Ramírez, R., del Río, A. y Flores, P. (2018). Mathematical talent in Braille code pattern finding and invention. *Roeper Review*, *40*(4), 255-267
https://doi.org/10.1080/02783193.2018.1501782
Submitted version.

Mathematical talent in Braille code pattern finding and invention

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Abstract

The recognition of patterns as well as creativity are two characteristics associated with mathematical talent. In this study, we analyze these characteristics in a group of 37 mathematically talented students. The students were asked to find the pattern the Braille code had been built upon and reinvent it with the aim of making its mathematical language become more functional. Initially, the students were unable to identify the formation pattern of Braille, but after experiencing the difficulties that blind people face when reading it, they recognized the generating element and the regularity. The results were in contrast with those of a control group, and it is noted that the students with mathematical talent were more effective in using visualization to identify the regularity of the pattern and their invention proposals were more sophisticated and used less conventional mathematical content.

Keywords: Braille, Creativity, Giftedness, Mathematical Talent, Pattern finding

Introduction

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Research on support for talented populations encourages teachers to develop teaching strategies that address the needs of gifted students and highlights the need for systematic processes that allow teachers to design enrichment tasks according to the detected characteristics (Delcourt & Renzulli, 2013; Fox, 2013).

Identifying a student's specific characteristics of mathematical talent is a complex task for which mathematically appropriate instruments are required (Pitta-Pantazi & Christou, 2009). A topic of interest in research forums on mathematical talent education is to contribute to the development of non-psychometrics tools to identify these characteristics (Pfeiffer & Blei, 2010; Singer, Sheffield, Freimann & Brandl, 2016). Specifically, the role of using visualization when solving problems is a topic that requires further research (Presmeg, 2006). In mathematics education, the need to design appropriate teaching techniques to favor the use of visualization has been emphasized (Arcavi, 2003; Bishop, 1980; Gutiérrez, 1996; Hershkowitz, 1990). Numerous research studies have shown that students with mathematical talent demonstrate little willingness to use visual methods and do not show a better use of visualization over other students (Krutetskii, 1976; Lee, Ko & Song, 2007; Neria & Amit; 2010; Presmeg, 1986; Ryu, Chong & Song, 2007). From a psychological approach a connection between giftedness and visual-spatial talent has been established and classroom activities that highlight the use of visual images are demanded (Kalbfleisch & Gillmarten, 2013; O`Boyle, 2008). However, several renowned collections of articles focused on the role of visualization on gifted students, point out that there exist few programs for individuals to express their visual-spatial talent (Kalbfleisch, 2013), despite their significance for STEM (Science, Technology, Engineering, Mathematics) studies (Andersen, 2014).

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Taking on tasks that bring graphic and symbolic representation systems into play requires the use of visualization. Tasks associated with searching for and creating patterns have these characteristics and, therefore, become an opportunity to detect the use of visualization in talented students. The search for patterns is associated with characteristics of mathematical talent, such as the ease of generalization and a higher level of mathematical reasoning (Amit & Neira, 2008; Lee & Freiman, 2006). Presenting such tasks in visual contexts allows teachers to detect students' different ways of using visualization (Duval, 1999; Rivera, 2007; Vale, Pimentel, Cabrita, Barbosa & Fonseca, 2012). In addition to being considered important skills in any type of education (National Council of Teacher of Mathematics, 2000), the study and creation of regularities and patterns play a fundamental role in the mathematics discipline.

This creation of regularities is also connected with mathematical talent, since creativity is considered to be one of the main components of giftedness (e.g., Sternberg & Renzulli, 2004). In the mathematical domain, creativity supposes a specific, clear and distinct ability to create in this area (Piirto, 1999). Originality is one if the aspects that characterize this creativity (Torrance, 1974) which can be evaluated according to the conventionality of a solution. Leikin and Lev (2007) showed that the creativity of gifted students was higher than that of regular students on every type of task. And that their creativity differed from that of their expert non-gifted counterparts on non-routine tasks only. On those real applied problems of non-routine tasks, creatively gifted students have an unusual ability to generate novel and useful solutions, and creates a sense of ownership of the content (Ali, Akhter, Shahzad, Sultana & Ramzan, 2011; Chamberlin & Moon, 2005). Open-ended problems promote that

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these students cultivate divergent thinking and develop their creative thinking skills (Matsko & Thomas, 2014; Kwon, Park & Park, 2006). In school mathematics, depending on the curricular contents of the various proposals, inventing patterns can be an instrument to determine mathematical creativity. Students are certainly capable of offering new insights of inventing pattern's creativity (Sriraman, 2005).

In order to provide evidence in the identification of mathematical talent and to contrast the answers of these students with those of non-talented students, we must first contextualize the tasks regarding the recognition of patterns in the Braille code. Initially, the understanding of the logic upon which Braille is built does not require mathematical knowledge or complex numerical properties, thereby avoiding a bias towards students with mathematical talent. The cognitive perception perspective in patterning supports strategies of both constructive generalization and deconstructive generalizations (Rivera & Becker, 2008) allows the detection of different cognitive levels regarding visualization (Barbosa & Vale, 2015). In addition, code is useful because it provides relevant information on pattern searching, which gives a prominent role to the contextualization of a task.

In this paper, we describe how talented students between 14 and 17 years old recognized the pattern upon which the Braille code is built and how they created their own mathematical language for visually impaired people. To identify information relevant to mathematical talent, the answers from the group studied were compared with those of a control group of the same age. We focused on two characteristics of mathematical talent, namely creativity and the search for patterns. We extrapolate on issues relevant to the following questions: What characteristics do mathematically talented students

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have in being able to visually recognize regularities and identify patterns? How is creativity used in inventing patterns?

Theoretical Background

The characterization of talent as a student's potential allows us to consider the process of acquiring these qualities (Gardner, 2001). In relation to a high level of mathematical ability, and with the intention to consider potential capabilities, we use the term "mathematical talent" in the sense defined by Marland (1972), referring to students who have demonstrated specific skills in the area of mathematics. For this author, gifted and talented students are those who,

> (..) by virtue of their outstanding abilities, are capable of high performance. These are children who require differentiated educational programs and/or services beyond those normally provided by the regular school program in order to realize their contribution to self and society. Children capable of high performance include those with demonstrated achievement and/or potential ability in any of the following areas, singly or in combination:: 1) general intellectual ability, 2) specific academic skills, 3) creative or productive thinking, 4) leadership skills 5) visual and performing arts, 6) psychomotor skills (Marland, 1972, p. ix).

With the purpose of defining these specific skills, several studies have listed particular traits that indicate talent in the area of mathematics. Following the exhaustive pioneering studies of Krutetskii (1976), which determined the different components of mathematical talent, other studies have

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documented various characteristics of high mathematical ability, one of which is pattern finding (Freiman, 2006). Other closely related qualities of talent are generalization, the manipulation and organization of data (Greenes, 1981), flexibility and creativity (Miller, 1990), and paying attention to detail in order to locate solutions to problems and develop efficient strategies (Freiman, 2006). Mathematical work with patterns can develop in two different ways: recognizing collections that have similarities and recognizing and sorting objects' sequences according to a regularity (Castro, 1995).

