THREE PARADIGMS IN DEVELOPING STUDENTS' STATISTICAL REASONING

Dani Ben-Zvi
University of Haifa, Israel - Faculty of Education, LINKS I-CORE

Abstract: This article is a reflection on more-than-a-decade research in the area of statistics education in upper primary school (grades 4-6, 10-12 years old). The goal of these studies was to better understand young students' statistical reasoning as they were involved in authentic data investigations and simulations in a technology-enhanced learning environment entitled Connections. The article describes three main paradigms that guided our educational and academic efforts: EDA, ISI, and Modeling. The first, EDA, refers to Exploratory Data Analysis – children investigate sample data they collected without making explicit inferences to a larger population. The second, ISI, refers to Informal Statistical Inference – children make inferences informally about a larger population than the sample they have at hand. The third, Modeling – children use computerized tools to model the phenomenon they study, and simulate many random samples from that model to study statistical ideas. In each of these three paradigms, we provide a short rationale, an example of students' products, and learned lessons. To conclude, current challenges in statistics education are discussed in light of these paradigms.

INTRODUCTION

Data is everywhere nowadays and is increasingly used, in large quantities, in all areas of life, to make important human decisions. Statistical reasoning is required in many disciplines and professions, and raises interesting issues and challenges. Therefore statistical literacy must be a key goal of education at all levels. The teaching and learning of statistics has been a focus for research in many areas. A majority of these research studies (reviewed by Garfield and Ben-Zvi, 2007) suggest innovative ways of teaching and learning statistics that differ from the traditional classroom practices. This article offers three approaches (paradigms or models) to teaching statistics in the early age of primary school: EDA, ISI, and Modeling. They can be used by statistics educators in a variety of ways to a) better conceptualize their enterprise; and b) as starting points of theory and design for deep learning (Sawyer, 2014) of statistics to better develop students' statistical reasoning.

We shall first present statistical reasoning, and then provide the context in which all the studies described in this article took place – the Connections learning environment. This will be followed by three paradigms that enabled us design learning trajectories: EDA, ISI, and Modeling. For each paradigm, we provide a rationale, a quick outline of the intervention, empirical example, and brief list of the lessons learnt from that stage. Finally, we conclude with a few thoughts on current challenges of statistics education.

Statistical reasoning

Although statistics is now viewed as a unique discipline, statistical content is most often taught worldwide in the mathematics curriculum (K–12) and in departments of mathematics (college level). This has led to exhortations by leading statisticians, such as Moore (1998), about the differences between statistics and mathematics. These arguments challenge statisticians and statistics educators to carefully define the unique characteristics of statistics and in particular, the characteristics of statistical literacy, reasoning and thinking (Ben-Zvi & Garfield, 2004). We focus in this article on statistical reasoning.

Statistical reasoning is the way people reason with statistical ideas, consider how to collect data, create, select and interpret sets of data, graphical representations, and statistical summaries, and make sense of statistical information embedded in context. Statistical reasoning may involve connecting one concept to another (e.g., center and spread, sample and population) or may combine ideas about data and chance to make inferences and interpret statistical results. Statistical reasoning also means understanding and being able to explain statistical processes, and being able to interpret statistical results. We see statistical reasoning as the mental representations and connections that students have regarding statistical concepts. Underlying statistical reasoning is a conceptual understanding of important statistical ideas such as: variability, distribution, center, association, uncertainty, sampling, inference and probability (Garfield, 2002). We move on to describe the studies' settings.

The Connections Project

The Connections learning environment is built upon the principles of socio-constructivist theory (e.g., Cobb, 1994) and is designed for young learners (age 10-12). It is a design and research project which started in 2005 (Ben-Zvi, Gil, & Apel, 2007) to develop students' statistical reasoning in an inquiry and technology-enhanced learning environment in primary schools in Israel.

The project extends for five weeks (six hours per week) each year in grades 4-6 during which students actively experience some of the processes involved in statistics experts' practice of data-based inquiry. Students conduct data and statistical modeling investigations through peer collaboration and classroom discussions using TinkerPlots (Konold & Miller, 2011), a computer tool for dynamic data and modeling explorations. By playing a role in helping students learn new ways of representing data and develop statistical reasoning, TinkerPlots gradually becomes a thinking tool for these students; it scaffolds their ongoing negotiations with data, statistical ideas, inferences and their meanings (Ben-Zvi & Ben-Arush, 2014).

