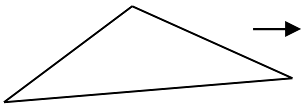
WORKSHEETS FOR PAPER AND PENCIL TASKS

Name…………………………………...……Date…….......Session...…...Group…….

SUM OF THE ANGLES OF A REGULAR POLYGON

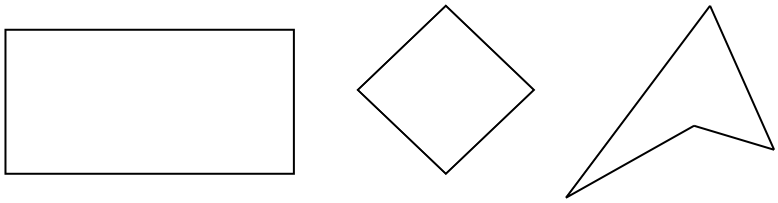
Marta and Pablo play with polygons like the following and they are wondering how much the interior angles of each of them add up to. They don't have any tools to measure the angles, but they know that the sum of the angles of a triangle is always 180º. *Can you find a way to get the sum of the interior angles of each polygon without measuring each angle?*

Triangles

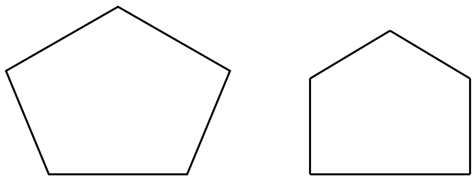


→ The sum of the 3 angles of a triangle is always 180º

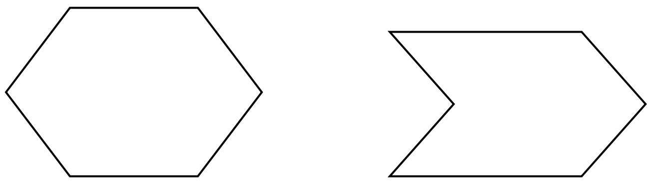
Quadrilaterals



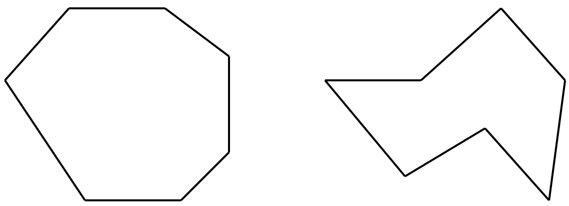
Pentagons



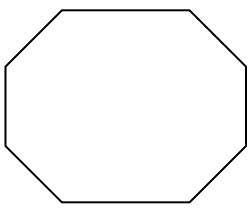
Hexagons



Heptagons



Octagons



Can you obtain a formula that allows you to calculate the sum of all the angles of any polygon knowing only the number of sides?

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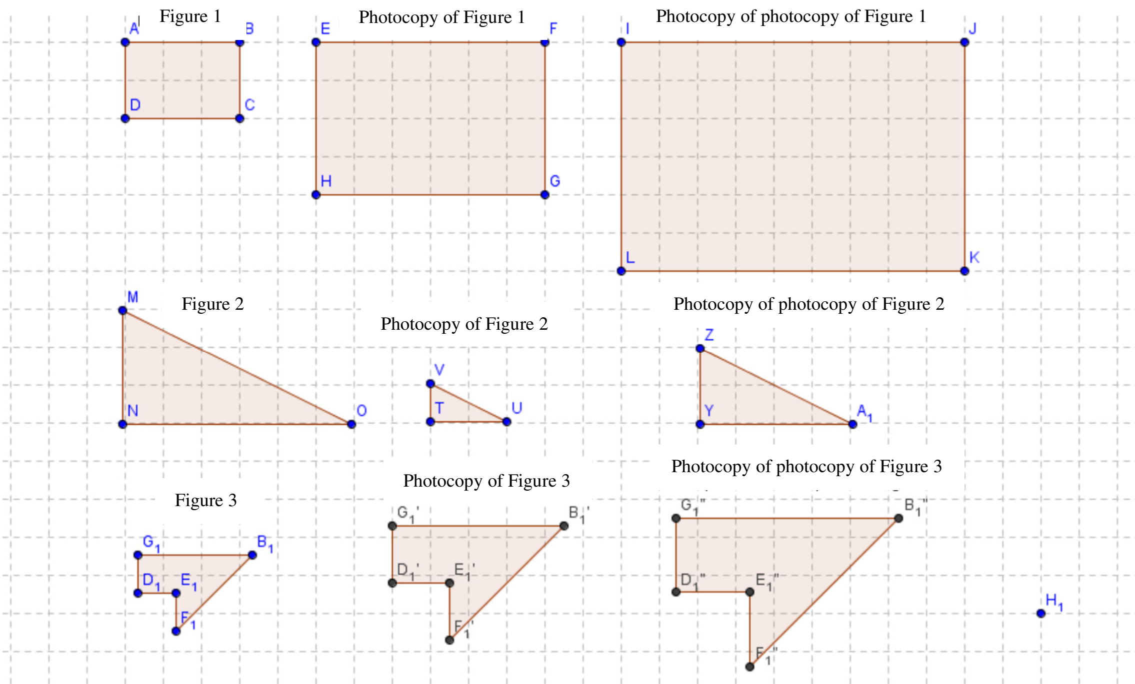
SIMILAR FIGURES