The study and creation of regularities and patterns is an integral part of the mathematics curriculum and enables the intuitive understanding of expressions and relationships that can be used in further mathematics studies. An interesting aspect of mathematics lies in creating and recognizing patterns to solve problems, thus trying to create general laws from particular cases and their combinations (Polya, 1966). The characteristic of "pattern finding" is intended to help students find what is common, what is repeated regularly at different events or situations, and what is expected to be repeated again. In experimentation with patterns, the connection between algebra and generalization has been particularly emphasized and patterns are seen to promote the development of algebraic thinking. Most studies have focused on linear thinking, while only a few have studied non-linear thinking (Amit & Neira, 2008).

Prior to making a generalization, at the pattern recognition stage students must either recognize an invariant property or relationship within the pattern, find something in common, or establish a regularity (Rivera & Becker, 2008). In this initial phase, visualization, understood as the organization of a chain of units (words, symbols and propositions), implies understanding the relationships of a structure (Duval, 1999). Seeing a pattern is necessarily the first step in searching for a regularity. Thus, the use of a visual aid in presenting problems involving the search for patterns can lead to the application of different approaches to achieving generalization (Barbosa & Vale, 2015).

However, the effective use of visualization in the search for non-numerical patterns, such as dot configurations, is not clearly associated with mathematical talent. The pioneering studies of Krutetskii (1976) found that the ability to make generalizations about mathematical objects, relationships and calculations is a component of mathematical talent. Nonetheless, the study did not find that the ability to visualize abstract mathematical relationships or the ability to visualize geometric spatial concepts were necessary skills in students with mathematical talent. Traditionally, research has highlighted the low preference for visual methods by students with mathematical talent (Presmeg, 1986). However, more recent research has shown significant evidence for the relationship between visual perception ability and mathematical ability (Rabab'h & Veloo, 2015; Rivera, 2011). Reasoning in visual tasks requires the cognitive ability to work within a space that has no safe routines, and this characteristic hinders visualization (Arcavi, 2003). In the case of patterning tasks that involve figural cues, the study of cognitive perception allows us to investigate how students see aspects of patterns they find relevant. Cognitive perception goes beyond the sensory when individuals see or recognize a fact or property in relation to the object (Rivera, 2007).

The study of visualization skills to understand the mental or physical actions the images are involved with is often included in mathematical education (Gutiérrez, 1996). Del Grande (1990) compiles seven skills of spatial perception that have operational results to describe the visual processes

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used by students with mathematical talent in task resolution (Ramírez, 2012; Ryu, Chong & Song, 2007): eye-motor coordination, figure-ground perception, perceptual constancy, position-in-space perception, perception of spatial relationships, visual discrimination and visual memory. In this paper, we consider these skills when describing the visual processes necessary in searching for regularities in a task.

On the other hand, in school mathematics, creativity is closely related to talent, especially regarding the finding of original solutions to a given problem (Liljedahl & Sriraman, 2006; Stenberg, 1997), however there are multiple perspectives about the nature of this creativity (Mann, 2006). For example, divergent thinking (Guidford, 1967) is described as flexible thinking and involves the creative generation of multiple answers to the problem. On the other hand, Torrance (1974) defined fluency, flexibility, elaboration and originality as the main components of creativity. In a mathematical context we need to distinguish between creativity as a quality of advanced research mathematicians and creativity in school mathematics, where students are not expected to produce extraordinary works, but offer new insights of their own previous experiences and comparing with the production of their classmate (Sriraman, 2005). This author defines mathematical creativity at school level as a process that results in original (insightful) solutions to a given problem and/or approaches to an old problem from a new perspective. In order to evaluate these solutions we will use Torrance's conception of creativity as divergent thinking. In problem solving, Silver (1997) characterized fluency as generating multiple ideas to a problem, flexibility as generating new solutions when at least one has already been produced, and novelty as exploring many solutions to a problem and generating a new one. Originality, in relation to novelty, has been highlighted in several studies as the strongest component in determining

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creativity (Leikin, Levav-Waynberg & Gubermann, 2011, Vale et al., 2012). In this paper, we analyze novelty to identify the creativity in the students' responses in pattern creation tasks.

In order to analyze the originality from a mathematical point of view, (Leikin, 2009a) it points out two categories that had previously been used in research on creativity in posing and solving problems: sophistication and alignment to the curriculum. Based on Ervynck's (1991) levels, the higher stage, employs sophisticated methods, usually based on assumptions embedded in the problem, and makes use of the problem's internal structure. As for aligning with the curriculum, Leikin (2009a) uses in his notion of solution spaces, the idea of unconventional to consider the use of mathematical content not required by the school curriculum.

Based on the research findings presented above, we can expect students with mathematical talent to be more capable at recognizing algebraic patterns and to show greater creativity. However, there is no consensus that students should show greater potential in the recognition of visual patterns.

The aim of this paper is to provide information about the ability of students with mathematical talent to visually recognize regularities and create patterns. A qualitative analysis was carried out to show how students with mathematical talent solved tasks related to the recognition and creation of patterns. To do this, we initially described the process upon which the Braille code had been built, and then analyzed the visualization skills necessary for the visual recognition of regularity. Subsequently, the research process describing how the tasks were presented is described. For each of the tasks, the answers from students with mathematical talent were compared with those from a control group. To understand the differences, we analyzed the categories related to the recognition of regularity, the identification of the repetition pattern, and the novelty in the construction of a new code. Finally, we conclude with an overall discussion and recommendations.

Methods

Participants

For the design of the session and the analysis of the results, we conducted a teaching experiment with a single episode of teaching involving two researcher teachers and an observer researcher (Steffe & Thompson, 2000). The same teaching session took place at two different times with the two groups of students—the mathematically talented students and the control group. The mathematically talented students consisted of high school students in the ESTALMAT Project (mathematics talent stimulation) in Eastern Andalucía (Spain).

In this project, selected mathematically talented students were enrolled in the program through a math test consisting of original and varied problems divided into sections by levels of increasing difficulty. The test valued students' aptitude and attitude over mathematical knowledge. Students received three-hour enrichment sessions on Saturdays outside of school hours. The group consisted of 37 Caucasian students (24 boys and 13 girls) between the ages of 14 and 17, with an average age of 16.1 . In this study we are not considering contributions related ethnicity or gender.

The control group consisted of 37 students selected from two classes at the same high school in Granada (Spain), with the criterion that the range, the proportion of boys and girls, and the mean age

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should be similar to that of the talented group. None of these students were considered to be mathematically talented.

