



Research in Mathematics Education: Numerical Thinking

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Mathematics is part of compulsory education in every country and makes a broad contribution to cultural development, to preparation of the individual, and to social integration. This can be confirmed from a number of viewpoints. First, mathematics is a discipline which has responded to social and scientific needs in all civilizations over the course of history, providing instruments for constructing an intelligible world, based on reason, giving mathematics a predominant role in the cultural tradition of our society. Additionally, the very nature of mathematics, its essential characteristics, the notions and structures it comprises, contribute in a unique way to educating persons and to developing their faculties and abilities, as well as to cultivation of character. Finally, mathematics provides tools for research, for economic development and innovation, for creating wealth, for preparing qualified professionals ready for the world of work, making mathematics an important part of a society's heritage and of its tools for training and preparing its citizens.

These qualities make it perfectly sensible to speak of mathematics education both at the level of personal education and improvement as well as for its enrichment of society. Mathematics education proclaims the principle that all citizens should reach, through mathematics, the highest possible development of all their abilities – individual and social, intellectual, cultural and emotional (Rico & Lupiáñez, 2008).

Mathematics education is the object of research for the community made up of mathematics teachers and specialists in Mathematics Didactics. This research can revolve around diverse priorities, of which we highlight three that can be analyzed from the student's or the teacher's point of view, or from the contexts in which learning takes place (English, 2009). These priorities are:

- Life-long democratic access to powerful ideas¹,
- advances in research methodologies, and
- the influences of advanced technologies (p. 10)

When these priorities are analyzed from the three perspectives mentioned above, certain issues come to light that define the research agenda in mathematics education for the coming years. These issues have to do with, for example, characterizing the knowledge and skills that schoolchildren need in order to cope effectively in society; specifying the knowledge and skills teachers need in order for their students to develop this knowledge; the methodological options that can be incorporated in mathematics education, and the role played by new technologies in students' development of basic mathematics skills (pp. 12-17).

¹ Democratic access has to do with the idea of mathematics for everyone, but in every sphere where educational practice is played out. Powerful ideas have to do with mathematical knowledge that, as we said before, structures disciplined thought and has cultural value, which encourages personal development, and supplies tools for managing in society (English, 2009, p. 11).

In this monograph, we have gathered examples from some of today's research that explores these issues and others of interest to mathematics education, all with a common thread of numerical thinking². The papers address different viewpoints and contexts, the study of learning-related phenomena and school children's understanding of different notions in mathematics, difficulties in learning, problem solving, teaching techniques, use of educational resources, treatment of mathematics in textbooks, teacher training and psychological foundations of numerical thinking.

Fernando Hitt and Christian Morasse present research on advanced numerical and algebraic thinking in students from secondary education. The authors direct their attention to the concept of covariation as a step prior to the concept of function, they explore how students learn this notion, and demonstrate the importance of teaching techniques and resources to bring about this learning. The paper by M. Consuelo Cañadas, Encarnación Castro and Enrique Castro, within the context of problem solving, focuses on development of inductive thinking, presenting a model that analyzes such cognitive development and identifies areas which are more complex for school children.

José Luis Villegas, Enrique Castro and José Gutiérrez describe research carried out with students in the final year of a Mathematics undergraduate degree program; the purpose was to explore how students make use of different representation systems when solving optimization problems. The authors find evidence that relating and moving between different representation systems becomes essential to solving this type of problem. Regarding learning difficulties, the study by José Luis González, Luis Rico and Jesús Gallardo addresses the problem of numerical understanding in the transition from natural numbers to integers, taking relative natural numbers as an intermediate step.

In the article by Marta Molina, Encarnación Castro and Enrique Castro, the authors carry out a study with primary school children in order to explore the development of their understanding of equalities and numerical sentences. To do so, they present the children with tasks consisting of open equalities and true/false numerical sentences. Next, within the context of new technologies in the mathematics classroom, Maria del Mar García and Isabel Romero analyze how these impact problem-solving techniques and attitudes of students in secondary education. Bringing together arithmetic and geometry, Elena Fabiola Ruiz and José Luis Lupiáñez explore research on the notions of reason and proportion, and the learning difficulties involved; they carry out an empirical study which reveals some of these difficulties in primary school children in Mexico.

² Several of the authors belong to the research group "FQM193 Mathematics Didactics: Numerical Thinking", which forms part of the Andalusia Plan for Research, Development and Innovation, Regional Government of

Francisco Ruiz, Marta Molina, Jose Luis Lupiáñez, Isidoro Segovia and Pablo Flores focus their study on teaching innovations carried out in pre-service teacher training for primary mathematics teachers, in order to adapt it to the European Higher Education Area. The authors give examples of some of these innovations in working with future arithmetic teachers. Pedro Gómez also explores how teachers-in-training learn, but in this case for secondary education. He presents an instrument to help these future teachers develop their planning competencies. Based on the results of implementing this instrument in a training program, implications are described for designing and launching such programs, and for the role of the teacher trainer.

Isidoro Segovia and Enrique Castro focus their study on numerical thinking in estimation, and describe different related studies which were carried out in the research group FQM193. These studies reveal the importance of estimation for developing numerical thinking in school children. The paper by Alexander Maz and Luis Rico deals with the History of Mathematics, analyzing the treatment of negative numbers in Spanish textbooks used in the 18th and 19th centuries. This study reveals the status of Spain in this era in the context of science in Europe. Finally, the study by J. Domingo Villarroel addresses the complexity of numerical thinking from a multi-disciplinary perspective, and describes how it is possible to analyze and describe this complexity from the standpoint of experimental psychology, neuroimaging and developmental psychology.

All these studies help us to put into context the large number of phenomena which mathematics education entails, revealing that studies can be carried out that explore, characterize and analyze these phenomena from the perspective of Mathematics Didactics.

References

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Andalusia. The group's structure, composition, priorities in research, along with some of their results, can be accessed at: http://prensa.ugr.es/prensa/investigacion/grupos/index.php?accion=ver&id_grupo=199/