We are going to enlarge and reduce these three figures on the photocopying machine. *Look at the original figures, write down their measurements and do the same with the photocopies.*



1.- *For figures 1 and 2, compare the measurements of each original figure with those of the first photocopy. Then, compare the measurements of the first photocopy with those of the second photocopy. What do you observe? Write down the calculations and your answer.*

2.- *For figures 2 and 3, also measure the angles and write down the measurements. Compare the angles of the original figures with those of the first photocopy. Then, compare the angles of the first photocopy with those of the second photocopy. What do you observe? Write down your answer.*

Objects that are enlargements or reductions of other objects are called similar objects. *What must the sides and angles of similar figures be like? Explain your answer in your own words.*

3.- If we compare figure 1 and its first photocopy we see that the ratio between the sides is $\frac{EF}{AB} = 2$

The quotient obtained by dividing homologous sides of two similar figures is called **the similarity ratio**. *Find the similarity ratios of each figure and its first photocopy and also calculate the perimeter of all of them. Write all calculations and results in the table below*:

	Figure1 and its first photocopy		Figure 2 and its first photocopy		Figure 3 and its first photocopy	
Perimeter						
Similarity Ratio $=\frac{Photocopy\_side}{Figure\_side}$	$\frac{EF}{AB} = 2$		$\frac{VU}{MO} =$		$\frac{G1'B1'}{G1B1} =$	
Perimeter Ratio $=\frac{Photocopy\_Perimeter}{Figure\_Perimeter}$						

Do you observe any relationship between the similarity ratio and the perimeter ratio of each figure and its photocopy?

4.- *Find the areas of each figure in the table below and complete the table. Write down the calculations you make to find the areas:*

	Figure1 and its 2nd photocopy		Figure 2 and its 2nd photocopy		Figure 3 and its 2nd photocopy	
Area						
Similarity Ratio $=\frac{Photocopy\_side}{Figure\_side}$	$\frac{IJ}{AB} =$		$\frac{VU}{MO} =$		$\frac{G1''B1''}{G1B1} =$	
Area Ratio $=\frac{Photocopy\_Area}{Figure\_Area}$						

Do you observe any relationship between the similarity ratio and the area ratio of each figure and its photocopy?

