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An experience of teacher education on task design in Colombia

GEMAD

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We describe an experience in task design within an in-service secondary mathematics teacher education program in Colombia. Following a model known as didactic analysis, a team of researchers, educators, mentors and practicing teachers worked together in designing, implementing, assessing and reformulating secondary school mathematics tasks. We present here the main features of the framework on which the program is based, identify some of the characteristics of the experience lived by trainees, educators and researchers on task design during the first implementation of the program, and analyse the trainees' assessment on their own proposals of tasks and on the contribution of the program on their task design competencies.

Keywords: teacher training; task design; task implementation; cross-communities

Law sets curriculum autonomy in Colombia. Schools and teachers are fully responsible for curriculum design and development in all areas. Schools are expected to produce curriculum planning for each course and academic period and teachers are usually autonomous for designing and implementing the lessons they are in charge of. They often do so by producing what is known as "teaching guides": sets of tasks that they design or copy from different resources, and propose to students. Most pre-service teacher education programs in Colombia do not prepare future teachers in task design nor other practical questions; instead, they are based on theoretical approaches to education.

In this paper, we describe an experience in task design that emerges from an in-service teacher education program in Colombia, known as MAD (Master in Didactic Analysis). It is based on a model—didactic analysis— that enables trainees to design, implement and assess sequences of tasks on specific topics for which a constructivist view of students' learning is assumed (Gómez, 2007). Based on this model, a group of researchers, educators, mentors and in-service mathematics teachers have worked together in MAD. We use the term task as "anything that a teacher uses to demonstrate mathematics, to pursue interactively with students, or to ask students to do something" (ICMI Study 22, 2012, p. 10).

In what follows, we describe the main features of the framework on which the program is based, identify some of the characteristics of the experience lived by trainees, educators and researchers on task design during the first implementation of MAD, and analyze the trainees assessment on their own proposals of tasks and on the contribution of MAD on their task design competencies. In the final section, we reflect on the role of the different agents in the program.

Framework

MAD is a master's degree in mathematics education for in-service secondary mathematics teachers. We assume a functional view of school mathematics in MAD. This vision puts the focus on the usefulness of the mathematical concepts for solving problems in a variety of contexts. Students are expected to use their mathematical knowledge for this purpose. They are expected to develop their own cognitive strategies, manage different representations of the mathematical concepts, choose the best solution strategies, argue about their decisions and communicate fluently their thinking processes. This functional view of school mathematics is coherent with a constructivist approach to students' learning and can be implemented with different pedagogies. MAD does not explicitly promote any of them, since each trainee and his context impose their own restrictions. Nonetheless, there are some implicit methodological principles: it is considered that the good tasks are those that promote the active implication of students, imply the development of strategic knowledge for problem solving in a diversity of contexts, and require that students make decisions and justify them.

MAD is based on a model known as didactic analysis (Gómez, 2007; Lupiáñez, 2009). This model is a conceptualization of the activities that the teacher has to do in order to design tasks that seek to promote students' learning on a mathematics topic. It is organized around four interrelated analyses: subject matter, cognitive, instruction, and performance analysis. The didactic analysis begins with the identification of the student's knowledge for the topic at hand (see Figure 1). With this information, and taking into account the global planning of his course, the teacher determines the mathematics content he wants to work on and the goals he wants to achieve (Box 1 in Figure 1). The next step involves the subject matter analysis (Box 2), in which the teacher stresses the relationship among concepts, highlights its multiple representations, and distinguishes the phenomena from which they emerge. This information is used in the cognitive analysis, in which the teacher describes his hypothesis about how students construct their knowledge. The cognitive analysis involves the establishment of learning expectations, and the identification of the skills, reasoning, and strategies necessary to achieve those expectations, and of the difficulties, mistakes and obstacles students might face. This information allows the teacher to carry out an instruction analysis: the identification and description of the tasks that can be used in the design of the teaching and learning activities that will compose the instruction in class (Box 3). During the implementation, these tasks should mobilize students' knowledge in order to generate cognitive conflicts and promote the construction of meaning using the materials and resources available (Box 4). In the performance analysis the teacher observes, describes, and analyzes students' performance in order to produce better descriptions of their current knowledge (Box 5). After this process, the teacher can review the planning in order to improve the sequence of tasks for future implementations.

