Nuances of the philosophy of mathematics

Nuances of the philosophy of mathematics

C E Flórez-Pabón[1]
[1] Grupo de Investigación Conquiro, Universidad de Pamplona, Pamplona, Colombia

E-mail: ceflorez@unipamplona.edu.co

Abstract. Historically, the term philosophy of mathematical education is associated with the term philosophy of mathematics. However, mathematics as a mathematical education has its own foundations, schools, currents, and authors. Thus, it is possible to visualize from different research that one's own research with foundations in the philosophy of mathematics is stronger than in the philosophy of mathematical education. In order to base the present research on the philosophy of mathematics education, we have made use of a mixed method, which involves a quantitative analysis of the references made in Scopus databases through their search algorithms, in addition to a qualitative design under the contributions of the onto-semiotic approach and the Kantian synthetic analysis. When trying to particularize the identity of the philosophy of mathematical education as an area of research in development, it is evident among the main approaches: (i) the identity process of the philosophy of mathematical education is still under construction; and, (ii) recognize the decoloniality of mathematical knowledge, imposed for centuries by Western mathematics, and backed by a single philosophy of mathematics.

1. Introduction

Interpreting mathematics involves interpreting different nuances of what it represents. Thinking about the training of mathematics teachers leads us to rethink the role that mathematics will play in their disciplinary education. In the search for an adequate balance for the comprehensive training of the future mathematics teacher, the curriculum will value contributions mainly from mathematics, pedagogy and didactics. This alludes to a dialogue of decades ago that brings to the debate the representations of both mathematics and education. On the one hand, mathematics traditionally turns to a logistic philosophy in order to solve this debate, giving more importance to an abstract identity of science. On the other hand, mathematical education exposes its identity from the understanding of the socio-cultural practices of those who participate in a training process in view of an adequate professional work.

Therefore, if both mathematics and mathematical education have a unique identity as an area of knowledge, then would it make sense to speak of a philosophy proper to both mathematics and mathematical education? This paper intends, based on the observations of B. Russell [1], to understand the particularities that refer to these areas of knowledge. For this we will use a mixed research method. On the one hand, we start from a quantitative approach referenced in the Scopus databases, which identifies the state of the art of the subject to be dealt with. And, based on this, we apply a qualitative design under the contributions of the onto-semiotic approach and the Kantian synthetic analyses.

2. Theoretical framework

The first consideration that we must keep in mind in the referential framework is that this idea of the article is based on the famous distinction developed by Russell in 1919, relying on the distinction between mathematical philosophy and the philosophy of mathematics, think that this paper is out of

place in the mathematical philosophy. Idea that makes us think of another possible distinction in this area of knowledge. That is, on the one hand, to propose the philosophy of mathematics which points to applied mathematics, and it is closer to "mathematics philosophy" [1] and on the other hand, to propose a philosophy of mathematical education that reflects on practical philosophy in the classroom that looks at the philosophy of mathematics, according to the Russellian proposal of the twentieth century.

Secondly, it is not necessary to agree with this nuance that we are proposing in the investigation. Neither It is not necessary to agree with what he there suggests as to the readjustment of the field of philosophy by the transference from it to mathematics of such problems as those of class, continuity, infinity, in order to perceive the bearing of the definitions and discussions that follow on the work of "traditional philosophy" [2]. If philosophers cannot consent to relegate the criticism of these categories to any of the special sciences, likes mathematics, it is essential, at any rate, that they should know the precise meaning that the science of mathematics assigns to them, in which these concepts play so large a part [3-5].

Thirdly, if, on the other hand, there be mathematicians to whom these definitions and discussions seem to be an elaboration and complication of the simple, it may be well to remind them from the side of philosophy that here as elsewhere, apparent simplicity may conceal a complexity which it is the business of somebody, whether philosopher or mathematician, or, like was Bertrand Russell [1], both in one, one mystery to unravel; and that allows us to raise the idea of specialized knowledge of the professor of mathematics (mathematics teacher specialized knowledge (MTSK)) [6] as Carrillo and colleagues will suggest in another perspective later in the history of mathematical practices today. But above all, which is allowed to propose this study is that update and differentiation of this area of knowledge proposed by Russell, and wondering what it would be today, where it comes from and what affects areas especially in philosophy [7] because it must be clear that in the mathematical perspective part of the mathematical education, as you can see in Figure 1. That is, to recognize the particular identity that the study of the philosophy of mathematical education deserves.