The tasks undertaken by Connections students are a series of open-ended real data investigations that provide students with rich and motivating experiences in posing statistical questions, collecting, representing, analyzing and modeling data, and formulating informal inferences in authentic contexts, which result in meaningful use of statistical concepts (Ben-Zvi, Aridor, Makar, & Bakker, 2012). The data are obtained from a questionnaire designed by the research team, teachers and students, and administered by students in their school. The Connections classroom is conceptualized and organized as a learning community (e.g., Bielaczyc & Collins, 1999) that supports collaboration, argumentation, sharing and reflection. This is done physically in the class and virtually in a website that includes all educational materials and supports, students' reflective diaries, and peer and teachers feedback. Students are usually highly motivated to present and discuss their work in short presentations during the project and at the Statistical Happening, a final festive event with their parents.

In the Connections learning environment, statistical concepts are initially problematized—that is, rather than first teach students directly about these concepts, then ask them to apply them in investigations, the investigations themselves are designed to raise the need to attend to these concepts, hence deepening students understanding of both their relevance and application. Additional strategies are used in the design of the educational materials such as growing samples, which is a pedagogical heuristic (e.g., Bakker, 2004), in which students are gradually introduced to increasing sample sizes that are taken from the same population. For each sample, they are asked to make an informal inference and then predict what would remain the same and what would change in the following larger sample. Thus, students are required to reason with stable features of distributions or variable processes, and compare their hypotheses regarding larger samples with their observations in the data. They are also encouraged to think about how certain they are about their inferences. Beginning with small samples, students are expected to experience the limitations of what they can infer about this current sample. This is a useful pedagogical tool to sensitize and slowly introduce students to the decreasing variability of apparent signals in samples of increasing sizes.

Ben-Zvi (2006) found that the growing samples heuristic combined with “what-if” questions not only helped Connections students make sense of the data at hand, but also supported their informal inferential reasoning by observing aggregate features of distributions, identifying signals out of noise, accounting for the constraints of their inferences, and providing persuasive data-based arguments. The growing awareness of students to uncertainty and variation in data enabled students to gain a sense of the middle ground of ‘knowing something’ about the population with some level of uncertainty and helped them develop a language to talk about the grey areas of this middle ground (Makar, Bakker, & Ben-Zvi, 2011).

Connections students gained a considerable fluency in techniques common in exploratory data analysis; use of statistical concepts; statistical habits of mind, inquiry-based reasoning skills, norms and habits of inquiry, and TinkerPlots as a tool to extend their reasoning about data (e.g., Ben-Zvi, Aridor, Makar, & Bakker, 2012; Gil & Ben-Zvi, 2011). In a longitudinal mixed methods study (Gil & Ben-Zvi, 2014), long-term impact of teaching and learning was sought among ninth graders, three years after their participation in the three-year Connections intervention. Students from two groups – those who have / have not taken part in the program were compared throughout three extended open-ended data inquiry tasks and took a statistical reasoning test. Connections students had significant gains in terms of their conceptual understanding of aggregate view of a distribution and informal statistical inference. They used statistical concepts in a more meaningful, integrated and accurate manner in their explanations, were more fluent considering uncertainty involved in a generalizations from random samples, and supported their inferences with data-based evidence.

During the more-than-a-decade project, our academic passion is geared to the following three basic research questions:

1. What can young students understand about data and do with data?
2. How does students' statistical reasoning develop?
3. How can we nurture students' statistical reasoning?

EDA: exploratory data analysis

The Connections project was initially based on the exploratory data analysis (EDA) pedagogic approach (Shaughnessy, Garfield, & Greer, 1996). This paradigm is based on Tukey's thought and innovation.