Annex G: Worksheets for paper and pencil tasks and for GeoGebra tasks

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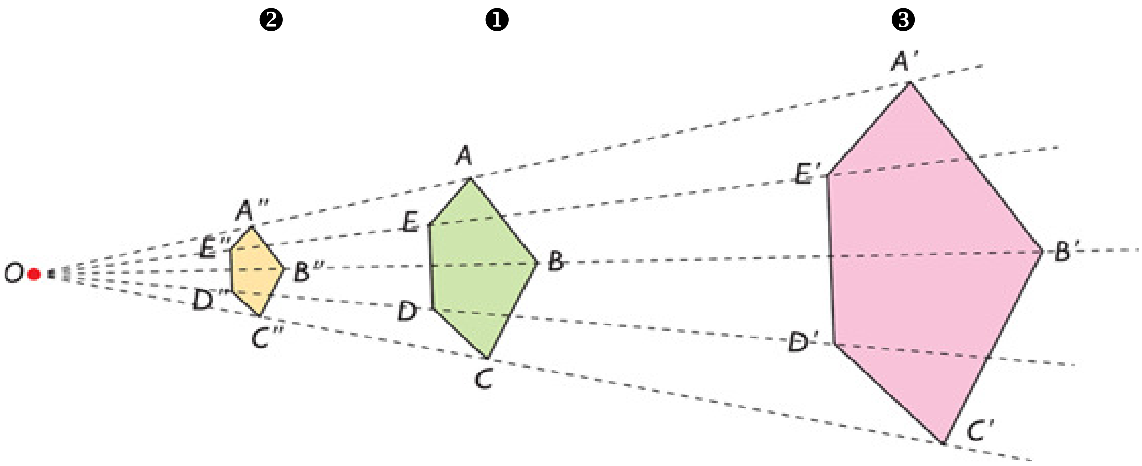
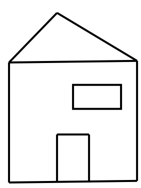
PROJECTION METHOD FOR CONSTRUCTING SIMILAR FIGURES



<u>1.-</u> *Find the similarity ratio between the figure ❷ and the original (figure ❶). Find the ratio between the distances OA" and OA. What do you observe?* (Hint: measure the sides of each figure and divide them to find the similarity ratio; measure the distances OA" and OA and divide them).

<u>2.-</u> *Find the similarity ratio between the figure ❸ and the original (figure ❶). Find the ratio between the distances OA' and OA. What do you observe?* (Same as previous exercise).

<u>3.-</u> *Employing the projection method, draw a similar figure to the following, enlarged three times, and another one also similar but reduced by half.* (You can place the point O wherever you want).



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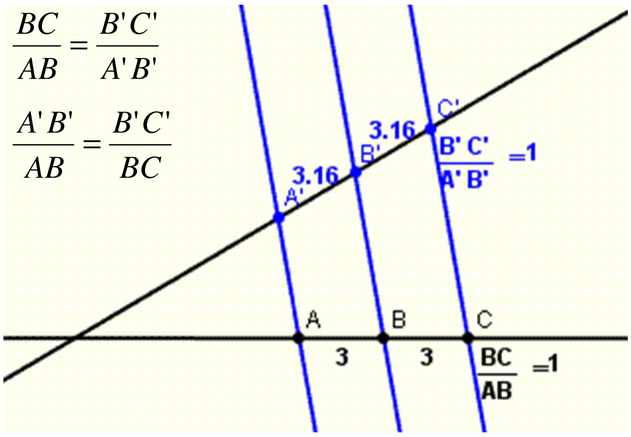
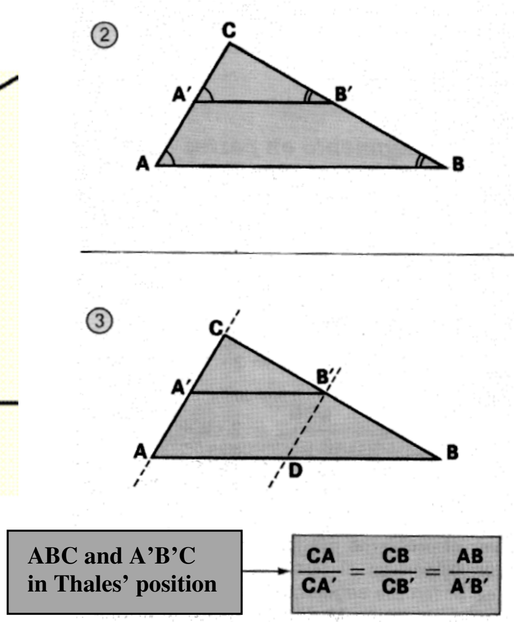
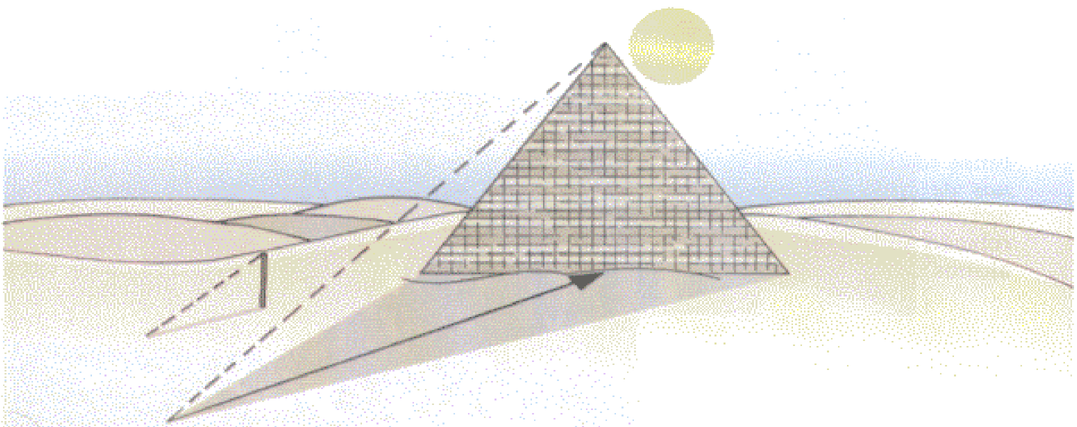
THALES OF MILETUS

