

PROSPECTIVE ELEMENTARY SCHOOL TEACHERS' WAYS OF MAKING SENSE OF MATHEMATICAL PROBLEM POSING

Olive Chapman

The study tackled prospective teachers' sense-making of mathematical problem posing and the impact of posing different contextual problems on their learning. Focus was on the generation of new problems and reformulation of given problems. Participants were 40 prospective elementary teachers. The findings provide insights into possible ways these teachers could make sense of problem posing of contextual mathematical problems and the learning afforded by posing diverse problems. Highlighted are five perspectives and nine categories of problem posing tasks to support development of proficiency in problem-posing knowledge for teaching.

Keywords: Contextual problems; Mathematical problem posing; Prospective elementary teachers; Sense making

Modos en que futuros profesores de primaria dan sentido a la invención de problemas matemáticos

El estudio indagó sobre los modos en que futuros profesores de primaria dan sentido a la invención de problemas matemáticos y el impacto de plantear diferentes problemas contextualizados en su aprendizaje. El foco fue la invención de nuevos problemas y la reformulación de otros dados. Los participantes fueron 40 futuros maestros de primaria. Los resultados proporcionan elementos sobre posibles modos en que estos maestros dan sentido a la invención de problemas matemáticos y el aprendizaje que ofrece plantear diversos problemas. Se destacan cinco perspectivas y nueve categorías de tareas en la invención de problemas para apoyar el desarrollo de la competencia de plantear problemas en la enseñanza.

Términos clave: Dar sentido; Futuros profesores de primaria; Invención de problemas matemáticos; Problemas de contexto

Chapman, O. (2012). Prospective elementary school teachers' ways of making sense of mathematical problem posing. *PNA*, 6(4), 135-146.
Handle: <http://hdl.handle.net/10481/20053>

This paper is based on a larger, ongoing project that investigates mathematics teachers' sense-making of contextual problems, problem solving, and problem posing and their development of problem-solving proficiency and knowledge for teaching. The project involves prospective and practicing elementary and secondary school teachers. The focus here is on prospective elementary school teachers and their mathematical problem-posing knowledge for teaching.

Problem posing, like problem solving, is promoted as an important way of learning and teaching mathematics (Kilpatrick, Swafford, & Findell, 2001; National Council of Teachers of Mathematics [NCTM], 2000). But whether or how this view gets implemented in the classroom will depend on the teacher and how he or she understands it. It is therefore important to understand teachers' sense-making of problem posing and ways to help them to develop meaningful problem-posing skills. This study contributes to this through the investigation of prospective elementary teachers' sense-making in posing word/contextual problems and the impact of posing various types of problems on their learning.

RELATED LITERATURE

Since the 1980s, there has been increased attention in promoting problem posing as an important aspect of school mathematics. The NCTM (1989, 2000) has proposed increased emphasis on problem-posing activities in teaching mathematics. Kilpatrick (1987) and Silver (1993) have suggested that the incorporation of problem-posing situations into mathematics classrooms could have a positive impact on students' mathematical thinking. Brown and Walter (1983) have also identified important aspects of problem posing in mathematics. Many benefits are gained from problem posing, such as enhancing problem-solving ability and grasp of mathematical concepts, generating diverse and flexible thinking, alerting both teachers and students to misunderstandings, and improving students' attitudes and confidence in mathematics (English, 1997a; Silver, 1994). Problem-posing activities reveal much about the understandings, skills and attitudes the problem poser brings to a given situation and thus is also a powerful assessment tool (English, 1997b; Lowrie, 1999).