> For a long time I have thought I was a statistician, interested in inferences from the particular to the general. But as I have watched mathematical statistics evolve, I have had cause to wonder and to doubt … All in all I have come to feel that my central interest is in data analysis. (Tukey, 1962)

According to the EDA paradigm, students are encouraged to become "data detectives" who use critically the PPDAC cycle – Problem, Plan, Data, Analysis, and Conclusion (Fig. 1) in order to make sense of data. These stages follow the logical order of an investigation starting from understanding and defining the problem, making a plan, and proceeding to data collection, organization, and interpretation in order to come to a conclusion. Real research, however, seldom proceeds in this orderly fashion due to all sorts of reasons, but the main reason is the interdependency of these research phases (Konold & Higgins, 2003).

> In these respects, data analysis is like a give-and-take conversation between the hunches researchers have about some phenomenon and what the data have to say about those hunches. What researchers find in the data changes their initial understanding, which changes how they look at the data, which changes their understanding, and so forth. (p. 194)

We designed the Connections EDA learning trajectory keeping this complex picture of data analysis in mind. This meant that we wanted students to stay focused on the data and what they have to tell, be attentive to the context from which the data were taken, and use data analysis tools for collecting, graphing, and summarizing in a creative and critical way.

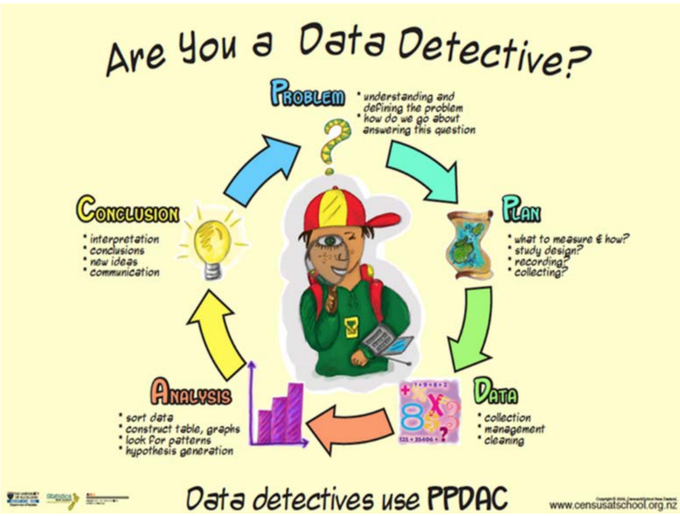

Figure 1: Data detectives use the PPDAC data investigation cycle (www.censusatschool.org.nz).

A typical example from our EDA studies is brought from a ten-hour experiment with second graders. The students studied data they had collected on the baby teeth lost by children in kindergarten to grade 3. The students were capable of making sense of the problem, the data collection and the organization of the sample data in appropriate inscriptions, some more concrete with names (Fig. 2) and some more abstract (Fig. 3). To develop and deepen their statistical reasoning, we used the growing sample strategy and “what if” questions. Further details will be given during the lecture.

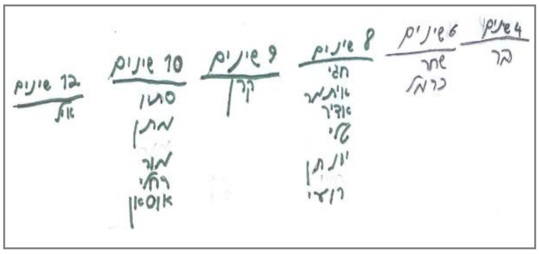

Figure 2: An inscription of baby teeth lost in grade 3.

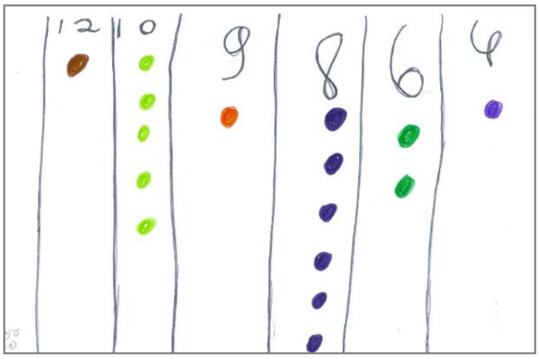

Figure 2: A graph of baby teeth lost in grade 3.