Instruments

Braille is a writing system similar to the written language in which specific signs are used to represent letters. These signs are formed in a 3x2 matrix of dots, some of which are marked in relief (Figure 1).

FIGURE 1: Title block Braille generator

The Braille alphabet has a clever logical design. Once the first ten letters (A through J) are built, the second row with the following 10 letters (from K to T) is obtained adding dot 3 to all the first ten title block; and the final letters from U to Z (except W) are built adding dot 6 to the previous.

FIGURE 2: Braille alphabet; identification of regularity

At the same time, there is also a logic on the construction of the first ten letters. They are known as "upper-cells" configurations, as they only involve dots in positions 1, 2, 4, and 5 in such a manner that each column can be empty or filled with one dot or two. It is then a sequence based on the pattern 1, 1-2, 2 on the first column, 4, 4-5, 5 on the second column and subsequent combinations of these two columns. This way we obtain 15 symbols (Figure 3), from which 5 of them are discarded by its ambiguity when being read by the touch, as in Braille it is impossible to distinguish the reference title block.

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FIGURE 3: First series of Braille code formation

For the writing of digits 1, 2, … 9, 0, braille code uses the characters of the 10 first letters of the alphabet (A to J) preceded by the number indicator (Figure 4).

FIGURE 4: Number indicator. Relationship between letter a and the number 1

Contextualized in Braille code, students were asked to solve the following tasks

Task R1: (each student is given a Braille alphabet).

a) How was the Braille alphabet code built?

b) Write the following message in Braille[1]. What difficulties did you find in writing it? What difficulties did you find in deciphering the message your classmate sent you?

Task R2: (each student is given digits 0-9 in Braille code and a selection of the most frequent mathematical symbols in Braille code).

a) How was the Braile code for digits built?

b) Write the following statements "according to your proposal" (explaining how you did it): a) 2014, b) 2 + 3, c) 7 * 3, d) $x^2 + 2x - \frac{3}{5} = 0$

[1] Each member of the pair was given a word to write or decipher. The word "*medias* (socks)" given to the first partner and the word "*duende* (elf)" was given to the other.

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Task R3: Create your own code of communication for the blind. The goal is to design a proposal, particularly concerning numbers and mathematical symbols that facilitates the understanding of mathematics for people who use Braille code.

The Braille code provides a suitable context for pattern finding because the understanding of the overall construction pattern can be approached in several ways, corresponding to the types of discursive apprehension (Duval, 1999), which involves seeing the figure as a configuration of several constituent gestalts or subconfigurations. The step of going from seeing the figure as a whole to seeing it as parts is an indication of a dimensional change in cognitive perception (Rivera & Becker, 2007).

From the perspective of seeing the figure as a whole to seeing it as parts, the student recognizes a certain regularity in the alphabet and discovers a sequence of 10 elements that are repeated in the first two rows of the cell. To get this, the singularity of the letter W (Figure 2) must be recognized. The visual recognition of this regularity implies using at least two visualization skills: visual discrimination, to identify similarities between groups and singularity, and figure-ground perception, to recognize the common elements of the first two rows by isolating them from each other.

By focusing on the parts of the whole, the student focuses on recognizing the transformation of each element of the series into the next, and then generalizes the logic of its construction. The understanding of the construction pattern is complex since it requires the student to consider functional aspects of the Braille language related to the ambiguity of certain configurations in reading. In the first

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three symbols, the black dots correspond to positions 1, 1-2 and 2 (Figure 3), representing the raised elements to which these symbols correspond. In the next three, the same combination is used, but in the second column, 4, 4-5 and 5. Combining 1, 1-2 and 2 of the first column with the three previous elements of the second column allows nine more representations to be obtained. Of these 15 symbols, combinations that could be confused with others and present ambiguity in reading are eliminated (marked by an X in Figure 3), creating a first series of 10 symbols (from a to j); this is associated with the first letters of the alphabet. Adding the dot 3 to this series, letters of the following series (from k to t) were obtained and the rest were obtained by adding dots 3 and 6.

The recognition of this formation pattern can be approached from purely numerical strategies, associating values to each letter, such as the corresponding values of the cell (1,1-2,2,4,4-5, ...) or other assignments. From a visual approach, the student can manifest the ability of Position-in-space Perception by locating positional variations on the dots of an element with respect the next one in the serie and Perception of Spatial Relationships by identifying relations of parallelism and perpendicularity between "lines” formed by two consecutive dots. As in the previous case, other skills such as Perception of Context and Visual Discrimination also come into play.

In order to recognize the first 10 digits, the student must identify the first 10 letters of the alphabet, preceded by an indicator (dots 3, 4, 5 and 6) (Figure 4). Visual discrimination, namely finding the similarities, and figure-ground perception, namely finding the indicator, come into play in this exercise.

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Once the skills related to determined tasks haven been associated, analytical categories can be established (Table I). We based these categories on the theoretical referents and on the results of a previous pilot study (Del Río & Ramírez-Uclés, 2013), where a temporal sequence (in tasks R1, R2 and R3) was designed for the process of finding patterns on the construction of Braille code.

Table I. Categories of the variable "pattern finding" in the Braille code

The task R1 was designed to help students recognize any regularity, such as the main series of the formation of 10 elements, the A-J (G2) series, and the construction of the others from these series, by adding combinations of the remaining dots (P3). It was necessary to identify the initial generators (G1) in order to rule out the combinations that could be ambiguous for blind readers, marked X in Figure 2 Meanwhile, they had to take into account the universality of the code despite the language used (Del Río & Ramírez-Uclés, 2013), generating exceptionalities with some letters.

Therefore, this presented an example of information in Braille and asked students to write and read a message. A priori, we considered this activity to be key to linking the functionality of the code with the formation pattern (P2). The material used included photocopies of the alphabet, numbers and some mathematical symbols used in the Braille code as well as cardboard to encode messages and an empty box of medicine where information was also displayed in Braille code. After the hands-on work, namely the tasks R1 and R2, students were expected to recognize the pattern formation of both the alphabet and the digits (P1, G3).

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In task R3, the mathematical content was emphasized, whereby students were expected to express their mathematical talent and creativity. In order to become familiar with the notation, they were asked to write certain operations and determine whether the language added difficulty to mathematical reasoning. In order to analyze the originality of the proposals in task R3 in relation to novelty, we used two categories: sophistication (C1) and unconventional (C2). The level of sophistication was based on whether the new solution would model the problem and gives a functional answer, making the mathematical language more operative and taking into account the difficulties of tactile reading. As for the unconventional we consider the use of mathematical content nor required by the school curriculum.

Procedure

Data collection was carried out at two different times. One session was held with the group of students with mathematical talent as part of the ESTAMAT project and another session was held with the control group at their school. Both sessions were taught by the same two researcher teachers and the written responses of the students and the observations of the researchers were used as records. Three research tasks were presented.

