Approaching theoretical thinking within a dynamic geometry environment*

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Abstract

In this paper we describe one classroom activity, part of a long-term project aimed at investigating the potentialities of dynamic geometry software, namely Cabri-Géomètre, in supporting students' production of conjectures and proofs in geometry at secondary school level. The paper focuses on the activity of a pair of students solving an open geometry problem in Cabri. The analysis shows that Cabri might be a support in bridging the gap between exploration (and conjecturing) and proof: first, exploration provides students with a wide range of local logical relationships between elements or properties of the figure; then, these local concatenations are to be globally rearranged in the proving phase, in order to construct a complete proof.

Key-words: *conjectures and proofs; dynamic geometry systems; open problems.*

Resumo

Neste artigo, descrevemos uma atividade de sala de aula, parte de um projeto mais amplo visando a investigar as potencialidades do software de geometria dinâmica Cabri-Géomètre, como suporte para alunos na produção de conjecturas e provas em Geometria no ensino secundário. O texto apresenta a atividade de uma dupla de alunos resolvendo um problema aberto no Cabri. A análise mostra que o Cabri pode servir como apoio, diminuindo a lacuna entre exploração (e levantamento de conjecturas) e prova: primeiro, a exploração propicia aos alunos um conjunto diverso de relações, lógicas e locais, entre elementos ou propriedades da figura; segundo, estas concatenações locais são reorganizadas na fase de prova, a fim de construir uma prova completa.

Palavras-chave: conjecturas e prova; sistemas de geometria dinâmica; problemas abertos.

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Introduction

Approaching theoretical thinking in mathematics requires giving up naïve empiricism, that is the common habit to gather conclusions and to justify conjectures on the basis of observations of particular cases. This habit is strong in human beings, as in everyday life we often need to gather conclusions from the observations of just a limited number of cases and experiences. This way of thinking is often transferred by students to situations for which it is not appropriate, as for example when they need to prove conjectures in geometry. Research has shown that students are usually satisfied with empirical verification of properties, for example using measures, and for only a limited number of cases (e.g. Chazan, 1993). In this way, they either validate or refute conjectures, without feeling the need for proving them.

In our opinion, naïve empiricism might be abandoned only if the didactic contract in the classroom makes explicit the role of mathematical proof as explaining why a conjecture 'works', other than just convincing (ourselves or a friend or an enemy) of the validity of a certain conjecture; precisely, explaining how a conjecture (proposition) is logically deduced from other propositions within a theory.

The concept of proof is a crucial issue of current international discussion among researchers in Mathematics Education at international level. The Italian contribution to this debate concerns both historical and epistemological analysis of the concept of proof and development of suitable learning environments, which can support students in the transition from explorations and conjectures to more formal hypothetical reasoning and proofs (Boero, Garuti & Mariotti, 1996; Bartolini Bussi, Boero, Ferri, Garuti & Mariotti, 1997; Furinghetti & Paola, 1998; Arzarello, Micheletti, Olivero, Paola & Robutti, 1998 (a); Arzarello, Olivero, Paola & Robutti, 1999). Results from this research show that providing students with problems in the form "prove that..." may prevent them from being able to attempt proving. On the contrary, providing students with problems which require and support the production of conjectures may help them in the proving phase: the hypothesis is that a cognitive continuity in the transition from exploration to proving might be constructed on the basis of the production of conjectures. The research project[1] we have been involved in for some years now within the

1 We would like to acknowledge all the participants in the project: teachers, students, researchers and the co-ordinator Ferdinando Arzarello.

Mathematics Education group at the University of Turin is aimed at investigating the potentialities of dynamic geometry software, namely Cabri-Géomètre, (Baulac, Bellemain & Laborde, 1988; Laborde & Strässer, 1990) in supporting students' production of conjectures in geometry. Our hypothesis, based on classroom experiments, is that Cabri may be a strong support for students not only in the conjecturing phase, but also in the proving phase.

In the paper we will analyse one classroom activity involving the solution of a geometric open problem in Cabri. First, we will describe the context of the experiment; secondly, we will present a preliminary analysis of students' solution process, which shows how Cabri was useful in supporting the production of conjectures, the dynamic exploration of the situation and the construction of proofs.