The main challenges identified in students’ statistical reasoning during years of study in the EDA learning environment were: a) global (aggregate) views of data versus local (pointwise) views of data (Ben-Zvi & Arcavi, 2001); b) seeing the signal in the noise (Konold & Pollatsek, 2002); c) distribution as an entity (Ben-Zvi & Amir, 2005). Furthermore, we felt that doing statistics without inference misses the main idea and goal of the discipline. We searched for ways to bring statistical inference to the center of the educational arena.

Informal Statistical Inference (ISI)

One ultimate goal and use of statistical reasoning is to enable students to make sound statistical inferences. A statistical inference is a probabilistic generalization, based on data, about a phenomenon under investigation. More specifically, a statistical inference is a statement about a population or process, which is inferred from a sample, along with an explicit level of confidence. Researchers in statistics education have been studying the foundations of students’ reasoning about statistical inference for several years (Garfield & Ben-Zvi, 2008, pp. 261–288). More recently, they have focused on students’ use of informal statistical inference in shaping these foundations (e.g., Pratt & Ainley, 2008).

While several definitions have been offered for informal statistical inference, its meaning is still fairly ambiguous. Makar and Rubin (2009) identified three features of both an informal and formal statistical inference: (a) a statement of generalization “beyond the data,” (b) use of data as evidence to support this generalization, and (c) probabilistic (non-deterministic) language that expresses some uncertainty about the generalization. Makar, Bakker and Ben-Zvi (2011) explained that the word informal is used to “a) make it clear that statistical inference is a broader concept than what is typically presented as hypothesis testing or estimation in an introductory statistics course; and b) emphasize that students are not expected to rely on formal statistical measures and procedures to formulate their inference” (p. 153). This attribution of informality agrees with studies of learning, which highlight the importance of informal, situated, and contextually rich instructional activities for supporting development of more formal knowledge (e.g., Hershkowitz et al., 2002).

Overall, students tend to have more difficulty with formal ideas of statistical inference than almost any other statistical concept (see Garfield & Ben-Zvi, 2008 for a summary of the research on this topic). We thought that informal inferential reasoning (IIR) may be an important goal on its own, especially with young children. Thus, following these ideas, we have moved in the Connections project from focusing mainly on EDA to focusing on a combination of EDA and ISI. Students drew informal inferences from real samples they had collected and investigated, and made informal inferences about a larger population.

One example that will be elaborated during the lecture is a pair of sixth grade students (age 12) studying the question, "How far do sixth and seventh graders jump?" To respond to this question, they analyzed several graphs of random samples taken from an unknown population of six and seventh graders in their school (an example of such a graph is shown in Fig. 4). A typical informal inference they made was, "based on these samples, it is possible to infer that six graders usually jump farther then seventh graders."

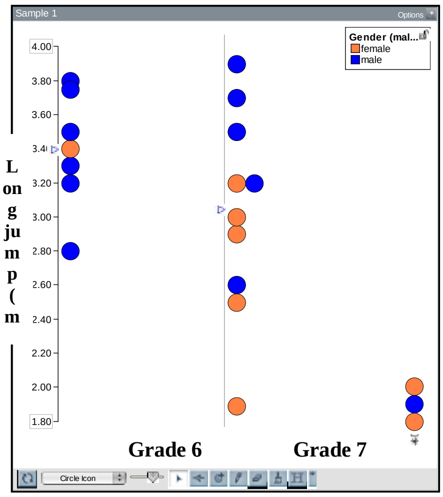

Figure 4: Long jump by grade (color indicates gender, and the blue triangles indicate the mean values).

Guiding students along a continuum from the informal to the formal reasoning may be an effective way of helping them build better foundations of statistical inference (e.g., Garfield & Ben-Zvi, 2008). However students faced many challenges in the study of ISI: a) integration of data and chance ideas; b) quantifying uncertainty using probabilistic language; and c) understanding sample-population relations. To address these challenges we have sought a new direction by integrating data modeling in the ISI learning trajectory.

Modeling

To foster students' appreciation of the power of their informal inferences, a model-based perspective has recently been added to the Connections learning trajectory, in which students build a model (a probability distribution) for an explored (hypothetical) population, and produced data of random generated samples from their model using TinkerPlots.