Initially, all research tasks were carried out individually; afterwards, the participants were grouped into small groups of five or six students that were asked to agree on an answer. After collecting the individual and group answers, the whole group came together and the researcher teacher moderated the students' answers while the students evaluated the contributions of their peers.

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For the analysis of the research tasks, the data collection was completed through the teacher's observation sheets and students' activities sheets, in which the students had to submit their written responses. In the retrospective analysis, the manifestation of the variable "pattern finding" and "novelty" were analyzed through a descriptive approach. As a unit of analysis (Krippendorff, 1990) we use both individual and group written tasks. In this way, we have a complete record of each student were the presence or absence of the 6 categories of analysis of pattern detection (P1, P2, P3, G1, G2, and G3, in tasks R1 and R2) and the 2 categories of novelty (C1 and C2, in task R3) are collected. In the case of presence of a category we it describe explicitly and we analyze if the category has been manifested incompletely

Results

We describe the results obtained for each task. These data are accompanied by descriptions of the main differences in the development of the two sessions between students with mathematical talent (talent session: TS) and the control group (control session: CS).

Both sessions were carried out according to the prepared script, with students generally acting as anticipated. Seven groups were formed, namely five groups of five students and two groups of six. In the responses they had to agree on a single answer.

Pattern finding

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Following the analysis of task R1 and R2, we summarized the results of the variable pattern finding in Table II:

Table II: Identification of regularity and patterns in task R1 and R2

In task R1, section (a), no student with mathematical talent individually located generators or iterative patterns of the full code. It should be noted that of the 37 participating students, only 22 found the regularity in the first two groups of 10 elements, showing the G2 category incompletely. Moreover, nine students partially recognized some of the elements of the original generators, declaring G1 to be incomplete, when identifying the similarities in sequences C, D, E and F, G, H when adding a dot to the first column. The remaining students focused on finding some regularity between specific letters without recognizing generators or iterative patterns. For example, two students believed that the vowels were built first, assigning one dot to A, two dots to E and I, and three dots to O and U. Some noted that D and L have three dots that are symmetrical two by two, and that all the letters have at least one dot in the left column.

In the small groups' work, it was observed that discussions were focused on finding sequences within the proposed alphabet. Two groups showed G2 and P3 categories in an incomplete way. One group identified the A-J repeat sequence (G2) in the K-T symbols by simply adding a dot below and to the left (P3). The other group, though not recognizing the complete generator, found a pattern, claiming that a series is constructed from the previous, adding a dot to various previous elements (P3). The

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remaining groups did not reach a consensus on the formation pattern, believing the individually proposed ideas to be inconclusive.

In the control group, no student identified the regularity in the groups of 10 elements nor a pattern of formation. In the group discussion, a student classified the letters according to the number of dots: a single letter with a dot (A), five letters with two dots (B, C, E, I, K), etc., and the group tried to look for some criterion to see what the letters of each set have in common without coming to any conclusion. In the remaining groups, individual ideas were put together, but they could not come to a conclusive answer.

In section (b), concerning tactile reading using the Braille alphabet, all pairs experienced difficulty when locating dots and separating the letters, and the behavior of the students in both sessions was similar. Students wrote and translated the proposed message by highlighting the corresponding dots on cardboard. In writing the selected message, they found that reading from a single dot led to ambiguity and made it difficult to write. Thus, they considered the functionality of the Braille code as one element to consider in finding patterns (P2). All students indicated at least some difficulties in reading the Braille text: determining the spacing of the letters, distinguishing the separation of the dots and their position relative to a reference, and the significance of font size when reading with the thumb. In the large group pooling, difficulties raised by the students concerning ambiguity were collected, whereby the interventions that focused on daily-life activities stood out, such as reading a drug label in Braille to discover information regarding its composition.

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After reading the message, in the TS, as expected, the key moment of previously recognizing the difficulties associated with reading and writing encouraged the whole group to recognize the pattern, not only of the sequence of numbers, but of the complete code (G1, G2, G3, P1, P2 and P3). In the CS, several students presented intuitive ideas about the elements that were eliminated by ambiguity, but they were not able to explain the complete construction process.

In both sessions, the process of building the alphabet was explained before being asked to complete task R2. In task R2, all the students recognized that numbers are the letters A through J, preceded by an inverted L-like symbol (G3) and performed all of the sections of task R2, displaying their understanding of the added difficulty that the writing of mathematical expressions has.

Creativity

Following the analysis of task R3, we summarized the results finding in Table III:

Table III

Regarding sophistication, in the TS, it is noteworthy that 15 students came up with a simple proposal that did not counter Braille logic yet eliminated the indicator and assigned special combinations of dots to the digits. For example, a proposal consisted of adding a dot below and to the right of the letters A through J to represent numbers (Figure 5), assuming a modification of G3 and P3. With a simple dot, this proposal allows one to jointly write numbers and letters, thus facilitating the writing of equations. New functional elements appeared to be considered in pattern formation. In addition to conflicting with other symbols of Braille besides letters, the use of “operators” to set the digits facilitates the identification (numeric or literal) of the context in which the speech is expressed.

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Thus, at the onset of a matrix that precedes the digit, the reader is prepared to “capture” it. However, with the added dot, they must identify new characters (23 letters, plus the 10 digits) as different codes.

FIGURE 5: Example of proposal for the first numbers

The rest of the groups in the TS made proposals based on a combinatorial process (P1 and P3), without abiding by the pattern’s one-to-one correspondence with the meaning or with other ideographic symbols such as Arabic digits, as happens with letters and their respective Braille codes. None of them were notable in providing greater functionality or operability, and contributions were based on variations of possible combinations, the use of different indicators, and permutations in letter assignments. However, the proposals showed that the students had integrated the restrictions of the code given by the difficulties.

In the control group, the proposals were limited to small variations in the assignment of symbols, resulting in even less operable and functional cells, such as expanding the number of dots in the cell to reproduce the spelling of the numbers, replacing the dots with dashes or adding more indicators. Most of the proposals showed that the students had not integrated some of the reading difficulties noted, such as the ambiguity of reading from a single dot and the confusion between numbers and letters when the indicator is not used.

In relation to the unconventional nature of the mathematical contents used in the TS, i.e., the mathematical content and the characteristics of the numbering system, all students identified as being talented used additive and multiplicative systems. Two students proposed a non-positional system

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using operations to represent the number 2014, one using prime factorization (2x19x53) and another through additions (2000+14). The remaining students maintained a positional system, which had the value of the number depending on the place that it occupied in the written number. Although these proposals stood out due to the complexity of their content, they were not very operable from the viewpoint of mathematical writing.

With respect to the bases, most proposed working in base ten, that is, proposing that 10 digits and 10 units be grouped within a given order to form a higher-order unit. Again, the combinatorial logic (there are many more combinations than signs they have to use) prevailed and was ideographic and based on social conventions, not on analogies. Moreover, 10 students represented the number 0 by using all the dots of the cell, assigning the number a different role from the rest of the proposed digits. Three students represented the numbers using other bases, such as binary or base six.