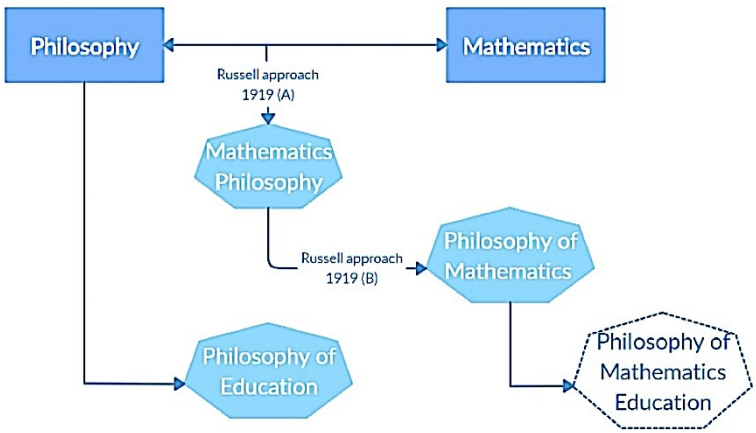

Figure 1. Philosophy's mathematics approach.

Fourthly, we will think that this differentiation and approach that we are considering from the reading of philosophers who were mathematicians, and mathematicians who were philosophers, allows us to raise the concept of truth in these assumptions, typical of mathematics but also of philosophical epistemology, remembering Kürbis' assumptions [8]. Having rejected all these possibilities, Kürbis resorts to the last one, which he considers the only viable possibility: to add falsehood to the proofs of truth he considers. He is convinced that the theory of proof is based, at least implicitly, on the concept of truth, and that we are free both to add falsehood and to embrace it without committing ourselves to embrace the explicit semantics of the functions of truth. And if this is so, in the constructions of language and its differentiation as discourse, new approaches are made that in this discourse seek truth. Truth that we can build as a foundation for a better exercise of teaching mathematics from philosophy, reviewing

our practices, knowledge, pedagogies, didactics, experiences and above all attitudes and beliefs that lead to a quality education with an integral sense [9].

Ideas that have to question the teaching work of the math teacher. His role in society, inside and outside the classroom, but above all his role as citizen educator. As Ernest and Skovsmose proposes, through a series of concerns which are focused on the notions and future of critical mathematical education, which are: mathematics, students, teachers, and society. Since mathematics can be put into action in technology, in production, in automation, in decision making, in management, in economic operation, in daily routines, in information, communication processes, in security procedures, etc. This implies that the factual mathematics in action play an important role in all areas of life, and this becomes a concern of fundamental mathematical education to address mathematics in its very different forms of applications and practices. There are no qualities - such as objectivity and neutrality - that can automatically be associated with mathematics. Mathematics-based actions can have all kinds of qualities, as Professor Skovsmose recalls, being risky, reliable, dangerous, suspicious, deceptive, expensive, brutal, benevolent, the action based on mathematics, which can serve any type of interest. As with any form of action, so math in action is also in need of being carefully criticized. This applies to any form of mathematics: mathematics, daily engineering mathematics, academic mathematics, as well as ethno mathematics, among many other varieties [9].

Ideas that have to question the teaching work of the math teacher. His role in society, inside and outside the classroom, but above all his role as citizen educator. As Skovsmose proposes, through a series of concerns which are focused on the notions and future of critical mathematical education, which are: "mathematics, students, teachers, and society". Since mathematics can be put into action in technology, in production, in automation, in decision making, in management, in economic operation, in daily routines, in information, communication processes, in security procedures, etc.

This implies that the factual mathematics in action play an important role in all areas of life, and this becomes a concern of fundamental mathematical education to address mathematics in its very different forms of applications and practices. There are no qualities - such as objectivity and neutrality - that can automatically be associated with mathematics. Mathematics-based actions can have all kinds of qualities, as professor Skovsmose recalls, being risky, reliable, dangerous, suspicious, deceptive, expensive, brutal, benevolent, the action based on mathematics, which can serve any type of interest. As with any form of action, so math in action is also in need of being carefully criticized. This applies to any form of mathematics: mathematics, daily engineering mathematics, academic mathematics, as well as ethno-mathematics, among many other varieties [10,11].

However, as Ernest proposes, these applications involve philosophy as a substantive body of knowledge [9]. In fact, philosophy, mathematical education and other areas of knowledge encompass research and practice processes, personal knowledge, and representations of knowledge. These are not simply substantial entities in themselves, but complex relationships (of power) and interactions between people, society, social structures, representations of knowledge and communicative, discursive practices, among others as Valero and colleagues also suggests [12] Thus, an additional broader sense of the philosophy of mathematical education includes the applications of philosophical processes, methods and modes of critical thinking which makes it constitute a different branch of the philosophy of mathematics and fully configure itself as a philosophy of mathematics education.