An Integrated Modelling Approach (IMA) was developed to guide the design and analysis of a learning trajectory aimed at supporting students IIR (Manor & Ben-Zvi, forthcoming). It is comprised of data and model worlds to help students learn about the relationship between sample and population. The data world is designed to foster in sample reasoning and the model world is designed to foster between samples reasoning.

In the data world, students collect a real sample by a random sampling process to study a particular phenomenon in the population. In this world, students choose a research theme, pose questions, select attributes, collect and analyze data, make informal inferences about a population, and express their level of confidence in the data. However, they may not account for probabilistic considerations, such as the chance variability that stems from the random sampling process.

In the model world, students build a model (a probability distribution) for an explored (hypothetical) population and generate random samples from this model. They study the model and the random process that produces the outcome from this model. The details vary from sample to sample due to randomness, but the variability is controlled. Given a certain distribution of the population, the likelihood of certain results can be estimated.

In the IMA learning trajectory, students iteratively create connections between the two worlds by working on the same problem context in both worlds and by using TinkerPlots, which includes a “Sampler” that allows learners to design and run probability simulations to explore relationships between data and chance, by means of one technological tool.

Our hypothesis is that the IMA can support students’ development of reasoning with uncertainty when making ISIs by experimenting with transitions and building connections between the two worlds. Two main features of the IMA that may support students’ reasoning with uncertainty are: 1) working on the same problem context in both worlds; 2) the support of the researcher’s guiding questions (e.g., what is the minimal sample size needed to draw conclusions about the population with certain confidence, or “what if” questions on optional real data results while exploring model generated random samples).

By analyzing generated random samples and comparing them with the suggested model, students can learn about the relationships between samples and populations. However students face many challenges in this reasoning process. Further details shall be provided in the lecture.

DISCUSSION

The goal of this article was to follow the development of an experimental curriculum that aimed at developing young students’ statistical reasoning. We have started from the pedagogical model of data analysis (EDA), in which the emphasis is on exploring authentic real data at hand. We have moved to the ISI paradigm, in which the relationship between sample and population is emphasized while making informal statistical inferences. The goal was to tighten the relations between data and chance (statistics and probability), but we concluded that a third paradigm had to be added, data modeling, to enable students better achieve this goal. Thus a new design emerged (IMA) that integrated reasoning about models and modeling with reasoning about inference.

The three paradigms, EDA, ISI and Modeling, present the progress we made in our study and theory of developing young students' statistical reasoning in the Connections project. Conceptually, we strive for better understanding of young students' abilities to develop statistical reasoning. We show the power of informal ideas of statistics in building students' conceptual basis for future development in their studies at later age. Pedagogically, we wish to develop sound design ideas for a learning environment that changes the way we treat the what (content), the how (pedagogy), and the assessment methods.

A learning environment perspective (Ben-Zvi, Gravemeijer, & Ainley, forthcoming) can guide statistics educators and researchers to view, design and assess statistics teaching and learning. A learning environment is a complex and dynamic educational system, composed of multiple factors: key statistical ideas and skills (content), engaging tasks, real or realistic data sets, technological tools, classroom culture including modes of discourse and argumentation amongst students and between students and teachers, norms and emotional aspects of engagement, and assessment methods. Integrating all these factors in order to reform the way statistics is learnt and taught is a challenging endeavor. A learning environment perspective can support the intentional transformation of an educational setting based upon conjectures about how the integration of features of the designed setting will support learning statistics.

The field of statistics education faces nowadays many complex challenges worldwide. Some of them are "classical": Poor teachers' statistical knowledge, lack of appropriate curricular materials and time at all age levels, shortage of technology and access to data and resources, and more. Recent developments in the emerging discipline of *data science* establish new kinds of challenges regarding the goals of statistics education, the interdisciplinary nature of our discipline, the need to develop new and free technology for schools everywhere, and more. Current research in statistics education must consider these new challenges, and be open to new themes, such as, data science in school, data science in future learning spaces, and data science in citizen science. I hope that these new directions will help us better prepare our school students to the information-based society of the 21st century.

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