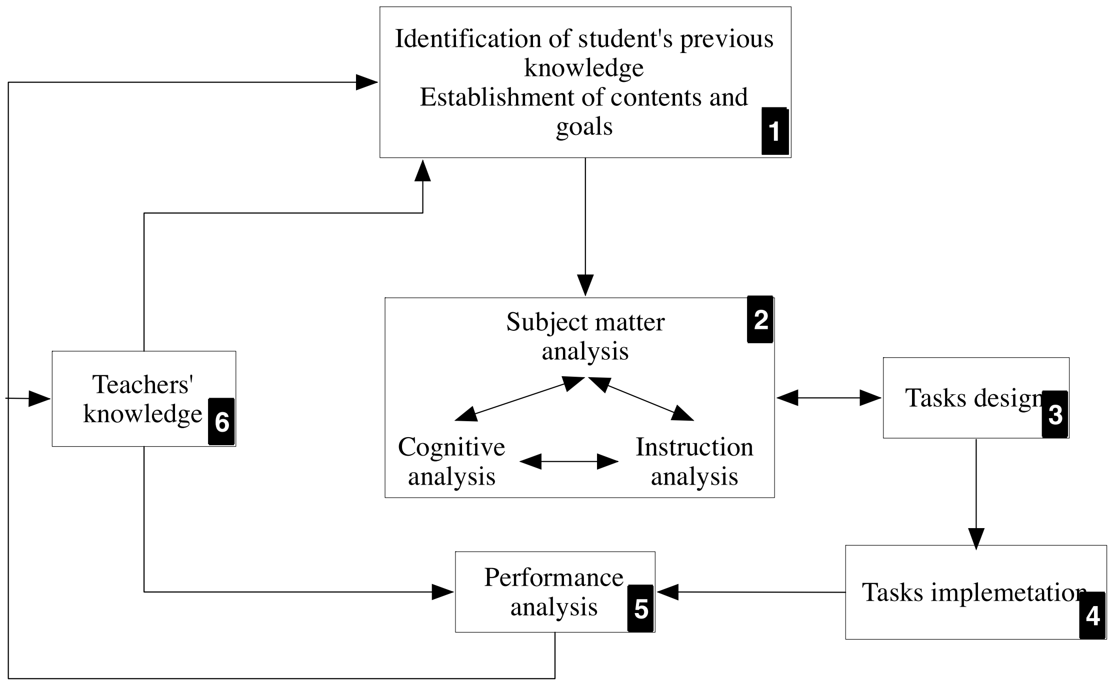

Figure 1. Diagram of a didactic analysis cycle

In this approach, trainees are expected to develop a deep enough knowledge of the topic so that they can support the choices and decisions they make for their lesson plan (Charalambous, 2008). This is a topic-specific knowledge that trainees are expected to develop by performing a series of activities during their training and contributes to the development of their didactic knowledge (Box 6).

Didactic analysis is a cyclic process in which trainees analyze a school mathematics topic with the purpose of designing tasks that provide the learning opportunities required for students to achieve the learning expectations. Trainees make decisions in different moments and with different purposes. When they describe in detail the topic from the mathematics point of view (the concepts and procedures involved, the forms of representing those concepts and procedures and the ways in which the topic organizes the phenomena that give sense to it), they produce and organize information about the topic that allows them to make decisions about those aspects that they consider relevant, about how to formulate and specify the learning expectations, about the capacities that can be used for characterizing those learning expectations, and about the mistakes that students can make when solving tasks related to the topic. The information that trainees produce form those decisions are the basis for further decisions about their anticipations about how students' learning can develop when they solve the tasks (Gómez & González, 2009). This process is based on a procedure that allows trainees to produce the learning paths of the tasks. Tasks' learning paths are a useful tool for assessing the effects of reformulating or extend the original tasks. For instance, trainees can make decisions about the material and resources that can be more effective for achieving the learning expectations, when they analyze the implications of their use in the learning paths of the new tasks. Based on the information about the capacities that characterize the learning expectations, trainees can establish the complexity of the tasks proposed and make decisions about how those tasks align with students' previous knowledge and about how to sequence the tasks. Trainees can also make decisions about the most effective ways of grouping the students and about how to foresight the teachers' performance when students begin solving the tasks and encounter difficulties. In summary, didactic analysis provides trainees with a systematic procedure for analyzing a school mathematics topic and sequentially making decisions that enable them to deepen in the different aspects of the topic and design and assess the tasks with which they pretend to contribute to the achievement of the learning expectations.