He was born around 640 BC in Miletus (Asia Minor, now Turkey) and died around 560 BC in Miletus. Thales was an essentially practical man: merchant, skilled engineer, astronomer, geometer... He is traditionally included among the Seven Sages. As a merchant, it is said of him that one year, foreseeing a large production of olives, he monopolised all the olive presses to make the oil, thereby making a splendid profit. As what we would now call an engineer, he was in charge of hydraulic works and is said to have diverted the course of the Halis river by building dams. He was more famous as an astronomer, he predicted the total sun eclipse what was visible in Asia Minor, as well as he was considered to be the discoverer of the Ursa Minor constellation. He is also believed to have known the sun's path from one tropic to another. He explained sun and moon eclipses and he believed that the year had 365 days. His famous THEOREM states that if two intersecting lines are cut by parallel lines, the segments formed are proportional:





Thales of Miletus used it to find the height of the Cheops Pyramid (Egypt):



Looking at the picture, can you find out how he did it? (Hint: he knew how long the pole was, He knew that the angle of incidence of the sun's rays on the Earth at the same time is the same, and he measured the shadow of the pole and the shadow of the pyramid at the same time).

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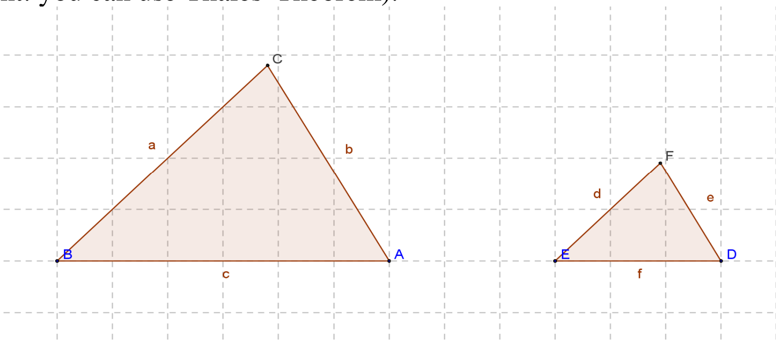
SIMILARITY CRITERIA

Thales' Theorem states that two triangles in Thales' position are similar, but even if two triangles are not in Thales' position, it is not necessary to prove that all the angles are equal and all the homologous sides are proportional to affirm that they are similar. We are going to see that it will be enough that only some of these conditions are fulfilled for any two triangles to be similar. Those conditions that, if they are fulfilled, give us the certainty that the triangles are similar are known as Similarity Criteria.

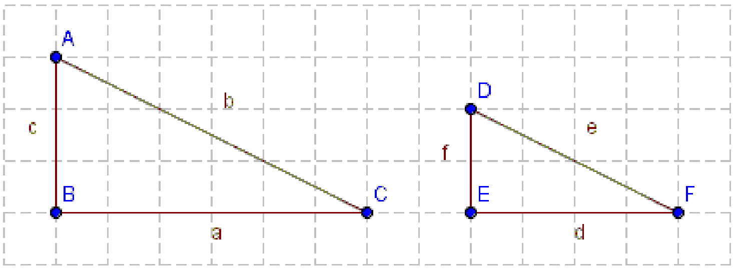
FIRST SIMILIARITY CRITERIA

Imagine you have two triangles like the following, where you can measure the angles but can't measure the sides. How would you know if they are similar only by considering the measure of the angles? (Hint: you can use Thales' Theorem).



