DISCUSSING A TEACHER MKT AND ITS ROLE ON TEACHER PRACTICE WHEN EXPLORING DATA ANALYSIS

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This article considers teacher knowledge in managing mathematically critical situations and the role of what can be termed a mathematical summary in the analysis of a teaching episode, viewed from the perspective of Mathematical Knowledge for Teaching (MKT). The analysis is based on an episode of content review, from a perspective which aims to understand the teacher's logic rather than merely identify gaps in their knowledge. We discuss the importance of approaching mathematically critical situations in order to contribute to eradicating mathematical innumeracy (statistics) and to promote a kind of practice which is "mathematically demanding" as well as "pedagogically exciting".

Keywords: Data analysis; Mathematical knowledge for teaching; Mathematically critical situations; Teacher's practices

Discusión del conocimiento matemático para la enseñanza de un profesor y su papel en la práctica docente cuando se explora el análisis de datos

Este artículo considera el conocimiento del profesor al gestionar situaciones matemáticamente críticas y el papel de lo que puede denominarse un resumen matemático en el análisis de un episodio de enseñanza, visto desde la perspectiva del conocimiento matemático para la enseñanza. El análisis se basa en un episodio de revisión del contenido, desde una perspectiva que trata de comprender la lógica del profesor en vez de simplemente identificar lagunas en su conocimiento. Discutimos la importancia de abordar las situaciones matemáticamente críticas con el fin de contribuir a erradicar la incompetencia en matemáticas (estadística) y promover un tipo de práctica que sea "matemáticamente exigente" así como "pedagógicamente interesante".

Términos clave: Análisis de datos; Conocimiento matemático para la enseñanza; Prácticas del profesor; Situaciones matemáticamente críticas

Only in recent years in Portugal has greater attention been given to the contents which make up the topic of data analysis. This increased attention can be seen in the explicit inclusion of the topic in the new *Programa do Ensino Básico* (Basic Teaching Syllabus) (Ponte, Serrazina, Guimarães, Breda, Guimarães, Sousa et al., 2007). The chief goal in teaching this topic is specified as "developing students' ability to read and interpret data presented in tabular or graphical form, and enabling them to collect, organise and represent data so as to find solutions to problems in various contexts relating to their daily lives" (p. 26). The inclusion of this topic, coupled with little (if any) training on the part of the teachers in the field, has unsurprisingly led to varied degrees of success in dealing with the topic in the classroom.

The treatment it has received is directly related to the teachers' mathematical knowledge for teaching and the way they put this into operation—statically or dynamically—. In this paper we conceptualise such mathematical knowledge following the systematisation of the research group led by Ball (Ball, Thames, & Phelps, 2008; Hill, Rowan, & Ball, 2005), in particular their conceptualisation of Mathematical Knowledge for Teaching (MKT). Within this framework, it is essential that teachers possess a full, sound knowledge of the content they intend to impart if they are to ensure a corresponding comprehension on the part of their students.

This paper analyses a sample of actual classroom performance by an experienced teacher and considers the role played by MKT with respect to the opportunities made available—or not—to the students for developing their knowledge. As a result of the analysis, we hope to gain a better understanding of how this knowledge shapes their teaching and how its deployment influences the possible student outputs, with a view to consider the implications for teacher training.

MKT and Mathematical Summary

In the last few decades there have arisen various conceptualisations and ways of addressing the professional knowledge of mathematics teachers. Essentially, these originate in the three categories identified by Shulman (1986) focusing explicitly on content knowledge: subject matter knowledge, pedagogical content knowledge, and curricular knowledge. From among the various approaches to mathematics teachers' knowledge, necessary and sufficient to teach mathematics, that have emerged in recent years—some focusing more on issues relating to content, others on pedagogical questions—, we opted for that of MKT and its various sub-domains put forward by Deborah Ball and associates. The selection of this conceptualisation over others derived from the nature of our aim, which was to identify, from observed practice, what knowledge the teachers were deploying at each specific moment, and consequently the system for making this identification played a key role. Also advantageous was the fact that MKT em-

braces a focus of knowledge in action. We especially wanted to explore the kind of mathematical knowledge that teachers require to fully tackle every aspect of each topic, and to ensure that learning takes place.

