learn both procedural and conceptual fraction understandings through problem solving.

Re-examining the case

During interviews, Mrs. Miller expressed concerns that her students will procedurally and not conceptually learn the intended mathematics embedded in the project. Procedural knowledge includes understanding steps or actions that are used when operating on numbers. Conceptual understandings usually refers to students having an “integrated and functional graph of a number of mathematical ideas” (National Research Council, 2000), in this case the relationships among and between fractions, decimals, and percents.

These concerns are articulated through an excerpt culled from an interview with Mrs. Miller where she stated:

> I think they will know procedurally, but a lot of them did not know conceptually and that came out in the journal prompts. I normally do a lot more hands-on with [math instruction] where we’d be coloring squares and they see the answer is getting smaller. Because I was trying to let’s do it and let’s move on and apply it, and I don’t think I spent enough time that they understood conceptually.

This excerpt relates to students’ work—as described in Table 1—on the circle graph budget activity that tasks students with creating a circle graph representing their party budget. Mrs. Miller wondered if the activities and related facilitation of these activities could make the mathematics procedural rather than conceptual for the students. For example, if students use calculators to find the fractions, decimals, and percents for each line item in the budget and create a circle graph in a very straightforward way without Mrs. Miller considering and then facilitating the understanding of relational mathematical concepts between these units, then students will not learn the conceptual mathematics. It is essential that Mrs. Miller work to connect her growing understandings about PBL with her current content specific knowledge about teaching and learning. For this, she would need not only to plan and implement the PBL activities but also to maintain her two key TMPS roles: planning of valuable tasks and the classroom facilitation of these tasks, which are essential to students learning significant mathematical content.

The role of the teacher planning a valuable task

First consider Mrs. Miller's PBL using the planning a valuable task element of the teacher's role in TMPS. Recall that the planning of a valuable task requires the teacher to attend to some important task features. The teacher must consider the mathematics he or she would like the students to learn, and then choose a related task that requires students to engage with substantial mathematics in order to find a solution. At the same time, a real-world context is a paramount consideration in PBL. In TMPS a real-world focus is beneficial for student engagement but is a secondary consideration to determining if a task is mathematically challenging. In order to link these considerations together, a teacher must consider if their real-world project is also mathematically challenging. In other words, this element of a teacher's role in TMPS suggests that the driving question, primary project activities, and/or resulting artifacts associated with a PBL approach still need to involve substantial mathematics in order to enable students to successfully find a solution. The target mathematics focus can be used to analyze the driving question and guiding activities of Mrs. Miller's project (Table 1). Mrs. Miller seeks to teach the relationship within and among fractions, decimals, and percents. The development of a deep and flexible understanding of this relationship is a complex endeavor for a student. Considering each individual concept and the interplay between the concepts should demonstrate this complexity. In learning fractions, students need to understand the relational nature of fractions (McNamara & Shaughnessy, 2010). Consider the use of the two digits, 2 and 3, first as the number 23 and then as the fraction $2/3$. In the case of the fraction, the digits represent a relationship or an underlying relative amount (McNamara & Shaughnessy, 2010;

Lamon, 2005), rather than the specific quantity reflected in the figure 23. While the magnitude of the two digits will not change, the meaning of $2/3$ is determined in part by the size of the units and the construct. For example, when considered as part of an area, the 2 represents the replications of the area that is a third of a whole. When $2/3$ is considered as a measure, the 2 is two iterations of the distance that is a third of the whole. If considered as a ratio, the fraction could refer to the probability of an event. The ratio might also be part-part or part-whole (Lamon, 2005). For example, the ratio $2/3$ could be the ratio of those wearing jackets (part) to those not wearing jacket (part); or it could be part-whole, meaning those wearing jackets (part) to those in the class (whole).

Another complexity arises in considering the idea of the size of the unit. When $2/3$ is considered as part of a set the 2 could mean two items, 4 items, 24 items, and so on, depending on the size of the entire set. Students should also understand issues of equivalency, namely that $8/12$, $4/6$, and $2/3$ represent precisely the same underlying relative amount. The differences in notation are merely a matter of the value of the denominator. Fraction, decimal, and percent notation are three different notational systems for rational numbers. A decimal is ultimately an alternative symbol system for Base-Ten Fractions. A percent is ultimately a way to express a fractional relationship out of 100. In order for students to learn the relationships among fractions, decimals, and percents, they need opportunities to apply their fraction knowledge to solve problems involving different contexts, settings, and relationships. These are complex understandings that need to be developed and strengthened over time with experience.