Studies on prospective elementary mathematics teachers have raised issues about their knowledge of problem solving. While such studies imply related issues with their problem-posing knowledge, this is an area that is under-explored. Studies on problem posing tend to focus on students at school levels. Such studies have increased attention to the effect of problem posing on students' mathematical ability and the effect of task formats on problem posing (Leung & Silver, 1997). Some studies have investigated the extent to which children generate problems (Lowrie, 1999; Lowrie & Whitland, 2000; Silver, Mamona-Downs, Leung, & Kenny, 1996). One finding is that unless children are encouraged to talk about problem solving (Lowrie, 1999) and share ideas during mathematical

activities (English, 1997b), they tend to pose traditional word problems that are variations of those found in textbooks. Lack of exposure to meaningful contexts for problems was also found to restrict students' ability to pose problems (Stoyanova, 1998). Since students grow up to become teachers, it is likely that prospective teachers maintain some of these issues that will then continue the cycle unless they are helped in appropriate ways.

PERSPECTIVE OF PROBLEM POSING

Duncker (1945), and more recently Silver (1994), described problem posing as referring to both the generation of new problems and the reformulation of given problems. Stoyanova (1998) defined it as the process by which, on the basis of concrete situations, meaningful mathematical problems are formulated. For English (1997b), generating new questions from given mathematical tasks is considered to be the main activity of posing problems. However, as Silver et al. (1996) explained, "The goal is not the solution of a given problem but the creation of a new problem from a situation or experience" (p. 294). Importantly, the problem poser does not need to be able to solve the problem in order for positive educational outcomes (Silver, 1995).

In this study, the focus is on the generation of new problems and reformulation of given problems. The relevance of this is associated with the teacher's role in selecting, creating, or posing appropriate problems to engage students in meaningful problem-solving experiences (NCTM, 1989, 1991). To promote diverse and flexible thinking for students, it is critical for teachers to be able to generate diverse problems. They need to be able to generate a broad range of problems to widely combine situations with mathematical concepts or solution methods. For example, for mathematics teachers to develop quality-structured problem-posing situations, they should be able to pose problems based on textbook problems by modifying and reshaping task characteristics; formulate problems from every-day and mathematical situations and different subjects' applications; restart ill-formulated or partially formulated problems and pose complex and open problems as well as simple problems.

Problem posers have to appropriately combine problem contexts with key concepts and structures in solutions along with constraints and requirements in the task. Thus, both contextual settings and structural features of problems are recognized as crucial. Comparison between problems is also important. As Gick and Holyoak (1983) demonstrated, similarity judgement between problems facilitated the induction of schemata, that is, general information about key elements and their relationships in the problems. In problem posing, it is important to identify key elements and their relationships embedded in problems (English, 1997b; Leung & Silver, 1997).

The preceding theoretical background about problem posing provided the basis for selecting problem-posing tasks used in the study and for framing the research method.

RESEARCH METHOD

Participants were 40 prospective elementary teachers (Grades 1-6) in the second semester of their two-year post-degree bachelor of education program. They were not required to take, and had not taken, any post-secondary mathematics courses. They also had no instruction or exposure to formal theory on problem solving or problem posing prior to this problem-posing experience. The class in which the study was conducted was their first course in mathematics education. This timing of the study was intended to capture their initial ways of making sense of problem posing. So, data was collected in the second class of the semester.

The problem-posing experience included comparing problems of similar and different structure and responding to problem-posing tasks involving posing a problem: (a) of their own choice, (b) similar to a given problem, (c) that is open-ended, (d) with similar solution, (e) related to a specific mathematics concept, (f) by modifying a problem, (g) using the given conditions to reformulate the given problem, (h) based on an ill-formed problem, and (i) derived from a given picture. Following are some examples of the problem-posing tasks.

Task 1. Create a “word problem” of your choice for students in a grade of your choice.

Task 2. Create a “word problem” that you think is open-ended.

Task 3. Create a “word problem” that you think is similar to the following problem: Tennis balls come in packs of 4. A carton holds 25 packs. Marie, the owner of a sports-goods store, ordered 1600 tennis balls. How many cartons did she order?

Task 4. Create three “word problems”, each related to a different meaning of multiplication of whole numbers.