The model developed by Ball and associates also provides a more specific classification, dividing Content Knowledge and Pedagogical Content Knowledge each into three sub-domains. The former is comprised by Common Content Knowledge (CCK), that is, typical 'schoolchild' maths, Specialised Content Knowledge (SCK) and Horizon Content Knowledge (HCK). The latter is formed by Knowledge of Content and Teaching (KCT), Knowledge of Content and Student (KCS), and Knowledge of Content and Curriculum (KCC).

The teacher should understand how the various mathematical areas relate to one another and how the requirements of any particular topic develop as students' progress up the school (HCK). Further, it is insufficient for the teacher to have knowledge of solely "how to do", they equally need to know "how to make understandable" (SCK). In other words, content knowledge needs to be complemented by an understanding of how to make said content accessible to students, and this includes knowing where and why students might encounter difficulties. In the case of data analysis, an example of CCK might be the knowledge concerning how to draw a pictogram incorporating a set of data, that there are impossible random generalisations, or that it's only possible to infer something when data comes from a representative sample of the population. With respect to SCK, on the other hand, the teacher has also a responsibility to understand the role of each variable in the pictogram so as to be able to teach the students to successfully construct their own. Amongst other things, SCK includes—in this instance—the knowledge on the effect of changing the scale employed in the pictogram, and the question of representativeness by which a sample approximates to the total population and how this affects the strength of inferences. They need also a knowledge related with proportionality, in order to know—be able to explain to pupils—the why of the characteristics of the sample in order to allow generalisations.

In addition to knowledge of content, teachers should also have a thorough knowledge of the curriculum and pedagogy. KCT corresponds to the type of knowledge which the teacher draws on in order to organise the different ways the students explore mathematical contents, such as determining the sequencing of tasks, choosing examples, and selecting the most appropriate representations for each situation. Regarding KCS, Ball, Thames, and Phelps (2008) relates it to the need for the teacher to anticipate what the students are likely to think, their difficulties and motivations as well as listening to and interpreting their comments. The teacher must be aware of the students' capacity to understand in such a way that it could allow him/her to go further in deepening the students' knowledge. With respect to KCC, the authors agree entirely with Shulman (1986, p. 10) that teachers should have a complete picture of the diversity of programs for teaching certain subjects and topics at a particular level/year group, and a variety of edu-

cational materials they can draw on. They should also be able to recognise the varying circumstances which suggest the adoption of one approach over another. In general terms, their curricular knowledge should be what can be termed both vertical and horizontal in its scope.

This knowledge, or its lack, has a direct influence on practice, and the use of different factors which can be included in its analysis, the richer the analysis. One aspect which can be revealing is what can be termed the mathematical summary. This summary can be explored through the components considered by Watson (2007) for analysis of practice. These components can focus on two aspects. First, they can be related with how the teacher regards their role—which also exteriorises the MKT they have or believe they have—; and second, being related to how to use the enunciation of the task of teaching that unfolding at each moment noting the role of the teacher and students during the course of the lesson and the type of interactions which take place (Ball et al., 2008; Thames, 2009), which can be expressed through dialogue, writing, and different forms of mathematical representation. Combining all these theoretical elements allows us to explore/focus on questions of mathematical content in the classroom and ways to approach such content, leaving aside other aspects such as management and behaviour.

Context and Method

This paper draws on data collected within the scope of a broader research project concerned with the professional development of teachers from the point of view of various facets of their professional knowledge. Here we look at the MKT of a primary teacher, Maria, with 18 years experience. It takes an instrumental case study approach (Stake, 2005) combined with a qualitative methodology, and includes consideration of the summary to an episode involving the review of a topic of data analysis in year four.

In the situation under discussion here, the teacher explicitly aims to review what she considers an inference from the data presented in a pictogram. From the analysis of this episode, we seek to deepen our understanding of the phenomena under consideration and to arrive at some kind of theoretical construct that can amplify our knowledge of practice, the factors which influence it, and how they influence it—with a view to also considering potential perspectives for improvement—.