Since the information that trainees produce with the didactic analysis is complex and plentiful, when they make decisions, trainees might give priority to some decisions over others. For instance, they might focus on the treatment of the students' mistakes, so that their decisions will focus on ways of facing students with those mistakes and helping them to overcome them. For instance, they might concentrate on the search of representations, resources and possible teachers interventions that can help students overcome their mistakes and difficulties. In the empirical experience that we describe below, we show the concrete decisions that were made by a group of trainees in MAD.

MAD is set up from a social perspective of the trainees' learning (Gómez & González, in press). Trainees are organized in groups of 4 or 5 teachers. Each group has a mentor that accompanies it during the program's two years duration. Each group selects a school mathematics topic on which it will work during the program. The program is composed of 8 modules, two modules per semester. Each module begins with one week of face-to-face instruction in which the educator in charge of the module presents its theoretical basis and introduces the four activities that the groups have to carry out. Each activity spans over two weeks and requires the groups of trainees to analyze or produce information on their topic from a given perspective or with a given purpose. For instance, in an early activity, the groups produce the information concerning the representation systems of their topic. Later in the program, they analyze, in another activity, the role that the teacher had during the implementation of the tasks. For each activity, the groups produce a draft of their work at the end of the first week. They then receive comments on this draft from their mentors, and produce a final version of their work that they present to their pairs, the educators and mentors at the end of the second week.

Experience

In this paper, we report on MAD's first implementation that took place during 2010 and 2011. The 26 teachers that participated in MAD were working in public and private schools of Bogotá (Colombia) and its surroundings. They were organized in 6 groups that worked on the following topics: integers, linear equations (2 groups), straight lines in the plane, and trigonometric ratios (2 groups).

The design, analysis and selection of tasks are processes that span over the whole program. It begins with a first selection of tasks that is refined and improved with new ideas and analysis proposed by educators and mentors. The different activities of the modules structure this process. Once the groups have a tasks sequence that is ready to be taken to class, the groups implement, collect information on its implementation, analyze that information in order to assess the sequence's design and implementation, and make improvement proposals for future implementations. In what follows, we show a summary of the process for the specific case of Group 5, whose topic was trigonometric ratios. This group designed a sequence of five tasks distributed in 12 lessons. The first selection of tasks was guided by activities in which the group set up the learning goals, the contents they expected to cover, the materials they wanted to use and the context (personal, scientific, etc.) in which they wanted to place their proposal. This group focused the design of their tasks in the use of materials and resources, some of which were elaborated by the group itself. The group used the materials and resources for bringing together the tasks characteristics that they considered important for students' learning:

> Materials and resources play an important role in our task sequence because they allow us to motivate students in working with mathematics; they facilitate the achievement of the learning expectations; they promote mathematical communication and the construction of arguments; and they put into play different systems of representation.

Their pedagogical decisions focused in two aspects: (a) the grouping of students and (b) the communication in class. They proposed to use the tasks with different types of students' groupings: heterogeneous groups of three students (with high, medium and low achievement), big group, and, less frequently, individual work. In order to promote the classroom communication, they decided to use the following strategy: at the beginning of each task, the teacher shares its goals; then he induces students to create their solving strategies in small groups; the groups present their strategies and argue in favor of them to the whole group; once the task is finished, the teacher gives students follow-up and feedback on their performance. These decisions were guided and founded on their functional view of school mathematics and on the group's aims of contributing to the development of students' argumentation and justification competencies.

In what follows, we show in detail the design process of one of the tasks, named The streetlight height. In the following excerpt, Group 5 describes the task's features based on the subject matter analysis they have previously realized.