Task 6. Create a “word problem” for the following situation. Some students held a bake sale to raise money for a local charity. They sold fudge, brownies, and cookies. Each type of treat was put into paper bags and the students were allowed to keep the leftovers. They started out with 110 cookies, 130 pieces of fudge and 116 brownies.

These problem-posing tasks were presented one at a time in an intentional sequence to minimize the influence of one task on participants’ thinking of another. Participants were also required to focus on their thinking as they created the problems in order to notice and document it. They were told to interpret “word problem” in flexible ways that made sense to them. It was not intended to mean

only traditional-style problems. This explains the use of quotation marks around word problem in stating the tasks.

Data sources were the participants' written work for the problem-posing experience and reflective journals of their thinking. Upon completing all tasks, they wrote journals describing what they learned in general and about mathematics, problem posing, problem solving, and teaching and learning mathematics. Six of the participants whose thinking seemed to be representative of different ways of making sense of problem posing were interviewed to further explore and clarify their thinking. Interviews were audio taped and transcribed.

Data analysis began with a process of open coding (Strauss & Corbin, 1990). In addition to the researcher's coding, two research assistants conducted this open coding independently of the researcher, and independently of each other. Only after initial categories had been identified were the results discussed and compared and revisions made where needed based on disconfirming evidence. Themes emerging from the initial coded information were used to further scrutinize the data and then to draw conclusions. There was triangulation among participants' problems posed, interviews, and journals. Coding included identifying the types and nature of the problems the participants posed based on guidelines developed from the literature and participants' sense-making and learning based on significant statements in their thinking and the knowledge implied in the context and structure of problems. The coded information was summarized and categorized for each participant and compared for similarities and differences in their thinking, knowledge, and learning. The final coded information was summarized and categorized for each participant and compared for similarities and differences to determine the themes in their thinking, knowledge, and learning. The themes associated with the participants' collective sense-making of problem posing on a general level across tasks were then grouped, based on comparison to theory, into five perspectives of problem posing. The other themes focused on the patterns in their thinking that formed the basis of the problems they posed, their ways of posing the word problems for each task, and the nature of the learning resulting from engaging in the problem-posing activities.

FINDINGS

The findings represent the participants' ways of making sense of problem posing prior to taking any mathematics education courses. The focus here is on their sense-making of problem posing in general and a sample of the problem-posing tasks and their learning from the problem-posing experience.

Sense-Making of Problem Posing in General

Collectively, the participants' thinking displayed the following five perspectives of posing "word problems" that related to their sense-making of problem posing. While these were partly influenced by the problem-posing task, they all emerged

in tasks where there was no problem to influence their choice or thinking (Task 1, Task 2) and prior to seeing the other tasks.

Paradigmatic Perspective

The paradigmatic perspective emphasizes problem posing as creating a problem with a universal interpretation, a particular solution and an independent existence from the problem solver. This was evident in some of the participants' problems of their choice and reflected their experience with traditional word problems. For example one participant posed the following problem in response to Task 1: "Suzie leaves school with 3 pencils in her pocket. Her friend John asks if he can have one to keep. How many pencils does Suzie have left?" She explained that she thought about "students, what all students can read and do the mathematics to get the answer." So she made it "easy to understand and applicable to the students."

Objectivist Perspective

The objectivist perspective is similar to the paradigmatic perspective but is highlighted as being specific in considering problem posing as creating an object involving a mathematical fact. Thus the goal is primarily to work backwards from the fact that needs to be computed or determined in the problem. For example, some participants started with a number sentence, like $2 \times 3 = 6$, and then clothe it with a context.

Phenomenological Perspective

The phenomenological perspective emphasizes problem posing as creating a problem that is meaningful from the learner/student's perspective and provides a lived experience, that is, students are allowed to interact with problem contexts in a personal way and produce personalized interpretations and solutions. This was common for the open-ended problem (see Task 2). For example, "John is going to the grocery store and needs 6 fruits total. How many apples, oranges, and pears did he buy?"