Review of Mrs. Miller's overarching driving question and guiding project activities finds that they do not necessarily represent a mathematical challenge that focuses on the relationship between fractions, decimals, and percents. First she considers the overarching driving project question, "Applying mathematical standards, what is the best way to plan and budget for a birthday party, while working within a fixed budget?" The key aspects of planning a birthday party while representing the real world are not necessarily mathematically problematic. The party theme, menu content, and decorations are considerations far more central to the problem. For example, a student can determine the party theme, menu, and decorations prior to creating a budget. After the party is planned a student can then fit that plan into a working budget. In other words, the mathematics is not integral in determining a solution.

Next consider the guiding project activities. What if the mathematical computations to complete the party budget and adjust recipes are purely procedural and cursory to what might be perceived as the other far more interesting challenges? Mrs. Miller plans for the students to create a spreadsheet representing the party budget for each line item in dollars, fractions, and the equivalent percent. Students will then use the line item percentages to create circle graphs that represent the party budgets. Will the students be engaged in data entry and procedural calculations, activities that are mathematically unproblematic? Although students will be working with numbers and performing computations, this is distinctly different from struggling with, making sense of, and justifying the responses to a problem with the described relationships of fractions, decimals, and percents at the center of the solution. This suggests Mrs. Miller should modify her overarching project question and/or rework her project activities so that the answers can only be determined through engaging in substantial mathematics. For example, she might consider the idea of incorporating mathematical comparisons of party budgets.

The role of the teacher facilitating a task

A second element for successfully TMPS is the teacher's facilitation of a valuable task. Recall that typical teacher facilitation in TMPS involves first letting students work on a solution and then facilitating the use of student solutions as fodder for a classroom discussion. Mrs. Miller could use this lens to examine her facilitation of guiding project activities.

An example taken from a classroom video shows Mrs. Miller facilitating—as described in Table 1—the circle graph budget activity. This was the activity that Mrs. Miller was referencing when she expressed concerns about the students not conceptually learning the mathematics. Mrs. Miller formed a small group of students by pulling one student from each group to model how to create a circle graph for a mock party budget. Using this information, these students later went back to their groups to create a circle graph representing their party budget.

Mrs. Miller began the small group instruction by asking:

> [Mrs. Miller]: "So, we need to figure out how to get sections of a circle graph so we need to change this information [the mock budget] into a percent. What do we know about setting up a fraction? What does the numerator mean and what does the denominator mean? [silence]. Which one is the part and which is the total?"
>
> [Allison]: "The total is the denominator" [Mrs. Miller]: "the part is the………" [Emily]: "…numerator."
>
> [Mrs. Miller]: "When we do food, what are we going to have on top?"
>
> [Jessica]: "100".
>
> [Mrs. Miller]: "…and the total is….." [Jessica]: "250".

[Mrs. Miller]: "How do I change that to a decimal?"

[Alex]: "use a calculator".

[Mrs. Miller]: "You can use a calculator but what are you going to do?"

[Alex]: "100 slash 250…"

[Mrs. Miller]: "What did you get?" [Alex]: "0.4".

Students continued calculating decimal values of all mock party budget line items using calculators. Mrs. Miller later moved to help students change calculated decimals to percents.

[Mrs. Miller]: "So, this is four tenths but as a percent that would be…" [Emily]: "40%".

[Mrs. Miller]: "40% of what?" [Emily]: "The entire circle".

[Mrs. Miller]: "What do we know about "of"?" [Multiple Students]: "It means times".

[Mrs. Miller]: "It means multiply".

[Mrs. Miller]: "What are we going to do to figure out degrees?" [Jessica]: "0.4 × 360".

During these procedural tasks, students continued figuring out the number of degrees of a circle graph representing each line item. Based on these figures, Mrs. Miller demonstrated how to create a hand-drawn circle graph using a compass and a protractor. Observations reveal that even if the task was mathematically problematic (as discussed in the previous section) her facilitation consists primarily of stepping in and guiding the students. TMPS would suggest that she first let students work on a solution and then facilitate the use of student solutions as fodder for a classroom discussion.

In order to offer an explicit example related to a more effective planning and implementation of the two integrated instructional approaches, the following hypothetical activity and student responses serve to build on the birthday party project context. The activity asks students to decide which group spent the most of a total party budget on food. The students are given data for the four groups, each with a different budget. Students are then tasked with comparing the group spending on the specific line item for food using different strategies and then asked to justify their responses (Table 2).

Table 2.
Budget for Each Group.