In accordance with the original proposal and highlighting the use of complex mathematical content, a student used base two both in the proposed symbols and in writing a specific number (Figure 6). The number 0 was represented only with the number indicator, while the numbers from 1 to 63 were represented by the sums of powers of two, giving each of these powers a place in the original cell. For larger numbers, the number indicator was used again in cells in which the next powers of two appear.

FIGURE 6: Example of proposal and writing of 2014 using base two

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All the proposals from the students in the control group were in base 10, positional, additive and multiplicative. Zero was represented by different combinations of dots or even by means of a blank cell. From the point of view of curricular alignment, it is remarkable that a student proposed reassigning dots on the letters' title considering the frequency of use, that is, if a letter appears very frequently would be represented with fewer dots. . Although the student argued for greater operability by reducing the number of written dots, the student recognized that he did not have the mathematical tools to carry out this proposal.

Conclusions

As shown in the results described here, the mathematically talented students who participated in this study mainly exhibited "pattern finding", which is one of the characteristics indicated as defining mathematical talent (Freiman, 2006). To understand how the students found patterns, we observed two processes: recognizing generator elements and identifying the regularity with which they repeat (Castro, 1995; Polya, 1966).

In the initial stages of identifying regularities, mathematically talented students showed greater efficiency in recognizing the sequences of 10 elements, showing that their cognitive perception goes beyond sensorial perception in order to discriminate relevant facts (Rivera, 2007). They also manifested an effective use of visualization (Barbosa & Vale, 2015; Casey & Wolf, 1989; Del Grande, 1990) when it was combined with other characteristics of talent, such as data organization and generalization (Freiman, 2006). In the less complex task of identifying digits, no differences were noted in relation to the control group; both groups showed the visual aptitude necessary to solve the tasks. In this sense, we believe that visual tasks such as searching for patterns require other elements of reasoning to be used, effectively showing the use of visualization and helping to understand how talented mathematics students use visualization (Kalbfleisch & Gillmarten, 2013; O´Boyle, 2008; Presmeg, 2006).

As for recognizing the generation of pattern elements, a key contributor to success was giving students a real situation of writing and reading, allowing them to discover the ambiguity of some letter and number combinations that the blind experience. Without this prior experience, students would not have recognized the pattern because they would not have considered the elimination of combinations that are ambiguous to touch. However, once these ambiguities were recognized, unlike the control group, all the talented students showed their ability to recognize the pattern and understand the process of generalization (Amit & Neira, 2007). We believe it is relevant to note that they did not perceive aspects of the problem beyond the mathematical challenge, even though other variables not presented to them regarding the real problem should have been taken into account.

Attending to creativity, we observe that the solutions given by gifted students are more novelty than those of control group, even if the interpretation about sophistications and unconventional is different. With respect to sophistication, most of the solutions given by gifted students model the problem and give a correct answer, showing that they understand the problem's internal structure (Ervynck's, 1991) and that they present original thinking (Leikin & Lev, 2007). However, in relation with unconventional, some gifted students prefer to use complex operations or mathematical contents

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not required on school curriculum, more than usefulness of their proposal. Some students of control group also presented original ideas, but they were not adaptable to the real demands of the problem.

We are aware of the weaknesses and limitations of these studies. For one, the results are limited to a specific group of mathematically talented students, although similar results were also obtained from the pilot study carried out with other mathematically talented students and other control groups (Authors, 2013). Even so, we believe that this study contributes to the identification and the attention to students with mathematical talent providing practical implications.

On the one hand, different responses to the approach of the same task have emerged from each group, talent group and control group, which entails implications in the characterization of cognitive processes of mathematically talented students. This differentiation makes it easier for teachers to identify the characteristics of mathematical talent, being these students more effective in pattern recognition and in the originality and sophistication of their answers.

On the other hand, the mathematical content of the proposed task has been comprehensible to all students, allowing it to be carried out in a heterogeneous classroom with a cluster of gifted students. For these students, curricular enrichment will allow for a greater level of depth in the more complex topics, providing teachers with strategies to address special educational needs.

With respect to identification, the posed task has shown significant differences between the control group and the mathematical gifted students. Students with mathematical talent have shown a more effective use of visualization recognizing the Braille pattern, while on the visual task of relate letters and numbers there were no differences. We interpret that pattern finding tasks that imply the use of complex mathematical reasoning could be proper tasks for these students to shown their

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visualization (Kalbfleisch & Gillmarten, 2013; O´Boyle, 2008) and to identify their mathematical talent (Amit & Neira, 2008; Lee & Freiman, 2006).

Regarding the design of mathematical talent attention programs, a key moment on the understanding of the pattern was the recognition of his usefulness, contextualizing the problem in a real situation. Even though, some of the proposed solutions, with a higher mathematical content, were not practical solutions to the real problem of design a more operative mathematical language for blind people. We consider that a more functional approach of his mathematical knowledge is needed (Ali, Akhter, Shahzad, Sultana & Ramzan, 2011;Chamberlin & Moon, 2005). We believe that it is valuable for students to focus their high ability on solving real-world problems because, on the one hand, this process gives meaning to the content and elements of learned mathematical reasoning (Matsko & Thomas, 2014; Kwon, Park & Park, 2006) and, on the other hand, it allows students to become aware of the added difficulty blind people face in understanding mathematical language. We believe that the context in which a task is presented supports the development of these students' talent and creativity (Olszewski-Kubilius, Subotnik & Worrel, 2016; Sriraman, 2005).

The experience obtained during the research presented in this article can be used to design educational activities that respond to the characteristics of mathematically talented students, both individually and in groups, thus providing a possible educational response to the demand for attention to this group of students (NCTM, 2000). The sequencing of activities aimed at solving an open-ended problem in a real-world context, the combination of autonomous work with moments of reflection in

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small and large groups, and the proposal of a final research project have proven to be methodological approaches of teaching that foster mathematical talent.

The research is part of the R+D+I project EDU2015-69731-R (Spanish Government/MinEco and ERDF)

References:

Ali, R., Akhter, A., Shahzad, S., Sultana, N., & Ramzan, M. (2011). The impact of motivation on student achievement in mathematics in a problem-based environment. International Journal of Academic Research, 3, 306–309.

Amit, M., & Neria, D. (2008). “Rising to the challenge”: Using generalization in pattern problems to unearth the algebraic skills of talented pre-algebra students. ZDM Mathematics Education, 40(1) 111-129.

Andersen, L. (2014). Visual–Spatial Ability: Important in STEM, Ignored in Gifted Education. Roeper Review, 36(2), 114-121.

Arcavi, A. (2003). The role of visual representations in the learning of mathematics. Educational Studies in Mathematics, 52(3), 215–241.