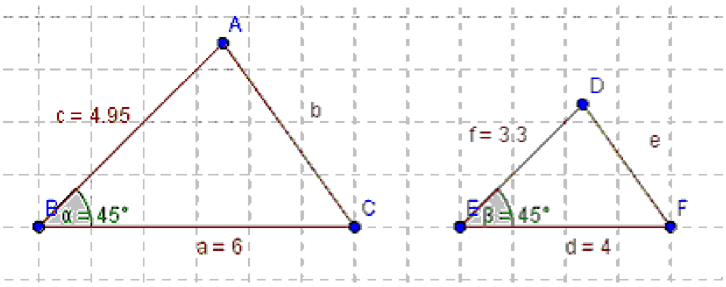
SECOND SIMILIARITY CRITERIA

Now let's suppose you don't know the measures of any of their angles, but you can get the measures of their sides. How can you find out if they are similar if you only know their sides measurements? (Hint: you can use Thales' Theorem)



THIRD SIMILIARITY CRITERIA

In this case, we have two triangles that have one of their angles equal and we know the measure of the sides that form them. How can we know if they are similar only with these data? (Hint: you can use the Thales Theorem).



Write in your own words the statement of each of the three similarity criteria.

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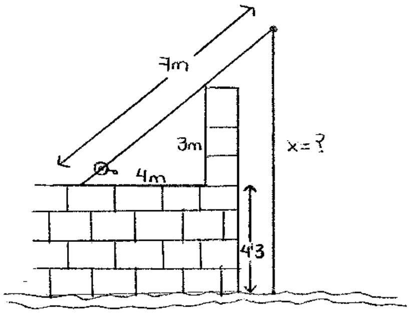
APPLICATION OF THE THALES' AND PYTHAGORAS THEOREMS

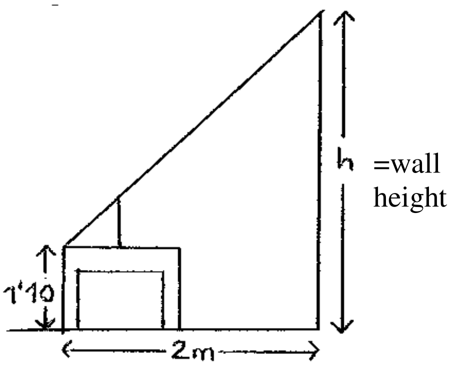
1.- Pepe and Blanca want to go fishing on Sunday at the San Sebastián pier, but they don't have a fishing rod. When they go to the shop to buy one, they realise that the price varies according to the reel on the rod, that is, the more metres of fishing line the rod has, the higher the price of the rod is. Not wanting to spend a lot of money, they decide to go to the pier to check the minimum amount of fishing line they will need to be able to fish. The pier location corresponds to the following drawing:



Which rod should they choose to be able to fish and spend as little money as possible? Why?

Price rod: -Rod A with 7 metres of line = 29,95 €
-Rod B with 10 metres of line = 49,95 €
-Rod C with 12 metres of line = 60 €

2.- We are going to use a bevel and a measuring tape to find unknown heights. We want to find the height of a room in our house, so we only need the bevel, the measuring tape and a table or a piece of furniture on which we can rest the bevel and which we can move easily. First we rest the bevel on the table and hold it with books. Then, we look through the bevel and we have to get the visual of the hypotenuse to end at the highest point of the wall, If we don't see that point we move the table until we see it. Now we have this situation:



We use the measuring tape to measure the height of the table (1,10 metres) and the distance from the table to the wall (2 metres), *can you find the height of the wall?* (Note: each leg of the bevel measures 30 cm). Explain how you do it.

WORKSHEETS FOR GEOGEBRA TASKS

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MOSAICS

In 1060, a vizier of Inb Habus, who was a great Berber king, built a residence on a hill in Granada, which was later rebuilt and extended to become the Alhambra, one of the characteristic monuments of the Moorish presence in Spain. In its palaces you can see the fundamental characteristics of Islamic art, such as the collections of mosaics.

A mosaic is a composition of tiles that reproduces a landscape or a figure, in which the tiles cannot overlap or leave gaps between them. When the tiles fill the plane based on symmetries, translations and rotations, we are dealing with a geometric mosaic.