> We expect students to find the streetlight height by using trigonometric ratios, without direct measures. The task covers a conceptual content that includes elements and properties of right triangles and trigonometric ratios. It involves also some procedures: (a) identifying regularities and patterns, (b) formulating equations, (c) using the functional language trigonometric ratios, and (d) situations solving. The task design includes working guides, goniometers made of set squares of 45º and 60º, protractor, calculator and a metric strip. The task refers to a personal situation.

In MAD, once the groups make the first task proposal, educators and mentors introduce new elements of analysis. That is the case, for instance, of considering the concrete capacities that can be activated with the task or the mistakes that students can make when solving the task. The groups characterize the task in terms of these new elements. For example, once Group 5 produced a list of 35 capacities and 12 mistakes for their two learning goals (that we do not have space to include here), they produced a table in which they related the learning goals, the tasks in their sequence, the capacities that each task could activate and the mistakes that students could make when solving each task. Table 1 shows an excerpt of this analysis for Group 5.

Goal	Task	Capacities	Mistakes
2	Streetlight	1, 7, 3, 8, 12, 14, 17, 33, 35	2, 5, 7.3

Table 1. Relationships among learning goals, task, capacities and mistakes

This information led them to modify the task's wording and to determine how to implement the task in class:

> Before starting working with the task, the teacher explains the use of the goniometer. In the first phase, the teacher asks students to go the location in the school where the streetlight is and to calculate its height with the instrument. The goals of this phase of the tasks are twofold: (a) that students recognize the use trigonometric ratios for measuring unreachable distances and (b) that they connect the elements of the instrument with the particular situation at hand in order to represent it and calculate the distance. The students are asked to record their observations in the working sheets provided, including the steps that they took for calculating the length and a graphical representation of the situation. They produce a poster to share their work with the class group.

Once they produced the final design of the tasks sequence, the groups implemented them in class. The following are two excerpts of the balance that Group 5 made of this phase of the process.

> One of the minor changes that we made during the implementation of the tasks sequence concerned the time foreseen for each session. For most tasks, the time required was greater than we expected. We can claim that the Streetlight height task was effective as it was designed because we verified that students activated the capacities that we expected. Furthermore, we observed that they also activated capacity 17 that was not expected by us. On the other hand, we established that capacities 12 and 35 were explicitly activated on a given moment of the task's development, but that they were also present along the task, since students permanently used the trigonometric vocabulary and verified the relevance of their results, when measuring and making calculations.

On the basis of this kind of analysis, Group 5 decided the following improvements for the Streetlight height task:

> (a) to include instructions for the construction of the goniometers; (b) to incorporate an activity for measuring lengths that can be found directly and to use the trigonometric ratios to corroborate the results; (c) to show the diversity of theodolites that can be found, with a brief explanation of each one; (d) to ask students to select the theodolite that they think is best suited for each situation, and (e) to ask questions that can lead students to look for tools that are different to those proposed and to include new strategies of solution.

Trainees' assessment of the program

In the final part of the program, the groups of trainees that participated in MAD performed a SWOT analysis (Strengths, Weaknesses, Opportunities, Threats) of their work and made a personal assessment of the program and their participation in it. We reviewed these analysis looking for the common themes that characterized what the groups of trainees appraised as the most salient features of their work and the most important influences of the program in their capabilities for designing, implementing and assessing tasks. We consider that the trainees' claims represent those features of MAD and their work that highlight the differences between what they usually do in class and what they actually did when planning, implementing and assessing their tasks during the program. We found several themes that were mentioned by most groups. In what follows, we identify and exemplify the most frequent ones.

Features of the work

All groups centred the assessment of their sequence of tasks in terms of its contribution to the achievement of the learning expectations and the overcoming of the students' errors and difficulties. They also claimed that the designed tasks were adapted to the context. The Colombian curriculum guidelines underline the importance of problem solving in context and, hence, of tasks that lead students to solve and interpret mathematical problems in a variety of situations. Trainees claimed that one of the salient features of their tasks was the fact that they were mathematical problems set in varied non-mathematical contexts that contributed to their students' proficiency in problem solving. As Group 1 explained: "the use of tasks in context favored the achievement of the learning expectations proposed because the situations in the tasks were close to the everyday life of the students." Similarly, all group of trainees mentioned the importance of the use of materials and resources (i.e., Cabri, Geogebra, Hands on Equations) in their tasks design and implementation. Their statements make us think that they do not usually introduce those resources in their teaching: "when designing the materials with which we developed our task sequence, we found that they have a great potential for other topics and school grades" (Group 5).