This participant explained that students could choose any amount for each fruit as long as the total was six. Another participant explained that students could decide who get how many in the following: "Gary received a package of jelly beans for his 9th birthday. He decides to share them with his 3 friends, Brad, Gilles, and Monica. If there were 26 jellybeans in total, how many would each person receive?"

Humanistic Perspective

The humanistic perspective is similar to the phenomenological one but is highlighted here as being specific in considering problem posing as creating situations directly related to personal aspects of the students' experience; like, their interests, meanings, creativity, and choices. For example: "There are 8 hockey

players on the ice. 4 are holding a hockey stick, 3 have a hockey puck and 2 have both a stick and a puck. How many are playing hockey?" This teacher explained: "I tried to think of something students would be able to visualize, something they know about and something they could reason with."

Utilitarian Perspective

The utilitarian perspective emphasizes problem posing as creating problems in terms of their worth based on their contributions to students' learning—mathematically or cognitively or socially—. For example, "How would you determine how many legs 8 spiders have altogether if there are 6 legs on each spider?" This participant explained: "I thought of something that would force the students to show their thinking and their work. Some would be able to show just by using the numbers, others would need to draw pictures or use manipulatives to explain their reasoning."

Another example that focuses on the social context of learning, "Raisins come in cartons of 25. Each sack of raisins holds 10 raisins. If Naomi ordered four cartons of raisins, how many raisins did she ordered?" This participant explained: "I tried to make it similar to that question [Task 3], but I wanted to add nutritious food into it."

Sense-Making of Each Problem-Posing Task

Only three of the tasks, which the participants considered to be the most challenging, are discussed here to highlight the uniqueness of their thinking. First, for posing an open-ended problem, their common thinking was that open-ended meant more than one answer but there was uncertainty about what this meant mathematically. One participant explained, "Open-ended means more than one answer, but when I think of math I can only think of one answer, so I couldn't provide an example." Some of the problems they posed were ill-formed, not mathematical, or lacking sufficient information, but not done intentionally or with awareness of these features. Other problems involved multiple operations—but not open—and potentially yes/no/don't-know answers. For some problems, open-endedness involved any interpretation or solution whether or not appropriate for the given conditions. Examples of these open-ended problems they posed were:

- ◆ If the population of the earth increases every year by 500,000, does the mass of the earth increase?
- ◆ A teacher creates a lesson on study of fish. Students are to observe the fish over the year. Will there be fish babies at the end of the year?
- ◆ How many times does Ben have to bounce his basket ball before he refills it with air?
- ◆ Given a large bag of candies and a variety of measuring tools—scale, ruler, scoop—, how would you discover how many candies are in the large bag without actually counting each candy individually?

Second, for the multiplication task (Task 4), 40% of participants created one problem, 40% created two problems and 20% created three problems. Collectively, they produced one meaning for multiplication—combining equal groups. The problems involved multiplication only, division only, or various combinations of two or three of the four arithmetic operations—i.e., addition, subtraction, multiplication, and division—. For many problems posed, the participants did not attend to relationships among numbers, operation, and context and whether the problem made sense structurally. Their thinking and problems indicated that they were unaware of their focus/interpretation/use of “times”—e.g., three times more; how many times; three times six; three times older—, which resulted in the various combinations of operations and not necessarily attending to the meanings of multiplication. Their problems included:

- ◆ How many books are on the shelf if each book is an inch thick and the shelf is 15 inches wide?
- ◆ A mouse has 3 babies in January. If a mouse is pregnant for 3 months at a time and has a litter of 3 every time, how many children will she have in September?
- ◆ If you had 3 flowers and bought 4 more groups of three flowers. How many flowers would you have?
- ◆ I am going to play tennis. I will need 5 tubes of balls each containing 3 balls, how many tennis balls do I have in total?