The Arabs were excellent creators of geometric mosaics. Islamic culture decided to reject any representation of living beings, in order to avoid confusion with other cultures such as Christian one, which depicted people and animals, but which allowed them to show their own identity as a people. For this reason, their creativity turned towards calligraphy and geometric drawings, in which they reached a level of beauty and complexity that would be difficult to surpass. Moreover, given the mathematical knowledge of mosaics at the time, it is striking to see that they knew every existing single type of mosaic. (17 groups of plane crystallographers).

Task 1: Let's suppose you are an arabic decorator and the grand vizier orders you to tile the floors of the Alhambra under two conditions: you must use equal tiles in each room, but the tile designs must be different in each room (you can use as many tile shapes as you can think of). You must show the vizier all the tile shapes valid to tile his rooms, so that he can decide which one he likes best for each floor or wall. Try as much as you can to please the vizier, since your salary and reputation as a decorator depend on it. *What shapes can the tiles have?*

Are any of these shapes or polygons suitable for tiling a floor or wall? Explain in which cases you have been able to tile and in which cases you have not and why.

What conditions must a tile fulfil in order to be able to tile a surface with it?

Annex G: Worksheets for paper and pencil tasks and for GeoGebra tasks

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<u>Task 2:</u> *What techniques have you used to build each mosaic: drawing on the vertices of the grid with the mouse, using translations, rotations, symmetries...?*

Are you able to repeat some of the mosaics you have already drawn using a different technique than the one you used before? <u>(Choose a type of tile and try to build the same mosaic using at least three different techniques, repeat this exercise with three different types of tiles).</u>

<u>Task 3:</u> I suppose that after trying different tiles, you know that any triangle and quadrilateral can be used to tile a surface, but *can you explain why?*

(Hint: I suggest that you first try different types of triangles, starting with the ones you find easiest to build mosaics with. Then, when you get an answer, try different quadrilaterals, also starting with the ones you find easiest to tile).

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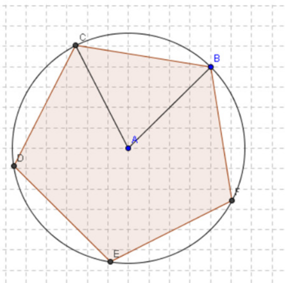
Task 4: Now we are going to focus on those tiles that have all sides and angles equal, that is, on tiles that are regular polygons, *are you able to build regular polygons of 3, 4, 5, 6, 7... sides? How do you do it?*



Task 5: The vizier suggests that you use tiles with all sides and angles equal to tile the floor of the "Chamber of the Abencerrajes" (this room is the Sultan's bedroom) in order to save money (these tiles are cheaper than those with irregular shapes). *What shapes can these tiles have?* (Hint: study with which of the regular polygons you have drawn in the previous task it is possible to build a mosaic and analyze what have in common the angles of the polygons you have been able to tile with).

Can you get more mosaics by using regular tiles with more sides? Justify your answers.
Hint: You may find it useful to find the values of the interior angles of each regular polygon, as shown in the table below:

Number of polygon sides	3	4	5	6	7	8	9	10	11	12
Value of the interior angle										

Task 6: *What techniques have you used to build each mosaic: drawing on grid vertices with the mouse, using translations, rotations, symmetries...? Are you able to repeat the 3 regular mosaics you have already drawn using a different technique than the one you used before?*