Barbosa, A., & Vale, I. (2015). Visualization in pattern generalization: Potential and Challenges. *Journal of the European Teacher Education Network*, *10*, 57-70.

Bishop, A. J. (1980). Spatial abilities and mathematics education: A review. *Educational Studies in Mathematics*, *11*(3), 257–269.

Casey, H. V. E., & Wolf, J. S. (1989). Developing Visual Literacy Among Academically Able Fifth-Grade Students, *Roeper Review*, *12* (2), 86-91.

Ramírez, R., del Río, A. y Flores, P. (2018). Mathematical talent in Braille code pattern finding and invention. *Roeper Review*, *40*(4), 255-267
https://doi.org/10.1080/02783193.2018.1501782
Submitted version.

Castro, E. (1995). *Exploración de patrones numéricos mediante configuraciones puntuales*. Granada, Spain: Editorial Comares.

Chamberlin, S. A., & Moon, S. (2005). Model-eliciting activities: An introduction to gifted education. *Journal of Secondary Gifted Education, 17*, 37–47.

Del Grande, J. (1990). Spatial sense. *Arithmetic Teacher, 37*(6), 14-20

Del Río, A., & Ramírez-Uclés, R. (2013). Invención de patrones para los dígitos del código Braille [pattern design for the digits of Braille code]. In L. Rico, M. C. Cañadas, J. Gutiérrez, M. Molina & I. Segovia (Eds.), *Investigación en Didáctica de la Matemática. Homenaje a Encarnación Castro* (pp. 151-158). Granada, Spain: Editorial Comares.

Delcourt, M., & Renzulli, J. (2013). The three-ring conception of innovation and a triad of processes for developing creative productivity in young people. In L.V. Shavinina (Ed.), *The Routledge international handbook of innovation education* (pp. 128-150). Abingdon, Oxon, USA: Routledge

Duval, R. (1999). Representation, Vision and Visualization: Cognitive Functions in Mathematical Thinking. Basic Issues for Learning. In F. Hitt & M. Santos (Eds.), *Proceedings of the 21st Annual Meeting of the North America Chapter of the International Group of Psychology of Mathematics Education* (pp. 3-26). Cuernavaca, Morelos, Mexico: PMENA

Ervynck, G. (1991). Mathematical creativity. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 42–53). Dordrecht, The Netherlands: Kluwer

Fox, L. H. (2013). Innovation education Perspectives from research and practice in gifted education. In L.V. Shavinina (Ed.), *The Routledge international handbook of innovation education* (pp. 185-200). Abingdon, Oxon, USA: Routledge

Ramírez, R., del Río, A. y Flores, P. (2018). Mathematical talent in Braille code pattern finding and invention. *Roeper Review*, *40*(4), 255-267
https://doi.org/10.1080/02783193.2018.1501782
Submitted version.

Freiman, V. (2006). Problems to discover and to boost mathematical talent in early grades: A Challenging Situations Approach. *The Montana Mathematics Enthusiast, 3* (1), 51-75.

Gardner, H. (2001). *La inteligencia reformulada*. Barcelona, Spain: Paidós.

Greenes, C. (1981). Identifying the Gifted Student in Mathematics. *Arithmetic Teacher, 28* (8), 14-17.

Guilford, J. P. (1967). *The nature of human intelligence*. McGraw-Hill: New York.

Gutiérrez, A. (1996). Visualization in 3-dimensional geometry: In search of a framework. In L. Puig & A. Gutiérrez (Eds.), *Proceedings of the 20th Conference of the International Group for the Psychology of Mathematics Education* (pp. 3-19). Valencia, Spain: Universidad de Valencia.

Hershkowitz, R. (1990). Psychological aspects of learning geometry. In P. Nesher & J. Kilpatrick (Eds.), *Mathematics and cognition* (pp. 70-95). Cambridge, England: Cambridge U. P.

Kalbfleisch, M. L. (2013). Introduction to the special issue. *Roeper Review, 35*,217-218.

Kalbfleisch, M. L. & Gillmarten, C. (2013). Left brain vs. right Brain: findings on Visual Spatial capacities and the neurology of Giftedness. *Roeper Review, 35*,265-275.

Krippendorff, K. (1990). *Metodología de análisis de contenido: teoría y práctica*. Paidós Comunicación.

Krutetskii, V. A. (1976). *The psychology of Mathematical Abilities in Schoolchildren.* Chicago, United States of America: University of Chicago Press.

Kwon, O. N., Park, J. S., & Park, J. H. (2006). Cultivating divergent thinking in mathematics through an open-ended approach. *Asia Pacific Education Review, 7,* 51–61.

Lee K., Ko, E., & Song, S. (2007). The analysis of activity that gifted students construct definition of regular polyhedral. In J. H. Wo., H. C. Lew, K. S. Park & D. Y. Seo (Eds.), *Proceedings of the 31st*

Ramírez, R., del Río, A. y Flores, P. (2018). Mathematical talent in Braille code pattern finding and invention. *Roeper Review*, *40*(4), 255-267
https://doi.org/10.1080/02783193.2018.1501782
Submitted version.

Conference of the International Group for the Psychology of Mathematics Education, *Vol. 1* (pp. 153-160). Seoul, Korea: PME.

Lee, L., & Freiman, V. (2006). Developing Algebraic Thinking through Pattern Exploration. *Mathematics Teaching in the Middle School*, *11*(9), 428-433.

Leikin, R. (2009a). Bridging research and theory in mathematics education with research and theory in creativity and giftedness. In R. Leikin, A. Berman & B. Koichu (Eds.), *Creativity in mathematics and the education of gifted students* (pp. 129-145). Rotterdam, The Netherlands: Sense Publisher.

Leikin, R. (2009b). Exploring mathematical creativity using multiple solution tasks. In R. Leikin, A. Berman & B. Koichu (Eds.), *Creativity in mathematics and the education of gifted students* (pp. 385-411). Rotterdam, The Netherlands: Sense Publisher.

Leikin, R., & Lev, M. (2007). Multiple solution tasks as a magnifying glass for observation of mathematical creativity. In Woo, J. H., Lew, H. C., Park, K. S. & Seo, D. Y. (Eds.). *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education, Vol. 3* (pp. 161-168). Seoul, Korea: PME.

Leikin, R., Levav-Waynberg, A., & Gubermann, R. (2011). Employing multiple solution tasks for the development of mathematical creativity: two comparative studies. In M. Pytlak, T. Rowland and E. Swoboda (Eds.), *Proceedings of the Seventh Conference of the European Research in Mathematics Education* (pp. 1094-1103). Rzeszów, Poland: University of Rzeszów.

Liljedahl, P., & Sriraman, B. (2006). Musings on mathematical creativity. *For The Learning of Mathematics*, *26*(1), 20–23.

Mann, E. L. (2006). Creativity: The essence of mathematics. *Journal for the Education of the Gifted*, *30*, 236–262.