3. Method

The focus of this research is mixed, because it starts from a quantitative approach to develop a qualitative differentiation, starting from a state of the art of the subject to be dealt with. Where the categories of analysis of this research were privileged through a search algorithm in the Scopus platform. That is to say: philosophy of mathematics and philosophy of mathematical education. As can be seen in Figure 2 of the document in question and statistical figuration was made by year and topic.

On the other hand, for the analysis of each one of these graphs it was thought from the onto-semiotic approach [13] was thought as an inclusive theoretical system that tries to articulate diverse approaches and theoretical models used in research on Mathematics Education based on anthropological and

semiotic assumptions about mathematics and its teaching. This thought allows us to qualitatively review the graphs from the perspectives of Kantian analysis and synthesis in the search for new exploratory knowledge. They take up again the analytical and synthetic part of what is called the combined regressive method as referred to in Figure 2.

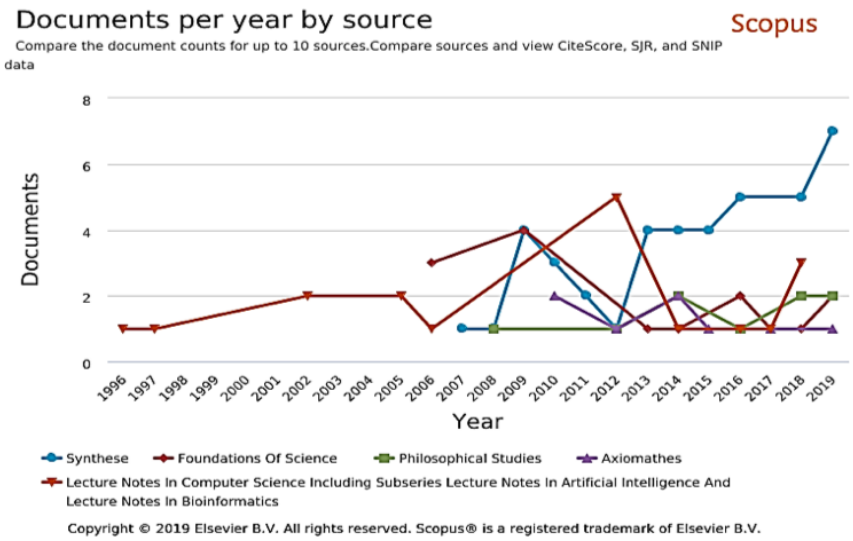

Figure 2. Graphic development of the search algorithm in Scopus on philosophy of mathematics and philosophy of mathematical education.

This analysis represented in Figure 2, shows the method of organization of mathematical practices as a synthesis. In other words, the constituent parts of Figure 2 are both the problem and the path to the solution of the problem, as an integrated cyclical problem. This is what we call 'synthesis" [14].

4. Results

Education is going through multiple changes, marked by different nuances of contexts, disciplinary areas, and research results. Mathematical education is not unconnected with this. In this sense, the results of this paper refer to the multiple nuances that the philosophy of mathematics has taken on and its implications within mathematical education and consequently, the particularization of specific knowledge (of mathematics) about the philosophy of mathematics education.

First, the philosophy of mathematical education does not yet have a defined identity of its own as an area of knowledge that depends on mathematics. However, approaches made by Skosmovse, Ernest and Valero, show the need to deepen the constitution of such identity.

Second, this work constitutes a basis for the development and innovation of research where innovative frontiers of successful experiences are proposed in other sciences such as mathematics to be applied to the humanities [15], but especially in the area of mathematical philosophy which responds to the philosophy of education and the philosophy of mathematical education. For this purpose, the "finished" knowledge is being deepened by closing the professional training cycle, but at the same time it implies that it be updated and adapted to what students of bachelor's degrees in mathematics as well as in philosophy, since they have within their own reasoning the capacity from these perspectives for the resolution of the problems raised. Either from mathematical logic, or from critical thinking. Which helps to build a new pedagogy [16], an integral professional, with quality and with a sense of educator of citizens. This is a task that not only belongs to the humanities teacher but is built with the contributions of the entire academy.

Third, thinking about the philosophy of mathematics, and the philosophy of mathematics education with proximity and community areas in the exercise of the teacher of mathematics or philosophy, contributes positively to improving teaching practices. Especially in mathematics, and allows to put into

perspective the nuances so that these differentiations are a pedagogical aid for the teaching of philosophy. Presenting a symbiotic relationship and creation of academic fabric. For the transversal axes of teaching seen as successful experiences mark the pedagogical exercise of both philosophy and other sciences. This proposes a change of mentality, and at the same time a specific knowledge in the respective areas of knowledge - mathematics and philosophy - so that they can perform properly and ask for help from the other area whenever it is needed in a dialogical sense.