Data collection took the form of audio and video recordings of lessons, with the focus on the teacher. The audio recordings were transcribed and complemented with video viewings, which enabled a fuller record of the teacher-student interactions to be made. Informal conversations were also conducted before and after each class, corresponding to the lesson image and an initial in situ self-analysis respectively. The transcriptions were divided into episodes (Ribeiro,

Monteiro, & Carrillo, 2009). Then, the episodes—each one associated to the teacher's immediate goals—and the MKT, along with the mathematical summary behind each one, were analysed.

ANALYSIS AND DISCUSSION OF THE ROLE OF MKT IN THE MATHEMATICAL SUMMARY OF PRACTICE

At an earlier point, Maria had constructed a pictogram with the aid of the students, using smiley faces to represent the preferences of the 12 class members with respect to visiting one of the continents. The episode began with the teacher asking the students how the distribution would be affected were the number of students involved four times that of those present. After a while, one of the students went to the board to explain how, in his opinion, the distribution would turn out indicating the number of smiley faces which would need to be added to each choice. The teacher and students then gave their confirmation that quadrupling the number of faces was correct. Other students then went to the board to show how they would solve the problem and the sequence repeated itself. This back and forth occupied most of the time devoted to mathematics. We show an extract from the transcription of this episode below, in which the teacher's declared aim is to draw an inference from the data represented in a pictogram. Each line of the transcription has been numbered for later identification purposes.

1409	*Teacher:*	How do we distribute the quadruple of these ones here?
1410		I'll ask someone who hasn't said anything, Ana, how would you do the
1411		distribution of the forty-eight?
…	...	…
1521	*Teacher:*	Seven! Let's be careful, Tiago. Make sure your partner is
1522		doing the distribution of the quadruple correctly.
1523		Have you placed any in America?
1524	*Students:*	No.
1525	*Teacher:*	Were there seven?
1526	*Students:*	Yes.
…	...	…
1564	*Teacher:*	Keep calm! Is the distribution she did for Oceania correct?
1565		(T indicates the number originally written for Oceania on the board)

1566		Twelve?
1567	*Students:*	It is!
1568	*Teacher:*	Why is it?
1569	*Students:*	Three times four is twelve.
1570	*Teacher:*	Three times four is twelve.
1571		(T indicates the number originally written for Oceania on the board)
1572		Everything OK then! It was the quadruple, three times four is twelve.

The joint analysis of the mathematical summary and MKT enables us to study the teacher's actions and what is emphasised during her teaching—the mathematical, or other, foci—as well as possible training needs in data analysis. It thus allows a detailed analysis of practice and the role of MKT in this teacher's practice.

With respect to the mathematical summary, this episode can be described as follows: (a) teacher elicits facts: What is the quadruple of 12?; (b) students find solution using procedure; (c) students "find solution" without knowing procedure, solution based on opinion; (d) teacher asks for definition, how to calculate the quadruple?; (e) teacher indicates the identification of relationships—between the actual and "required" number of smiley faces—; (f) teacher provides an explanation, asking students to locate the error, which is to be found in the count of 47 instead 48; (g) teacher requests student verbalization, explaining what they did in their own words; (h) teacher requests definition—multiplication—; (i) teacher provides summary of lesson. Although this sequence forms part of an episode in which the teacher's objective is to practise reading a pictogram and to derive from this "some sort of inference"—as stated in the interview before the lesson—, the mathematical summary of the episode illustrates that the mathematical content is confined to counting and how to quadruple a given number.

Looking at Maria's practice from the perspective of MKT, on the other hand, a certain (in)numeracy is in evidence, which will certainly lead to an incomplete understanding of data analysis on the part of the students. Some of this lack of knowledge is evident throughout the episode; in other cases; in other cases it is associated with specific moments—here referred to by the corresponding transcription line—. With respect to the task, and in terms of what can be considered "pure" mathematical knowledge, Maria shows that she knows how to calculate the quadruple of twelve and to interpret data represented in a pictogram. But in this respect, she reveals a certain innumeracy when she seeks to make inferences for another population (Line 1409), in that she assumes that the inference can be made using direct proportionality. A lack of SCK can also be perceived, in con-

junction with the CCK, whereby she appears ignorant that a sample should embody certain characteristics for generalisations to be made from it.