The classroom experiments

In order to investigate the main aim of the ongoing project, we have been carrying out classroom experiments, in secondary schools, in which students are asked to solve geometric problems in Cabri. Cabri-Géomètre[2] consists of a package for constructing geometrical figures. It deals with points, lines, circles and their relations and allows the user to do geometrical constructions. The most relevant feature of Cabri from the didactical point of view is the dragging function, that is the possibility of directly manipulating the constructed figures on the screen. If a figure has been correctly drawn, according to the rules and properties of Euclidean geometry, it keeps all its internal relationships whenever it undertakes dragging. Otherwise it loses its initial features.[3]

2 There are different versions of Cabri. In the activity we will describe, students used Cabri I.7.

3 For example if you want to draw an isosceles triangle, you can use the following construction:

1. Draw a segment
2. Draw its perpendicular bisector
3. hoose a point on this perpendicular bisector
4. Draw the segments from this point to the endpoints of the first segment.

The Cabri-construction, which translates the previous construction into Cabri commands, is shown below:

We observed one pair of students in each classroom and we collected the written material from all the students. Then we analysed the data collected with respect to students' cognitive processes, in order to find out the kind of support Cabri might give in the solution process.

The activities used in the classroom experiments are open problems, that is problems in which students can explore a situation, make conjectures and at first test them within the Cabri environment through the dragging function. Then they need to validate their conjectures within a mathematical theory, i.e. they need to construct a proof. The role of the proof is to show how the discovered properties, which are formulated in the conjectures, can be deduced from the axioms of the mathematical theory considered, in this case Euclidean geometry. Open problems can be tackled at different levels, according to students' knowledge and expertise. Students can perform different things, such as:

- exploring the most general case and looking for a general property which is always true under the given hypotheses;
- making conjectures and constructing proofs related to one or more particular cases;
- analysing an intermediate situation which is a generalisation from a number of cases and construct a proof for that;
- making many conjectures, but no proofs;
- constructing proofs for some cases without exploring the situation further.

The main potentiality of open problems is the fact they foster a wide mathematical production, in that all the students usually succeed in observing something from the situation they are exploring, while in problems of the form "prove that..." most students think a brilliant idea is needed in order to be able to provide a solution and so they get stuck.

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1. Segment

2. Perpendicular bisector (of the segment)

3. Point on object (on the perpendicular bisector)

4. Segment (repeated twice: the endpoints are the point on the perpendicular bisector and the endpoints of the first segment)

If you implement the previous construction then when you drag one of the vertexes of the triangle, it keeps the property of being isosceles. On the contrary if you only draw an isosceles triangle 'by eye', i.e. a triangle that seems to be isosceles but has not been constructed as such, it will loose its property as soon as you drag it.

The students are usually divided in pairs; each pair works at the computer with Cabri. We will present a detailed analysis of the solution process of a pair of students, who tackled an open geometric problem in Cabri. They belong to a classroom of a 'liceo scientifico PNI[4] (17 year old students). It is a mixed ability class[5] (there is group of high achievers, a few low achievers and another group in the middle). They are used to working in groups, both with paper and pencil and with the calculator (Cabri, spreadsheets, programming languages) and to sharing results, conjectures and proofs at the end of an activity. The teacher does not usually teach at the front, but his lessons are interactive and he usually co-ordinates a classroom discussion after group work. Working in groups, comparing and sharing results is now part of the classroom culture. As far as Cabri is concerned, this classroom has used Cabri since the first year of Secondary School (14-15 years old), both in construction problems and in exploration problems (as those described above). The teacher has introduced students not only to the technical features of the software, but also to the different modalities of exploration and dragging (finding invariants, limit cases and properties by exploiting different dragging modalities). The teacher plays a fundamental role in the introduction of Cabri in the classroom: in general, we observed that if the teacher makes explicit the different uses of dragging, students make a better use of Cabri, both in testing constructions and in exploring a situation, and they exploit many different dragging modalities, both at perceptive and theoretical level.

In a previous project, we identified different modes of dragging students use when solving geometric problems in Cabri, according to different aims (Arzarello, Gallino, Micheletti, Olivero, Paola & Robutti, 1998 (b); Olivero, 1999). The most frequently used are listed below:

- *Wandering dragging*, that is moving the basic points on the screen randomly, without a plan in mind, in order to discover interesting configurations or regularities in the figures;
- *Guided dragging*, that is dragging the basic points of a figure in order to obtain a particular shape;

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4 It is a scientific secondary school. Students attend 5 mathematics classes per week and they use new technologies in the mathematics class.