Ramírez, R., del Río, A. y Flores, P. (2018). Mathematical talent in Braille code pattern finding and invention. *Roeper Review*, *40*(4), 255-267
https://doi.org/10.1080/02783193.2018.1501782
Submitted version.

Marland, S. P., Jr. (1972). *Education of the gifted and talented: Report to the Congress of the United States by the U.S. Commissioner of Education and background papers submitted to the U.S. Office of Education. 2 vols* (Government Documents, Y4.L 11/2: G36). Washington, DC: U.S. Government Printing Office.

Matsko, V. & Thomas, J. (2014). The problem is the solution: creating original problems in gifted mathematics classes. *Journal for the Education of the Gifted*, *37*(2), 153-170.

Miller, R. C. (1990). *Discovering Mathematical Talent* (ERIC Digest E482). Reston, VA, USA: Council for Exceptional Children, ERIC Clearinghouse on Disabilities and Gifted Education.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA, USA: National Council of Teachers of Mathematics.

Neria, D., & Amit, M. (2010). Talented middle school students´ strategies and reasoning in solving analytic reasoning problems. In M. M. Pinto, F. Pinto and T. F. Kawasaki (Eds.). *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education, Vol. 3* (pp. 321-328). Belo Horizonte, Brazil: PME.

O´Boyle, M. W. (2008). Mathematically Gifted Children: Developmental Brain Characteristics and Their Prognosis for Well-Being. *Roeper Review*, *30*(3), 181-186.

Olszewski-Kubilius, P., Subotnik, R. F., & Worrell, F. C. (2016). Aiming talent development toward creative eminence in the 21st century. *Roeper Review*, *38*(3), 140–152.

Passow, A. (1993). National/State policies regarding education of the gifted. In K. Séller, F. Mönks and A. Passow (Eds.), *Internacional Handbook of Research and Development of Giftedness and Talent* (pp. 29-46). Oxford, England: Pergamon Press.

Pfeiffer, S., & Blei, S. (2010). Gifted identification beyond the IQ Test: Rating Scales and other procedures. In S. Pfeiffer (Ed.), *Handbook of giftedness in children: Psycho-educational theory, research and best practices* (pp. 177-198). Florida, USA: Springer.

Piirto, J. (1999). *Talented children and adults: Their development and education*. Upper Saddle River, NJ, USA: Merrill.

Pitta-Pantazi, D., & Christou, C. (2009). Psychological aspect: identification of giftedness in earlier ages. In M. Tzekaki, M. Kaldrimidou & H. Sakonidis (Eds.), *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education Vol. 1* (pp. 191-194). Thessaloniki, Greece: PME.

Pólya, G. (1966*). Matemáticas y razonamiento plausible*. Madrid, Spain: Tecnos.

Presmeg, N. (1986). Visualisation and mathematical giftened. *Educational Studies in Mathematics*, *17* (3), 297-311.

Presmeg, N. (2006). Research on visualization in learning and teaching mathematics. In A. Gutiérrez and P. Boero (Eds.), *Handbook of Research on the Psychology of Mathematics Education* (pp. 205–235). Rotterdam, The Netherlands: Sense Publishers.

Rabab'h, B., & Veloo, A. (2015). Spatial Visualization as Mediating between Mathematics Learning Strategy and Mathematics Achievement among 8th Grade Students. *International Education Studies*, *8(5)*, 1-11.

Ramírez, R. (2012). *Habilidades de visualización de los alumnos con talento matemático* [visualization abilities of mathematically talented students] (unpublished PhD). Granada, Spain: Universidad de Granada. Retrieved from http://fqm193.ugr.es/produccion-cientifica/tesis/ver_detalles/7461/

Ramírez, R., del Río, A. y Flores, P. (2018). Mathematical talent in Braille code pattern finding and invention. *Roeper Review*, *40*(4), 255-267
https://doi.org/10.1080/02783193.2018.1501782
Submitted version.

Rivera, F. D. (2007). Visualizing as a Mathematical Way of Knowing: Understanding Figural Generalization. *Mathematics Teacher*, *101*(1), 69-75.

Rivera, F. D. (2011). *Towards a Visually-Oriented School Mathematics Curriculum*. Mathematics Education Library 49, Springer Science+Business Media B.V.

Rivera, F. D., & Becker, J. R. (2008). Middle school children´s cognitive perceptions of constructive and deconstructive generalizations involving linear figural patterns. *ZDM Mathematics Education*, *40*, 65-82.

Ryu, H. Chong, Y. & Song, S. (2007). Mathematically gifted students´ spatial visualization ability of solid figures. In J. H. Wo., H. C. Lew, K. S. Park & D. Y. Seo (Eds.), *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education, Vol. 4* (pp. 137-144). Seoul, Korea: PME.

Silver, E. A. (1997). Fostering creativity through instruction rich in mathematical problem solving and problem posing. *ZDM Mathematics Education, 3*, 75–80.

Singer, F. M., Sheffield, L. J., Freiman, V., & Brandl, M. (2016). Research on and activities for mathematically gifted students. 1-41. DOI: 10.1007/978-3-319-39450-3_1.

Sriraman, B. (2005). Are giftedness & creativity synonyms in mathematics? An analysis of constructs within the professional and school realms. *The Journal of Secondary Gifted Education*, *17*, 20–36.

Steffe, L. & Thompson, P. W. (2000). Teaching experiment methodology: underlying principles and essential elements. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 267-306). Mahwah: NJ, USA: LAE.

Sternberg, R. J. (1997). A triarchic view of giftedness: Theory and practice. In N. Colangelo & G. A. Davis (Eds.), *Handbook of gifted education* (2nd ed., pp. 43–53). Boston: Allyn and Bacon.

Ramírez, R., del Río, A. y Flores, P. (2018). Mathematical talent in Braille code pattern finding and invention. *Roeper Review*, *40*(4), 255-267
https://doi.org/10.1080/02783193.2018.1501782
Submitted version.

Sternberg, R. J., & Renzulli, J. S. (2004). Lies we live by: Misapplication of test in identifying the gifted. In J.S. Renzulli & S. M. Reis (Eds.), *Identification of students for gifted and talented programs*.(pp. 53-62). Thousand Oaks, CA, USA: Corwin Press.

Torrance, E. P. (1974). *Torrance tests of creative thinking*. Bensenville, IL, USA: Scholastic Testing Service.

Vale, I., Pimentel, T., Cabrita, I., Barbosa, A., & Fonseca, L. (2012). Pattern problem solving tasks as a mean to Foster creativity in mathematics. In Tso, T. Y. (Ed.), *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education, Vol. 4* (pp. 171-178). Taipei, Taiwan: PME.

Table I

Table I. Categories of the variable “pattern finding” in the Braille code.