	Group 1	Group 2	Group 3	Group 4	Group 5
Food	125	156	175	192	210
Decorations	50	75	50	100	100
Paper Goods	75	69	125	108	190
Total Budget	250	300	350	400	500

Which group spent the most amount of money on food relative to their total budget? Explain your answer. As seen in Table 3, students might respond in different ways to the described task.

Mrs. Miller could first let the students work on the problem. Subsequent classroom discussion could focus on the different ways in which students made sense of the problem through the sharing of approaches used. Mrs. Miller could carefully choose what student methods would be beneficial to incorporate into a whole class example. The hypothetical shared student solutions (Table 3) might be chosen because they represent a variety of procedural approaches that provide windows into conceptual mathematical ideas through discussion. For example, student A states:

> In order to figure out which group spent the most on food out of their entire budget I started finding equivalent fractions. I thought that if I could make them all out of 100 they would be in percent form and easy to compare. I think group 2 spent the greatest amount on food.

Mrs. Miller could question the student:

Table 3.
Sample 3 Students' Hypothetical Solution and Explanations.

Student	Group	Student Solution	Student Explanation
Student A	1	125/250=62.5/125=12.5/25 (12.5+12.5+12.5+12.5)/100=50/100=50%	In order to figure out which group spent the most on food out of their entire budget I started finding equivalent fractions. I thought that if I could make them all out of 100, they would be in percent form and easy to compare. I think group 2 spent the greatest amount on food.
	2	156/300=78/150=26/50 (26+26)/100=52/100=52%	
	3	175/350=35/70=5/10 (5+5+5+5+5+5+5+5+5+5)/100=50%	
	4	192/400=96/200=48/100=48%	
	5	210/500=105/250=21/50 (21+21)/100=42/100=42%	
Student B	1	125/250 Divide top/bottom by 5 25/50	After I had divided every group's top/bottom by five I looked at each answer. Group 2 was the only answer that was more than half. So, I think they spent the most on food.
	2	156/300 Divide top/bottom by 5 31.2/60	
	3	175/350 Divide top/bottom by 5 35/70	
	4	192/400 Divide top/bottom by 5 38.4/80	
	5	210/500 Divide top/bottom by 5 42/90	
Student C	1	125	Group 5 spent $210.00 on food and that is more than any other group.
	2	156	
	3	175	
	4	192	
	5	210	

> What do you mean by equivalent fractions? Can you show or explain to me how 125/250 = 62.5/125?

This question would hopefully unearth a discussion about how the digits represent a relationship or an underlying relative amount (McNamara & Shaughnessy, 2010; Lamon, 2005). In other words the relationship is equivalent, not the digits.

Or, student B states:

> I divided every groups top/bottom by five because I knew that five was an easy number. I then looked at each fraction and tried to figure out how close that fraction was to half. Group 2 was the only answer that was more than half. So, I think they spent the most on food.

To which Mrs. Miller could ask:

> Can you explain what you mean by figuring out how close each fraction was to half?

This question could potentially facilitate student explanation of how they compared the amount of money spent on food out of the entire budget to a reference point of ½ (of the entire budget) even though each of the groups' total budgets varied.

Student C appears to not be thinking relatively. Instead, they are considering which of the food budgets is using the highest absolute amount of money. Mrs. Miller could use this solution in comparison to other groups to facilitate discussion about absolute and relative thinking.

The classroom discussion could continue with Mrs. Miller facilitating discourse that allows students to share solutions, justify their answers, and make sense of others solutions (Cai, 2003). Further, in this example notice that the artful use of facilitation serves to strengthen the guideline of keeping the mathematics central to the problem. Using the discussed considerations of TMPS allows a teacher to better understand how to facilitate the mathematics focused PBL environment in order to ensure that the intended mathematics content is learned by the students.

Conclusion

The purpose and contribution of this paper is to connect and discuss two robust instructional strategies within education: PBL and TMPS. The instructional approach specific to mathematics education, TMPS, discusses two elements of a teacher's facilitation role including the planning of valuable tasks and the classroom facilitation of this task. We described and explored a case of a sixth (6th) grade mathematics teacher named Mrs. Miller who incorporated PBL in her teaching. We then addressed Mrs. Miller's concerns and perspectives through related observations and connections made by the authors, organized using the essential elements of a teacher's role in TMPS. We believe these essential elements of a teacher's role in TMPS should be maintained when a teacher integrates PBL in his/her practices in order to ensure students learn significant mathematical content in a project. The same considerations for a teacher's role can be used within professional development, professional learning communities, and by individual teachers to plan PBL experiences with a focus on the embedded mathematics. Importantly, this allows teachers to connect a potentially new instructional strategy to which they are being introduced—such as PBL—with previous knowledge within the field of mathematics education.

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