5 In Italy all the classrooms at all school levels are mixed ability.

- *Lieu muet*[6] *dragging*, that is moving a point so that the figure keeps a discovered property following a 'hidden' path (*lieu muet*), even without actually seeing the path;
- *Dragging test*, that is moving a figure in order to see whether it keeps the initial properties it had: if so, then the figure passes the test; if not, then it means the figure was not constructed according to the geometric properties it was supposed to have.

A fine-grained analysis of students' cognitive processes

We have analysed the solution process of the pair Piero and Gervasio. They are high achievers. They are used to working together and interacting with each other in a productive way.

The problem that was given in the classroom was formulated as follows.

> You are given a quadrilateral ABCD. Construct a square on each of the sides AB, BC, CD and AD, outside the quadrilateral. Construct the centres of the squares and name them E, F, G and H respectively.
>
> 1. After reading the problem carefully, explore the quadrilateral EFGH in relation to ABCD and make conjectures (in the form if…then).
> 2. Prove some of your conjectures.

An external observer who took notes observed the two students. The students' written work was collected as well.

The analysis of the students' solution process shows two key points of students' cognitive activity in this context[7], such as:

a) The transition from **perceptive** (in particular within the Cabri environment) to **theoretical** (towards the logic of proof in geometry) practices, which is shown by:

- a change in the way dragging is used;
- the use of sketches;

6 *Lieu muet* is a French word for *dummy locus*.

7 Only some aspects of the analysis will be described in the paper.

• the transition from perceived[8] objects to generic[9] objects.

b) The continuity from the Cabri exploration to the proving process, which is shown by:

• the use of some elements of the exploration process as starting points for proving[10];

• students' use of language.

We divided the process in three episodes, and each episode is divided in shorter episodes which will be numbered 1.1, 1.2, etc.... The following part contains the protocol and the analysis for each of the episodes.

Episode 1

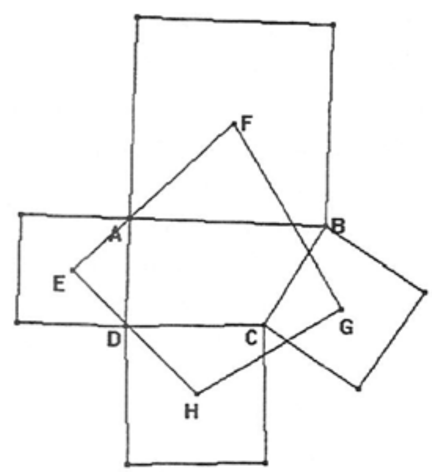

Figure 1 – A configuration showing point A on EF

8 A perceived object is a concrete object that students see, touch or read, i.e. an object they experience through their physical senses. For example, the drawing of a triangle on a sheet of paper or the diagram of a function, or a numer written on paper.

9 A generic object is an abstract object students think of; it has got all the properties of the class of the particular objects it represents. For example, a generic triangle or a function, or a number as a concept.

10 We distinguish proving from proofs. Proving is the process of constructing logical deductions to link some assumptions (hypotheses) with a final result (thesis). The product of this process is to be called proof, that can be either oral or written.