Third, the picture task (see Figure 1), which was intended to represent a comparison meaning of subtraction, was not interpreted this way by any one in terms of his or her thinking. The instruction for the task was: “Create a word problem using the following picture. Assume the circles represent marbles or any object you want to choose.”

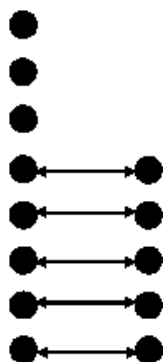


Figure 1. Picture task

The participants’ interpretation of this picture focused on pairing, “left over” and other interesting possibilities as in these examples:

- ◆ How much energy/force would be required to move the marbles in the left column to where they are in the right column?

- ◆ Mrs. C found 5 pairs of gloves and 3 toques in the lost and found. How many items did she find altogether?
- ◆ I have 13 players in a tennis tournament. I need 3 score keepers. How many games of tennis would be playing with the remaining players?

One example that implied the subtraction/comparison interpretation is the following: “There are two teams. One team has eight players. The other team has five. To play, each team must have five players on the court. How many subs [extra players] does each team have? Which team would be most tired?”

However, this participant’s thinking did not intentionally, or with awareness, focus on this interpretation, but on the context.

Learning From the Problem-Posing Experience

The participants focused on self-awareness in describing their learning. They became aware of what they could or could not make sense of, were uncertain of, and wanted to learn more about regarding problem posing and the mathematical concepts they encountered in the process. They developed awareness of the importance of context in problem posing. They realized that problem posing can be challenging and developed a different understanding of it and appreciation of its importance in learning mathematics. As one participant explained:

I learned how difficult it is to write math questions that are open-ended and require thinking rather than memorization.... I learned the differences between thoughtful questions and questions that I experienced that can make math stressful and boring for students.... I learned that math is not just memorizing multiplication tables and adding at the elementary level. It can be creative and have problem solving at a very young age.... I learned that by writing questions properly, students can be given the opportunity to share their own good ideas on how to deal with problems.... I learned how problem solving can be presented as more about memorization of skills, like the way I learned it, than about creating problem-posing abilities.

Participants also gained self-understanding of limitations of important aspects of their mathematics knowledge for teaching. The tasks required understanding of different mathematics concepts and provoked different ways of thinking about and reflecting on problem posing which allowed them to engage in mathematical thinking in a variety of ways.

CONCLUSIONS AND IMPLICATIONS

The participants’ sense-making of problem posing was dependent on their mathematical knowledge, imagination or creativity, and past experience with problem solving. They were challenged most by the tasks to pose questions that were

open-ended, related to a specific mathematics concept—meanings of multiplication—, and derived from a given picture of a mathematics concept—comparison subtraction—. These tasks conflicted with their prior experience that exposed them mainly to closed problems and one meaning of each arithmetic operation. Many of the participants were able to imagine and create interesting problem situations but, generally, their sense-making of posing “word problems” often excluded intentional or conscious consideration of mathematical structure or context of the problems or the relationship to the problem situation. The five perspectives of problem posing identified in the study—i.e., paradigmatic, objectivist, phenomenological, humanistic, and utilitarian—indicate ways of thinking about “word problems” and posing problem situations prospective elementary teachers can make sense of and thus provide a meaningful basis to build on to enhance their problem-posing skills for teaching. In spite of this range of perspectives, individually, the participants’ initial ways of making sense of problem posing on entering the education program was limited by their lack of experience with problem posing and exposure to mainly traditional ways of experiencing problems and problem solving.