Fourth, it is worth noting that socio-cultural approaches to mathematics (*i.e.* critical mathematical education [5]), allow to identify other limits of so-called "traditional" mathematics. Which implies recognizing the decoloniality of mathematical knowledge, imposed for centuries by Western mathematics, and backed by a single philosophy of mathematics. Consequently, mathematics education and its nuances have gained their own identity and need their own philosophy.

Fifth, one of the results shown by this research is that we have identified own elements to mathematics (field of knowledge) that influence the development of mathematical education (field of action of mathematical knowledge). However, this means that mathematical practices make invisible or minimize actions or research in mathematical education. For example, the philosophy of applied or pure mathematics is more recognized than that of mathematical education itself, because it is a field of action-research still in development.

Sixth, as shown in Figure 2 of this analysis, there is a clear trend towards publication and research on this subject. Figure 3 and Figure 4 represent the results produced by the algorithm when performing the search in Scopus on philosophy of mathematics and/or philosophy of mathematical education, both by affiliations and by territory.

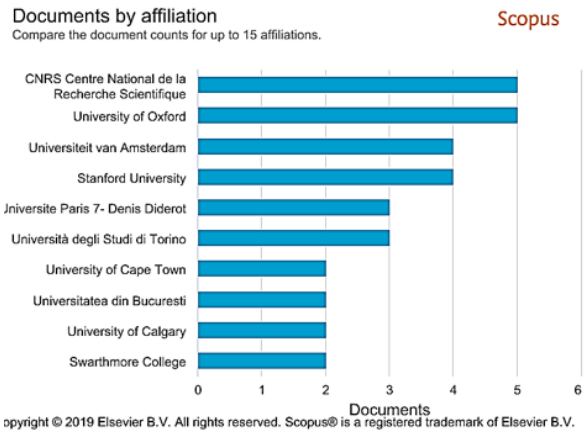

Figure 3. Graphic development of the search algorithm in Scopus on philosophy of mathematics and/or philosophy of mathematical education by affiliations.

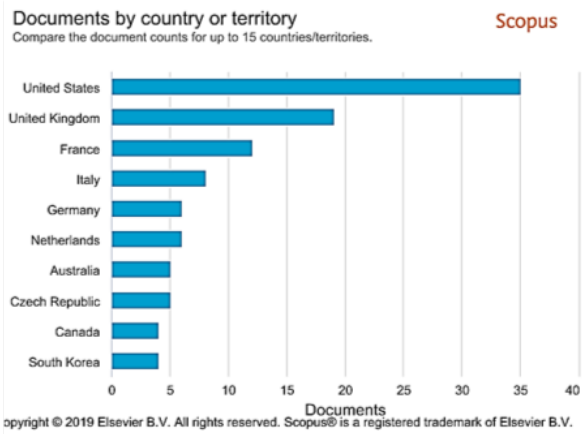

Figure 4. Graphic development of the search algorithm in Scopus on philosophy of mathematics and/or philosophy of mathematical education by territory.

This implies that other academies are thinking in a similar way, as can be seen in Figure 3 of this document. Ratifying the importance of the discussion of these topics that are trends in the academic world of mathematics but also of the human sciences as observed in Figure 4.

However, it should be noted that although there is a growing trend in this subject of mathematical education, of the philosophy of mathematics, by proposing a differentiation in the sense that we are updating B Russell [1], the philosophy of mathematical education does not appear clearly enunciated in spite of the fact that authors such as Paul Ernest or Skovsmose have already enunciated some principles that would support it. This conclusion allows us to recreate the recommendation of the work in this area of research.

5. Conclusions

On the one hand, we must conclude that through the use of algorithms developed by the Scopus platform, we can show how researchers currently associate the term philosophy of education with the philosophy of mathematics. However, these data do not demonstrate that a differentiation can be made in these subjects, but rather that they are treated as if they were the same area of work belonging to the epistemology of the sciences. Nevertheless, writings such as the present one attempt to draw a thin line that can distinguish these two areas that in other times were not perceptible to mathematical educators. Since everyone believed that talking about philosophy of mathematics and logic covered most of the topics that involved mathematical education in their professional development. But as we have seen throughout this paper, there are different approaches, schools, and authors that allow us to differentiate between these areas of research in mathematics and philosophy, of course.