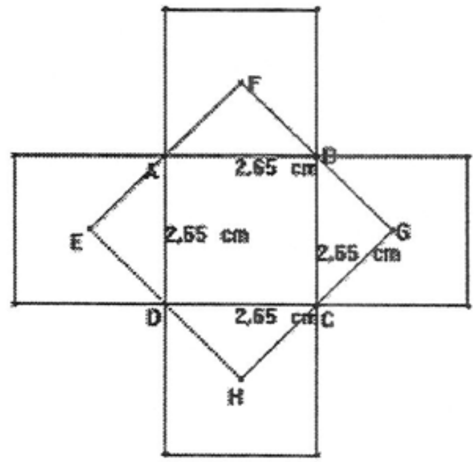

Figure 2: Measurements are added

1.1 The students construct a Cabri-figure. 1.2 They check if the construction is correct using the 'dragging test'[11] (in particular they check the construction of the four squares). 1.3 They begin to explore the situation moving the basic points randomly on the screen; they pass through particular and general cases, concave and convex quadrilaterals, moving quite fast. 1.4 While moving, they see that the vertices of ABCD happen to be either inside or outside EFGH and they realise that there is a moment in which one of the vertices of ABCD belongs to one of the sides of EFGH (Fig. 1). 1.5 Piero: "Maybe it is a square". 1.6 When the students observe that at some point one of the vertices of ABCD belongs to one of the sides of EFGH, they stop moving. Then they drag another point till they get a square (ABCD), they stop dragging and they observe the still Cabri figure. Then they put measurements on the sides (Fig.2). 1.7 They write down the conjecture "If ABCD is a square Þ EFGH is a square. The sides of EFGH pass through the vertices of ABCD".

The construction in Cabri shows a first transition from perceptive to theoretical activity (cf. key point 1.): the students realise they cannot construct a square just 'by eye' (as they tried to do at first), but they need

11 They check if the Cabri figure was constructed according to geometric rules. If the squares constructed on the sides of the quadrilateral stay squares when any vertex is moved, then the costruction is correct. If they mess up, then it means the construction is not correct.

to use geometrical properties. They succeed in constructing the four squares on the sides of ABCD and they check each square is correct through the *dragging test*. This mode of working (constructing a figure and checking the validity of the construction) might become a routine for the students if the teacher makes the opportunity of doing this explicit in the classroom.

In this particular case, the habit of checking each construction through the dragging test before going further with the exploration is part of the classroom culture, because it was promoted by the teacher. This has become a tool students are in control of and normally use. After the construction is completed, exploration begins (1.3)[12]. At first students move points randomly on the screen, doing *wandering dragging*. They drag one of the vertices of ABCD, trying to make ABCD change, considering particular cases and limit cases, concave and convex quadrilaterals. Students' observations about how the figures change on the screen are made at spatio-graphical level (Laborde, 1998), in that they only perceive different types of quadrilaterals, without relating them to one another. The students look at the continuous variation of the quadrilaterals on the screen (we named this way of observing the change in a Cabri figure, a 'film'), trying to perceive information, i.e. properties or invariants, from this. Dragging is quite fast and is aimed at seeing in the figure something that does not change while moving, something that is a 'good idea' to pursue. Such an exploration is typical of the Cabri environment and it could not be done with paper and pencil[13].

In the second phase, a new dragging modality is observed (1.4): while dragging students do not look at the figure as a whole, but they pay attention to single parts of it. Usually this dragging modality involves slower movements than before, and students aim at finding relationships between elements of the figure.

In this first episode, at some point students see that one of the vertices of ABCD belongs to EFGH (1.4); Piero anticipates (1.5) what still has to be seen on the screen: his thinking is quicker than the movement

12 The numbers in brackets refer to the protocol.

13 We observed that students who solved the same problem in paper and pencil did another kind of exploration: they started with very precise drawings of general quadrilaterals and carefully observed them. They did not have the variety of figures as in Cabri. At this point students very often got stuck and the teacher's intervention was needed in order to make them go further.

of the figure in Cabri. *"Maybe it's a square"* is an intuition at a perceptive level (it is difficult to say if it involves theoretical thinking as well). The conjecture is not yet formulated in a conditional sentence (if...then), because the idea of the square is part of Piero's thinking process, which is still in progress.

After Piero's anticipation, the students make ABCD a square, dragging all the vertices (1.6); then they stop dragging and they put measurements in order to check it is really a square. After that, they write down their conjecture (1.7) in a conditional form: *"If ABCD is a square ⇒ EFGH is a square. The sides of EFGH pass through the vertices of ABCD"*. The language has evolved towards a more standard mathematical form (if...then), which is part of the classroom culture.

Episode 2

2.1 Once they are in control of the square configuration, they switch to a rectangle.
2.2 They observe that the other quadrilateral is still a square (without moving anything in Cabri).
2.3 Gervasio: "let's transform ABCD into a parallelogram, then it is a general case. What figure is more general than the parallelogram? A trapezium, but with a trapezium it doesn't work any longer. Then we can prove the general case and the proof holds also for the other configurations".
2.4 The mouse, moved by Piero, follows Gervasio's suggestions and transforms ABCD from a rectangle into a parallelogram.
2.5 After that, the students need to slow down the Cabri rhythm: exploration has finished, so they keep the Cabri figure still on the screen and they start proving, looking at the figure on the screen. They do not immediately succeed in proving the parallelogram case, so they begin with the case of a rectangle and then they move to the case of a parallelogram.