To recognize the generator element, what is common, repeated (G)
The initial generating elements: dots 1, 1-2 and 2 and dots 4, 4-5 and 5 (G1)
The series of 10 symbols (G2)
The relationship between the series of 10 symbols and the digits (G3)
To identify the repeating pattern, the regularity with which it repeats (P)
The combination pattern of the initial generators (P1)
Ambiguous elements and the functionality of the language (P2)
The combination pattern of the series of 10 symbols with elements 3 and 3-6 (P3)

Table II

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https://doi.org/10.1080/02783193.2018.1501782
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Table II: Identification of regularity and patterns in task R1 and R2

	Mathematically Talented	Control Group
Task R1 Individual	22 students: G2 incomplete: only some elements 9 students: G1 incomplete: only some elements	No students
Task R1 Group discussion	1 group (5 students): G2 and P3 incomplete: only two series 1 group (5 students): P3 incomplete: only two series.	No groups
After R1 b)	All: G1, G2, G3, P1, P2 and P3	1 group (5): P2 incomplete
Task R2 (after alphabet explication)	All: G3	All: G3

Table III:

Table III: Identification of creativity in task R3

	Mathematically Talented	Control Group
Sophistication	3 group (15 students): eliminate the indicator, special combinations of dots.	No students
Unconventional	1 student: prime factorization 1 student: special additions	1 student: statistical frequency

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	3 students: binary or base six

Figure 1

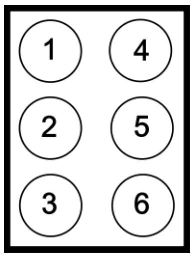

Figure 2

Ramírez, R., del Río, A. y Flores, P. (2018). Mathematical talent in Braille code pattern finding and invention. *Roeper Review*, *40*(4), 255-267
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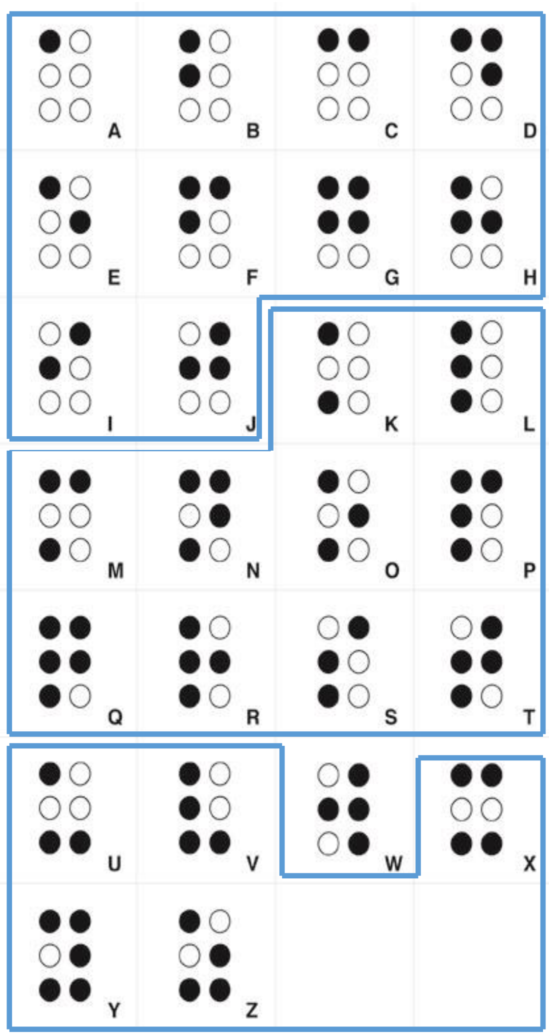

Figure 3

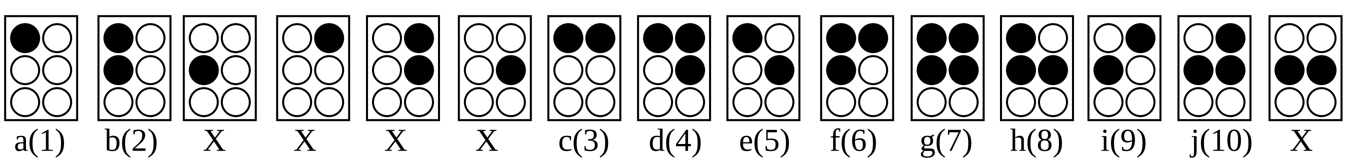

Figure 4

Ramírez, R., del Río, A. y Flores, P. (2018). Mathematical talent in Braille code pattern finding and invention. *Roeper Review*, *40*(4), 255-267
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Submitted version.

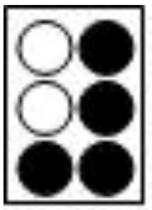
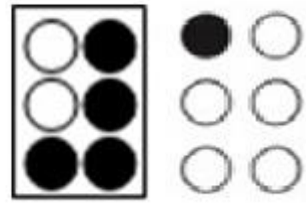
		
4a. Number indicator	4b. Letter A	4c. Number 1

Figure 5

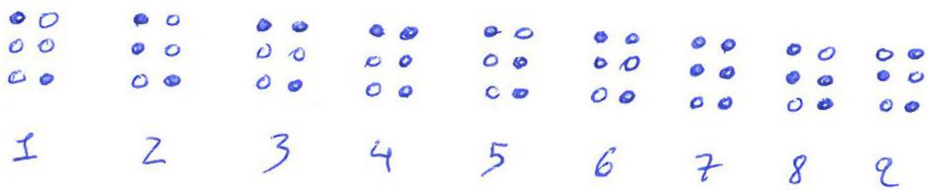
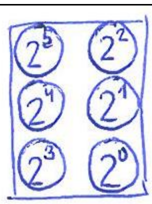
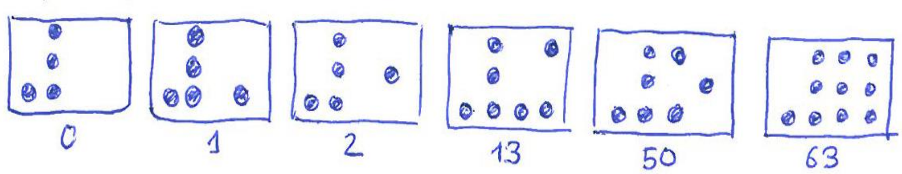
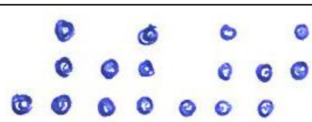


Figure 6

	
	2014 is 11111011110 in binary. It is represented in two cells preceded each of them by a number indicator. The last six digits are 011110 and are represented after the second number indicator (0+2+4 +8+16+0) The first five numbers 11111 are represented after the first number indicator that now represents powers from 6 to 11

Ramírez, R., del Río, A. y Flores, P. (2018). Mathematical talent in Braille code pattern finding and invention. *Roeper Review*, *40*(4), 255-267
https://doi.org/10.1080/02783193.2018.1501782
Submitted version.

	(64+128+256+512+1024 +0)