The study suggests the need to attend to the problem-posing knowledge prospective elementary teachers bring to teacher education in addition to addressing problem posing as an explicit topic in order to help them to build on, reconstruct, and extend their sense-making of it. The five perspectives of problem posing provide a basis to compare and unpack their ways of problem posing. All five need to be explored in order to allow the teachers to understand how each could support or inhibit students’ mathematical understanding and mathematical thinking. The nine categories of problem-posing tasks provide a meaningful basis of prospective teachers’ self-understanding and self-study of problem posing. The examples provided of the participants’ thinking for three categories of tasks—i.e., open-ended, meaning of a concept, and picture of a concept—draw attention to potential areas of concerns that are important to address explicitly in teacher education. These examples, linked to mathematics concepts, also imply that it is necessary for problem posing to be integrated as part of prospective teachers’ learning of the mathematics concepts they are expected to understand for their teaching. Their relational understanding of such concepts is needed to support their problem-posing knowledge and vice versa. This blending of the two could allow them to develop the flexibility to engage students in problem-posing not only in terms of being able to create and select worthwhile tasks, but also on an impromptu basis during mathematical discourse and teaching problem solving.

REFERENCES

- Brown, S. I., & Walter, M. I. (1983). *The art of problem posing*. Hillsdale, NJ: Lawrence Erlbaum Associates.

- Duncker, K. (1945). On problem solving. *Psychological Monographs*, 58(5), i-113.
- English, L. (1997a). The development of fifth-grade children's problem-posing abilities. *Educational Studies in Mathematics*, 34(3), 183-217.
- English, L. (1997b). Promoting a problem-posing classroom. *Teaching Children Mathematics*, 3, 172-179.
- Gick, M. L., & Holyoak, K. J. (1983). Schema induction and analogical transfer. *Cognitive Psychology*, 15(1), 1-38.
- Kilpatrick, J. (1987). Problem formulating: where do good problems come from? In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 123-147). Hillsdale, NJ: Lawrence Erlbaum Associates.
- Kilpatrick, J., Swafford, J., & Findell, B. (Eds.) (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.
- Leung, S. K., & Silver, E. A. (1997). The role of task format, mathematics knowledge, and creative thinking on the arithmetic problem posing of prospective elementary school teachers. *Mathematics Education Research Journal*, 9(1), 5-24.
- Lowrie, T. (1999). Free problem posing: Year 3/4 students constructing problems for friends to solve. In J. Truran & K. Truran (Eds.), *Making a difference* (pp. 328-335). Panorama, Australia: Mathematics Education Research Group of Australasia.
- Lowrie, T., & Whitland, J. (2000). Problem posing as a pool for learning, planning and assessment in the primary school. In T. Nakahara & M. Koyama (Eds.), *Proceedings of the 24th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 247-254). Hiroshima, Japan: PME.
- National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- Silver, E. A. (1993). On mathematical problem posing. In R. Hirabayashi, N. Nohda, K. Shigematsu, & F. L. Lin (Eds.), *Proceedings of the 17th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 66-85). Tsukuba, Japan: PME.
- Silver, E. A. (1994). On mathematical problem solving. *For the Learning of Mathematics*, 14(1), 19-28.
- Silver, E. A. (1995). The nature and use of open problems in mathematics education: mathematical and pedagogical perspectives, *International Reviews on Mathematical Education*, 27(2), 67-72.

- Silver, E. A., Mamona-Downs, J., Leung, S., & Kenny, P. A. (1996). Posing mathematical problems: an exploratory study. *Journal for Research in Mathematics Education*, 27(3), 293-309.
- Stoyanova, E. (1998). Problem posing in mathematics classrooms. In A. McIntosh & N. Ellerton (Eds.), *Research in mathematics education: a contemporary perspective* (pp. 164-185). Western Australia, Australia: Edith Cowan University.
- Strauss, A., & Corbin, J. (1990). *Basics of qualitative research*. Newbury Park, United Kingdom: Sage Publications.

This document was originally published as Chapman, O. (2011). Prospective elementary school teachers' ways of making sense of mathematical problem posing. In B. Ubuz (Eds.), *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 209-216). Ankara, Turkey: PME.

Olive Chapman
University of Calgary
chapman@ucalgary.ca