The students use dragging in order to get a precise figure they want to work with (*guided dragging*), that is a rectangle (2.1); they are at a spatio-graphical level and they observe the particular figure they selected, instead of the continuous 'film' of figures. When they get a rectangle they stop dragging (2.2): this shows a transition to a theoretical level, because at this point the students try to discover properties by observing the still figure, i.e. they look for geometrical relationships suggested by their own knowledge and not by Cabri.

In this episode exploration is much more controlled than in Episode 1: the students are in control of the situation and there is also a good synchrony with the Cabri environment.

Gervasio's intervention (2.3) contains a sequence of mental experiments at a theoretical level: he has moved from a perceptive level to a theoretical level, thinking in the geometry world. He is looking for the most general quadrilateral ABCD, which makes EFGH a square, so that he can construct a proof only for this general case that includes the other particular instances (provided more restrictive hypotheses are set). This shows a transition to the logic of proof. Since EFGH is a square when ABCD is either a rectangle or a parallelogram, the students want to generalise this conjecture to the case of a trapezium; however they see that the thesis does not hold in this case. So the conjecture to be proven is now: if ABCD is a parallelogram then EFGH is a square. Exploration is no longer useful, so the Cabri figure the students work on is still (2.5). Instead they use hand gestures and a ruler to point at segments and angles on the screen and they start proving their conjectures. The figure on the screen has now the status of a generic object, the students are working at a theoretical level and they work on that figure to construct a proof.

This episode shows some different phenomena happening at the same time:

- the transition from spatio-graphical to theoretical level, which is shown by Gervasio's words (2.3);
- the transition to the generic object, which is controlled by the students both in the mental experiments, that are the starting point for a new exploration in Cabri, and in the proving phase (2.3 & 2.5);
- the increasing control of the students over the situation.

This episode shows that Cabri is a support for students in the transition towards a mathematical theory because it helps them to see the figure from another point of view, that is to consider it as a generic object, from a theoretical point of view. Cabri supports students towards theoretical thinking. Moreover Cabri seems to influence students' attitude towards the problem, i.e. it avoids they get stuck without being able to go further in the production of conjectures and proofs. Thanks to Cabri, students approach the problem in a condition similar to experts (working without Cabri): dragging makes 'physically' possible all the dynamic explorations and transformations experts usually do in their mind. This situation is typical of students who are used to Cabri and have already internalised its practice through the teacher's intervention, as the software itself does not produce a transition to theoretical thinking.

Episode 3

3.1 Piero and Gervasio prove the parallelogram case, looking at the still Cabri figure on the screen (Fig.3).
3.2 At some point Gervasio makes a sketch on paper representing the parallelogram situation (Fig.4).
Observer: "Why did you feel the need to use paper and pencil?"
Gervasio: "Because I like it better, I can sketch things. I can't do this in Cabri. Constructing is faster in Cabri, better than using ruler and compass. When I start proving I may want to draw a sketch. If I need to discover other things then I go back to Cabri.
3.3 They go back to Cabri and they add the segments they had drawn on paper (diagonals of the squares).
3.4 They go to paper again and they sketch two triangles they want to prove congruent.
3.5 They look at the screen, using ruler and gestures to point at segments on the still Cabri figure.
3.6 Piero: "I know it! I can't remember by which theorem these two are equal…". He proves the property on the screen.
3.7 The students write down their proof for the rectangle and the parallelogram (Fig.5).