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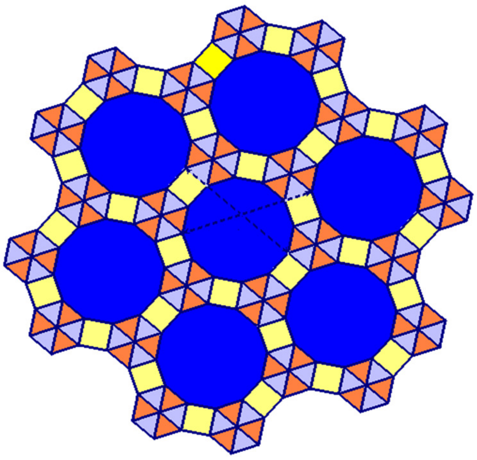
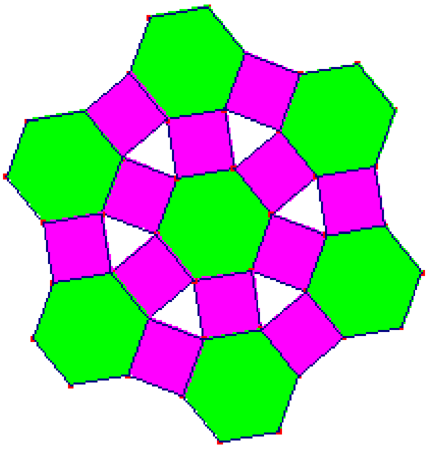
<u>Task 7</u>: The grand vizier has not been very satisfied with the mosaics that you have presented to him for tiling the floors, because he thinks that they are not aesthetically attractive. Therefore he suggests that using regular polygons to save money and your knowledge of mosaics, *you can make other more interesting designs* (try any regular polygon, not just the 3 that tessellate the plane). You can mix different types of polygons and use different colours.





The two previous mosaics are examples of "Semi-Regular Mosaics", as they are obtained by using several kinds of polygons, with the condition that the different polygons have all sides of the same length.

<u>Task 8</u>: Between the two mosaics of the previous task, there is a small difference: in the mosaic on the right it is fulfilled that in all the vertices of the mosaic there are the same polygons and in the same order (placed in the same way) and the mosaic on the left doesn't satisfy this condition. Polygons that meet this condition are called "Congruent Semi-Regular Mosaics".

The Grand Vizier, always eager to make the work easier for the workmen, suggests *you show him different designs of congruent semi-regular mosaics* so that he can choose one to tile the walls of the Hall of Comares or the Hall of the Ambassadors (this is the largest and highest room in the palace. This is where the sultan's private audiences are held with others who sit in the recesses in the walls). *There are only 8 possible designs! Try to find them.*

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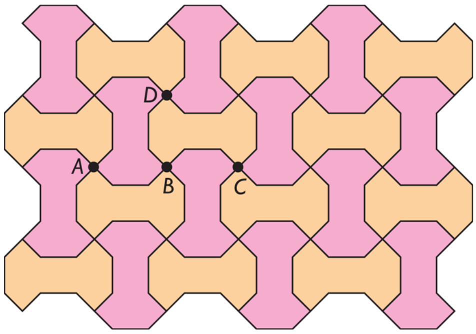
Task 9: We are going to study some transformations that we can make to the polygons that tessellate the plane, to obtain more original tiles that are still valid for tiling any surface.

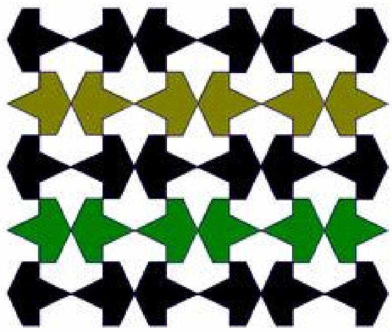
A very simple way to obtain mosaics is to deform a regular polygon, removing a part of it and adding it by means of a translation or a rotation to another side. To do this, using GeoGebra we must draw a polygon on one or several sides (deform one or several sides) and then we can:

1.- Translate this deformation to another side of the polygon.

2.- Rotate this deformation on another side using some vertex of the polygon.

I now propose you to make the opposite work of what you have been doing so far: instead of asking you to look for tile designs that can be used to make mosaics, *I give you two designs of very famous mosaics found in the Alhambra, so that starting from a square, you can draw them using GeoGebra*. (You will have to use translations, rotations or symmetries). The mosaic on the left is called the Bone Mosaic and the one on the right is called the Airplane Mosaic.





Task 10: We leave aside our facet as decorator of the Alhambra, to go back to our times. We are now going to transform ourselves into one of the most famous Dutch painters of the last century, Maurits Cornelius Escher, who used his mathematical knowledge of translations, rotations and symmetries for his paintings. Once you have been able to know part of his work, *I would like you to make some mosaics using an equilateral triangle and a regular hexagon as a basic tile and deform their sides by means of translations and rotations. After obtaining the mosaic you can embellish it using different colours or adding drawings inside the tiles. Be as creative as possible and use your imagination together with your mathematical knowledge!*
(Once you all have obtained your mosaics we will choose by vote the best of each type and upload it to our web page so that everyone can see it).