On the other hand, the fact that there is no differentiation between philosophy of mathematical education and philosophy of mathematics allows us to draw an erroneous conclusion in this area of knowledge. For, since there is no epistemology that differentiates these correlated concepts, it is not based on them as an area of work, making this area of knowledge imperceptible in the exercise of mathematical education. Consequently, the philosophy of mathematics and logic are stronger as an area of research than the work in mathematical education, epistemologically speaking, which would be an error caused by the lack of differentiation of these areas of knowledge.

This would be a mistake caused by the lack of differentiation of these areas of knowledge. In addition, it should be mentioned that thanks to these differentiating practices, it would be possible to validate the discussion of concepts such as professional identity, professional profile of the mathematics educator, the reconstruction of a mathematics curriculum not colonized by the West and open to other mathematical practices of the region, among others. This allows us to consider in the exercise of mathematics different shades of knowledge and not only one self-imposed by Western tradition.

Finally, we can express through the reading of the results of this research that education is undergoing multiple changes, and mathematical education is not alien to this phenomenon with the new times. This allows us to project ourselves to investigate the same sciences that we work on and to innovate the research processes so that they are better and effective in terms of what they are or are not concerned with, and to be able to export these experiences to other areas of knowledge, not to mention other areas that ask the same questions in different contexts in what concerns the work of the mathematics educator. Helping to form comprehensive and critical professionals in their thinking through the differentiations that are proper to their area of knowledge and that are built with the contributions of the different sciences, in this case philosophy.

References

[1] Russell B 2010 *Introduction to Mathematical Philosophy* (London: George Allen & Unwin Ltd.) p 361
[2] Ribeiro C M 2013 Del cero hasta más allá del infinito – algunas perspectivas desde el comienzo de la tesis doctoral hasta el futuro "también" a largo plazo *Investigación en Educación Matemática XVII* (Bilbao: Universidad del País Vasco) p 71
[3] Flórez-Pabón C E, Acevedo-Rincón J P 2018 Challenges and Perspectives of the Philosophy of Mathematics Education *Journal SYLWAN* **162(9)** 9
[4] Ravn O, Skovsmose O 2019 *Connecting Humans to Equations A Reinterpretation of the Philosophy of Mathematics* (Switzerland: Springer)
[5] Valero P Andrade M, Montecino A 2015 Lo político en la educación matemática de la educación matemática crítica a la política cultural de la educación matemática *Revista Latinoamericana de Investigación en Matemática Educativa* **18(3)** 3
[6] Carrillo J Climent N Contreras L Montes L Flórez-Medrano E Escudero-Ávila D Vasco D Rojas N Flores P Aguilar-González A Ribeiro M, Muñóz-Catalán C 2018 The mathematics teacher's specialised knowledge (MTSK) model *Research in Mathematics Education* **20(3)** 236
[7] Pierce C S 2010 *Philosophy of Mathematics Selected Writings Bloomington* (Indiana: Indiana University Press) p 335
[8] Kürbis N 2019 *Proof, Falsity A Logical Investigation* (Cambridge: Cambridge University Press) p 308

[9] Acevedo-Rincón J P, Flórez-Pabón C E 2019. TelEduc: A virtual learning environment for teaching and learning at the University of Campinas *J. Phys.: Conf. Ser.* **1161(012023)** 1
[10] Font V 2005 Reflexión en la clase de Didáctica de las Matemáticas sobre una "situación rica" *Unidades didácticas en Ciencias y Matemáticas* ed Badillo E, Couso D, Perafrán G, Adúriz-Bravo A (Bogotá: Editorial Magisterio) 33
[11] Stake R E 2000 Qualitative case studies *The SAGE Handbook of Qualitative Research* ed Denzin N K & Lincoln Y S (Thousand Oaks: Sage Publications) chapter 17
[12] Ernest P Skovsmose, O van Bendegem, J P, Bicudo M, Miarka R, Kvasz L, Möller R D 2016 *The Philosophy of Mathematics Education* (Switzerland: Springer)
[13] Godino J D 2002 Un enfoque ontológico y semiótico de la cognición matemática *Recherches en Didactiques des Mathematiques* **22(2-3)** 237
[14] Falkenburg B 2018 Kant and the scope of the analytic method *Studies in History and Philosophy of Science Part A* **71** 13
[15] Acevedo-Rincón J P, Flórez-Pabón C E 2019 *Lesson Study en Filosofia: Posibilidades y Desafíos para la Formación de Profesores* (Bogotá: Editorial Universidad Minuto de Dios) p 312
[16] Ball D L 2000 Bridging practices: Intertwining content and pedagogy in teaching and learning to teach *Journal of Teacher Education* **51(3)** 241