In the first part of the episode (3.1) the students work at a theoretical level, because they are constructing a proof for the case ABCD parallelogram, by observing the still Cabri figure. Then at some point (3.2) they feel the need to sketch a figure on paper: this usually happens when Cabri is no longer sufficient to prove an idea that comes to mind. Sketches are used to check what is being said. Sometimes sketches represent only a part of the figure (see fig.4). The use of sketches represents a kind of a-synchrony between Cabri and students' thinking flow: when they have an idea in mind they need to have a figure they can scribble on very quickly, which is possible with paper and pencil but not with Cabri (cf. Gervasio's intervention in 3.2). The sketch embodies students' thinking

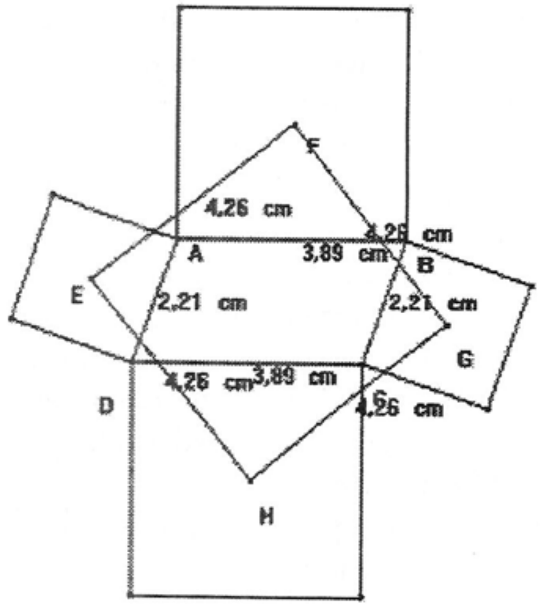

Figure 3 – Cabri figure on the screen

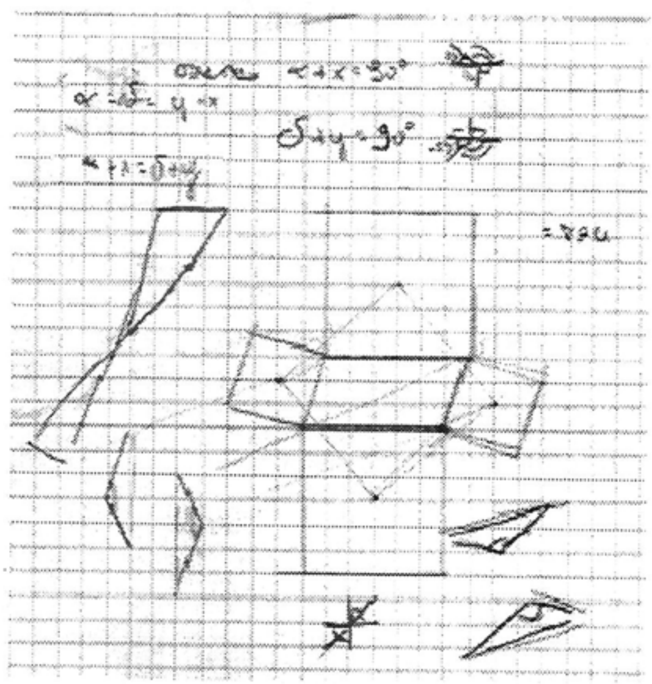

Figure 4 – Sketch on paper

flow, better than Cabri, at two levels. First of all the sketch is immediately available to students, in that there is not a mediator between their thoughts and such a visual representation (whereas in Cabri they have to interact with the software commands before producing an image). A sketch is drawn in real time, while a figure in Cabri is longer to be constructed. Secondly, at a theoretical level, a sketch can be seen by students as the representative of a set of objects, that is as a generic object. For example at some point the students draw on paper two triangles only (3.4), which are proven to be congruent (Fig. 4). Only when Piero is in control of the situation completely, he is able to produce a proof just looking at the screen (3.6), while a dialectic Cabri figure - sketches was crucial in the phase of constructing and looking for a proof (3.2 to 3.5). This dialectic proved richer in high achievers. If this dialectic supported students' proving activity then making it explicit in the classroom, particularly to low achievers, would be a goal to be pursued in a long-term process; the role of the teacher in this process is crucial.

As far as the written proofs are concerned, we observed that they contain bits of the Cabri exploration (cf. key point 2.). The whole protocol shows that students' success depends on the availability of a rich production in the exploration phase (Boero, Garuti & Mariotti, 1996). The starting point of the production of conjectures (the fact that one of the vertices of ABCD belongs to one of the sides of EFGH, see 1.4 to 1.6) is recollected in the proof for the rectangle (Fig.5): *"The sides of EFGH pass through the vertices of ABCD"*; it is then refined in the case of the square: *"The vertices of ABCD are the midpoints of EFGH"*. These two sentences are written down as theses of the conjectures about the rectangle and square, and they are to be proven. Students seem to feel the need to find a justification for what they discovered in the previous exploration phase, and they say: *"A belongs to EF because BAF=45°=EAD and DAB=90°"*. The students want to conclude that, if there are two 45° angles and one 90° angle, then the whole angle is 180° and E, A and F are on the same line.