All groups of trainees mentioned the importance in their tasks' design and implementation of using multiple ways of promoting collaborative work in class. They recognized the benefits of having students working in pair or groups and of generating class discussion among them. Some groups of trainees also mentioned the relevance of foreseeing the teachers' reactions to the students' performance in class, particularly to students' mistakes. The assessment made by Group 1 sustains such claim: "We acknowledge the benefits of the groupings proposed for students' work in class. For instance, as a consequence of the interactions produced, the students were able to strengthen their argumentative capacities for validating their results." Similarly, Group 2 claimed that "the tasks sequence involved a methodology that supports constructive learning of individuals and groups because it contributed to create a ZPD and strengthen the establishment of agreements when taking decisions concerning the challenges that were proposed."

The groups of trainees also mentioned some of the problems and deficiencies of their tasks. The most common shortfall referred to their mistakes when foreseeing the time required for implementing the tasks as mentioned by Group 5 above. On the other hand, some groups recognized that their students did not understand properly the wording of some of their tasks or that the tasks did not generate the student's performance that they were expecting. They recognized that, in some cases, they incorrectly assumed that the students had the previous knowledge required to face the tasks. For example, Group 3 acknowledged that "the wording of the instructions in one of the tasks was another weak point [of our task sequence]. This situation affected the time required for the task and the understanding that students developed when solving it." This assessment led them to propose new or modified tasks for a future implementation of the sequence.

Influences of the program

The groups of trainees highlighted the impact of MAD in their competencies for designing, implementing and assessing sequences of tasks. Trainees stated that the program provided them with tools for assessing how the tasks' design could achieve the planned learning expectations and how the tasks' implementation did in fact achieved them. Group 5 claimed that "the didactic analysis procedure lead us to be really conscious of the importance of planning a task sequence and assessing its relevance and effectiveness. We know better about the mathematics that our students should learn and how they should learn them. We recognize now that students' learning depends on the tasks they solve and that the teacher is the main responsible of that learning." In other words, the groups recognized that the program (a) provided them with a better preparation for developing teaching innovations based on a structured method; (b) encouraged them to reflect on their own practice; (c) showed them how to track the results of their lessons; (d) questioned their meanings of knowing and learning mathematics; and (e) motivated them to modify their role as teachers. The groups also reported that the program led them to introduce several data gathering instruments (some of them previously unknown to them—like the students' dairies and observation tables) that allowed them to properly assess the students' performance when solving the tasks. They also recognized that they did not have enough time for performing those procedures and analyze the information gathered.

Across-communities

The design and implementation of the program was a joint venture among researchers, educators, mentors and in-service teachers. Researchers have been working for several years on the development of the didactic analysis model and on a model for conceptualizing the trainees' learning of it. Some of the researchers were also educators in the program. Educators as a team have worked on the design of the program following the models proposed by researchers. They also implemented the program working hand in hand with trainees. This collaborative work was set up through a mentoring process in which mentors (the educators) interacted with their group of trainees weekly. Trainees' work informed researchers on the framework and principles, educators on their teacher education program design and implementation, and mentors on their performance. This joint venture has evolved beyond the program, as this paper shows. Teachers, educators and researchers have created a working group (that signs this paper), which continues working on mathematics task design an implementation. Some of the near-future results of this collaboration is a book with the reports of the groups' work in the program (Gómez, in press), the support of an international publisher for the publication of a set of teaching guides based on the groups' program's work, and the teaching of a course in the Colombian mathematics education congress.

MAD has generated several research studies in which researchers, educators and teachers have also collaborated. That is the case of studies in which we have explored, for instance, the teachers' learning of specific aspects of the program (Gómez & Cañadas, 2012; Suavita, 2012), the role of mentors (Arias, 2011), or the impact of the program in institutional planning (Gómez & Restrepo, 2012).

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