Regarding Pedagogical Content Knowledge, and specifically KCT, Maria considers it important that all the students verbalise their thoughts and ideas, attaching great importance to the students' ability to voice their opinion—even if this is based purely on their preferences—. In terms of the knowledge that can be considered to fall within KCS, she displays another lacuna in the instructions she offers the students, which will lead them to the idea that, if they want to make any inference about the quadruple number of students, then they should multiply each value in the pictogram by four, or randomly add three quarters of the forty-eight students, as the remainder are already to be found in the pictogram. Evidently, this lack of knowledge in terms of KCS relates intrinsically to those relating to CCK and SCK, and, depending on the analytical focus—teacher's content knowledge (explanation) or students' understanding—, this lack of knowledge can be considered in any of these sub-domains. This aspect illustrates why the sub-domains cannot be seen as hermetically intact parts—the whole is greater than the sum of the parts—, and highlights the complexity of the teaching process—and consequently teacher training—.

Such instances of (in)numeracy are mirrored in the quality of the students' learning. Nevertheless, they are unaware of this, as the teacher believes she is offering challenging tasks that will take their knowledge to a higher level. This knowledge is not realised as the premises on which it is based are for the most part false.

FINAL NOTES AND IMPLICATIONS

The lack of knowledge in terms of MKT leads to a mathematically limited exploration, directly or indirectly, of the prepared tasks—similarly as referred by Charalambous (2008)— in each instance. Maria focuses her classroom performance on obtaining quick answers to direct questions (Ferreira, 2005), prioritising, as the gaps in MKT show, objectives which might be "pedagogically exciting" but which are not always "mathematically demanding".

The treatment Maria gives to the topic reveals in itself how she approaches data analysis, and, in the context of aiming for mathematically competent students—with a good degree of statistical literacy—, the need for further training in this area is clear, as is the need for further studies into such mathematically critical situations. The identification of these critical situations, along with discussion of their associated mathematical summaries, aim to contribute to obtain a broader knowledge and understanding of such gaps in knowledge and the logic teacher applies. The objective is to understand the reasons behind such gaps and not only to identify areas where knowledge is lacking; so that, training can be improved. Fuller knowledge of these areas, and the situations in which they arise

in the classroom, would—one hopes—lead to a re-structuring of training programs to provide a specific focus on them, and also to teachers becoming more active and reflective professionals, better informed of their own MKT (Ribeiro & Carrillo, 2011), and so in a position to improve their practice. This implies, thus, a teacher's deeper understanding of the mathematical knowledge to teach it well (Ball, Lubienski, & Mewborn, 2001), and for that, a more focused attention from teachers trainers' to these aspects.

In our view such training is the most effective when it is based on reflection on actual practice—the teacher's own or that of others—and on such mathematically critical situations as have been identified, so that teachers genuinely feel the situations are their own and take ownership of the ensuing discussion (Tichá & Hošpesová, 2006). This awareness, drawing on difficult situations faced by others—and even oneself—, through the use of video recordings (Maher, 2008; Sherin & Hans, 2004) and/or students productions (Kazemi & Franke, 2004) and subsequent discussion, may promote reflections about their critical features and lead to an improvement in teachers' MKT and a more decidedly mathematical focus in the discourse on their practice. Such awareness—and overcoming of knowledge gaps—will promote the preparation and implementation of richer mathematical tasks and reduce teachers' fear of being asked "why", increasing their confidence to respond in a way that is both mathematically correct and understandable for the pupils (Ribeiro, 2011).

Both qualified and trainee teachers could benefit from this system of analysis, not only through the identification of the critical situations but also in terms of bridging theory and practice, and promoting a dialogue based on a common language and a shared understanding.

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