This shows that Cabri might be a support in bridging the gap between exploration and proof: first, exploration provides students with a wide range of local logical relationships between elements or properties of the figure, then these local concatenations are to be globally rearranged in the proving phase, in order to finally produce a complete proof.

When proving the case of the rectangle, the students exploit the previous observations to say that all the sides of EFGH are equal (because they are the sum of equal segments); however they still need to prove that the quadrilateral has got one 90° angle in order to say EFGH is a square. In order to do this, they use the fact that E, A and F are on the same line (a fact that was discovered in Cabri) and the fact that the sum

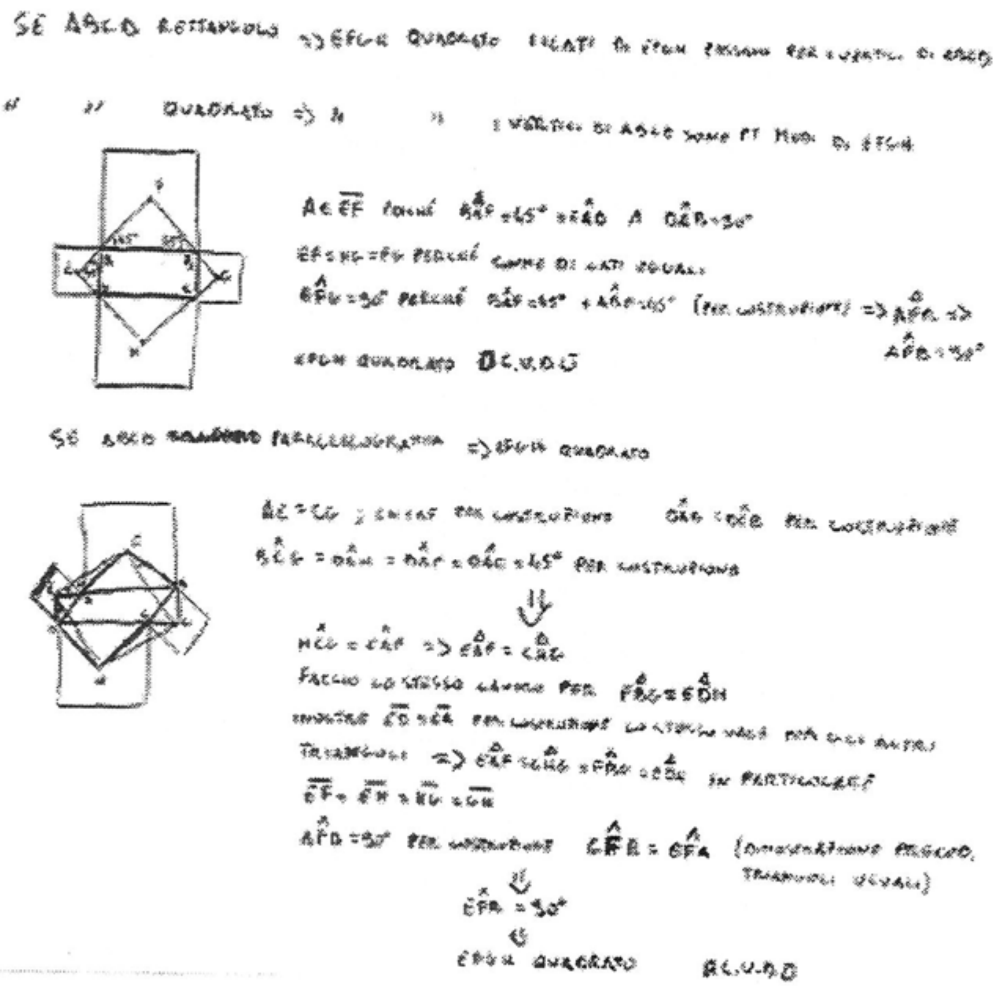

Figure 5 – The written proof

of the interior angle of a triangle is 180° (which comes from their geometrical knowledge); consequently their proof is based on their discoveries and conjectures.

The key point of the exploration phase is the fact that the point A can be moved so that it belongs to EF. This has got a dynamic function: dragging shows that A may or may not belong to EF, because A is on one side of EF first, then it belongs to EF, then it is on the other side of EF. The same fact reveals a key point in the following proving phase. The dynamic function is transformed into a logic function: the essential thing to be proven is that E, A and F are on the same line; then this can be used to prove that the sides of EFGH are equal and that EFGH has got one right angle, so it is a square.

Some Concluding Remarks

According to Balacheff (1998)[14] and Otte (1999)[15], mathematical objects are accessible only through their representations; therefore, having good representations becomes a relevant didactical problem. Laborde (1998) says: "Diagrams in 2D geometry play an ambiguous role: one the one hand they refer to theoretical objects whereas on the other hand they offer graphical - spatial properties which can give rise to a perceptual activity from the individual. This ambiguous role of diagrams is completely implicit in the traditional teaching of geometry in which theoretical properties are assimilated into graphical ones".

In our opinion, representations of geometric objects within dynamic geometry software, as Cabri, are a good way to foster the transition from perceptive to theoretical level. Cabri figures are a midway between empirical and generic objects. On the one hand, they can be manipulated as empirical objects and the effect of this manipulation can be seen on the screen as it happens (this constitutes a feedback, which is typical of the interaction of a subject with an external system)[16]. On the other hand, dragging figures in Cabri allows one "to see the one as a multitude, other than one among others" (Pimm, 1995, p.59).

In the same way, the Cabri figures are in between perception and theory. They can be directly manipulated, so they give information at a perceptive level. At the same time "they have an internal logic, which relates to their construction: the different parts of a Cabri figure are related

14 "Les objets mathématiques ne sont accessibles que par des représentations et la manipulation de ces représentations" (Balacheff, 1998).

15 "A mathematical object, such as a geometrical point, a number or a function, does not exist independently of the totality of its possible representations, but it is not to be confused with any particular representation either" (Otte, 1999).

16 "When the user drags one element of the diagram, this latter is modified according to the geometrical way it has been constructed and not to the wishes of the user. This is not the case in paper and pencil diagrams which can be slightly distorted by the pupils in order to meet their expectations. Computers diagrams are external objects whose behaviour and feedback is no longer controlled by the user as soon as they have been created. Their behaviour requires the construction of an interpretation by the pupils. Geometry is a means, among others, of interpreting the behaviour of these computer diagrams" (Balacheff & Sutherland, 1994).

to each other according to the way the figure was constructed" (Mariotti, 1996, p. 271).

Our classroom experiments have shown that the software itself does not grant the transition from empirical to generic objects, from perceptive to theoretical level. On the contrary, the teacher plays a very important role in students' approach to theoretical thinking. Technology itself cannot bring about an educational change. Very often there is the belief that if the technology used is *good*, then didactics will certainly improve. This assumption does not recognise that a computer based learning environment may be very complex, may need sometime to be usefully exploited (both by the teacher and the students), according to set learning objectives. Generally speaking, using new technologies in the classroom implies the redefinition of contents, methods and of the role of the teacher (Bottino & Chiappini, 1995; Noss, 1995). Simply making a software available does not mean that people will more or less automatically take advantage of the opportunities that it affords (Perkins, 1985).

For example, dragging in Cabri allows students to validate their conjectures; therefore the function of convincing (themselves, a friend or an enemy) proof has in mathematics is no longer useful. The work in Cabri is enough for the students to be convinced of the validity of their conjectures. If the teacher does not motivate students to find out *why* a conjecture (proposition) is true, then the justifications given by students may remain at a perceptive-empirical level: the proposition is true because the property observed on the Cabri figure stays the same when dragging the figure, given the hypotheses do not change. When such a belief is shared in the classroom, then Cabri might become an obstacle in the transition from empirical to theoretical thinking, as it allows validating a proposition without the need to use a theory. However, if the teacher makes explicit the role of proof in justifing, then students will be motivated to prove *why* a certain proposition is true (within a theory), after they know *that* it is true (within the Cabri environment). To paraphrase Polya (1954), first we need to be convinced that a proposition is true, then we can prove it.

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