The social production of school mathematical thinking

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Abstract
Studies of teaching and learning that aim to focus on mathematical thinking generally draw on notions of what might constitute the essence of the thinking of a mathematician. I will examine research that looks at how mathematical thinking is produced in the classroom and at how different forms of pedagogy have different outcomes for different groups of learners. I will report on a number of projects in which I have been engaged that address this theme. I will then consider how sociological and psychological perspectives might combine to inform research on teaching and learning mathematics.
Key-words: *Mathematical thinking; combining sociological and psychological perspectives.*

Resumo
Estudos de ensino e aprendizagem que visam enfocar o pensamento matemático utilizam, em geral, noções sobre o que pode constituir a essência do pensamento de Matemáticos. Neste artigo, examinarei projetos de pesquisa que consideram como o pensamento matemático é produzido na sala de aula e como diferentes formas de pedagogia têm diferentes impactos sobre diferentes grupos de aprendizes. Também discutirei como perspetivas sociológicas e perspetivas psicológicas podem ser combinadas para subsidiar pesquisas sobre o ensino e a aprendizagem da Matemática.
Palavras-chave: pensamento matemático; perspectivas sociológicas e psicológicas.

Introduction

In this paper it is my intention to engage with the 'bigger picture' in relation to researching mathematics teaching and learning. In doing so I will build on two recent publications in particular (Lerman, 2000a; 2001a) although my concern with the theories and perspectives used within the community goes back many years. I start from the notion

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that mathematical competence is defined and produced in school classroom interactions. Everyday activities may be harnessed by teachers and texts to serve the purposes of school mathematics. Explicitly mathematical activities may also be encountered outside school, usually through the actions of parents, other adults or siblings. One might also engage in debates about natural, innate human powers that develop into mathematical competencies. But what counts as doing mathematics and achieving mathematical success depends on what is valued in schools and different conceptions of what is important produce different kinds of mathematical thinking (Boaler, 1997).

I will first introduce some key issues to be taken into account in theory building including a distinction between 'mathematics' and 'school mathematics', political influences, and equity issues. I will indicate how sociological theory enables us to address these issues in ways that neither mathematics nor psychology as fields of knowledge-discourse permit. I will then describe briefly three projects in which I have been engaged in the last few years drawing on the sociological theory of Basil Bernstein (e.g. Bernstein, 2000). In the following section I will argue that an appropriate psychology is also required for educational research and I will discuss complementarity and compatibility in the search for an appropriate psychology. Finally I will present some thoughts on a unit of analysis that, again, builds on my previous publications and brings together the tools we require for study of school mathematics classrooms.

Key issues in researching the teaching and learning of mathematics

Studies of teaching and learning that aim to focus on mathematical thinking generally draw on notions of what might constitute the essence of the thinking of a mathematician. The questions they address are: what is mathematics; what do mathematicians do; how do mathematicians think? These are epistemological questions, to be answered by introspection by practising mathematicians (Polya, 1957; Hadamard, 1945; Davis and Hersh, 1981) or systematic survey (Burton, forthcoming) and we have some very well known and important texts on mathematical thinking based on this kind of approach (e.g. Mason, Burton and Stacey, 1982).

In any case as Burton (e.g. 2001) found, mathematicians describe a whole range of perceptions when asked questions about their practices; any hope for a unified view will not be satisfied. It is scarcely surprising that this proliferation of perceptions of mathematics exists: in these post-modern times we can suggest that fields of knowledge expand, build upon each other and themselves, fragment, proliferate, serve a range of purposes, defend their territory from attack of shrinking resources and changing statuses within Universities. Academics struggle for students, for grants, to get published, whilst teaching more students with greater range of preparedness for their studies.

In my recent work I have drawn on the assumption that school mathematics is different, although related, to mathematics as practised by mathematicians. One of the many things we have learned from postmodern theorising is to ask who benefits by any discourse, or where does the power lie in the power/knowledge carried by the discourse. Searching for the *essence* of what it is to know mathematics might serve the interests of conservative thinkers; they might find such debates useful in their efforts to (re)gain control of the uncontrollable, in this case the learning of mathematics. We see evidence, perhaps, in the Math Wars in California. Whilst many in the community might subscribe to the interpretations of mathematical thinking of Mason *et alii* (1982), for example, that view can be disputed by other mathematicians who argue for a focus on algebraic competence as the essence of what is required from school mathematics. After all, today's mathematicians (and mathematics educators!) went through just such a form of mathematical pedagogy themselves. In engaging with these questions, however, I propose that we agree to abandon the essentialist search and look instead at the sociological question "what does it mean to know *school* mathematics". This shift does not make the target more unitary: in different countries across the world and within countries themselves, school mathematics looks different. Traditional mathematics, skills, drill and practice, child-centred, problem-solving, authentic, reform, ethnomathematics, criticalmathematics are just some of the forms we can find. [We might conjecture that various versions of a 'traditional' pedagogy are the most common across the world although many countries are attempting to implement some kind of reform programme. This latter may be driven by broadly constructivist ideas of how children learn influenced perhaps by ideas of the role of the teacher in scaffolding learning, the Brunerian early version of Vygotsky's zone of proximal development. Here I am touching on theories and their use in research. I won't pursue this further (but see Lerman, 2000a; 2001a).] What the shift does is to change the intellectual field from epistemology to sociology, opening up the possibility of different kinds of systematic empirical study. This is not to deny the importance of epistemological studies of mathematics – they provide important support across a range of aspects of mathematics teaching and learning – but to indicate what can be gained by drawing on sociological theory. A key question for the community has to be why the same social group of students, those from low socio-economic backgrounds, fail in school mathematics no matter what form of pedagogy dominates. How can that be? What explanations are there for that? Sociology can offer explanations that neither mathematics nor psychology can offer. Both of these latter discourses can only talk of a student's failure or deficiency. Psychology is the study of normal behaviour and development and therefore inevitably whoever does not conform is deviant or deficient. I will return to the issue of sociology and psychology as different explanatory discourses below.

There are many agencies at work in the struggle for what should constitute school mathematics. In the UK one of the least powerful agencies in what Bernstein calls the unofficial field is that of teachers and researchers in mathematics education: this is not the case in the US nor in some other countries I expect. The key point is that what we as a community choose – when our relationship to official agents and agencies is such that we (the mathematics education community) are consulted or have some say in the matter – to teach in school is always a selection from what we (or whoever decides) perceive to be Mathematics (academic, in business/industry, etc.). Values are always associated with that choice, values as to what education should be all about, what and whose purposes it serves, and in particular what mathematics education should be all about (ethnomathematics, critical numeracy, traditional content, or whatever). These are political battles, as described so well by Michael Apple (1995), Stephen Ball (2002) and others. The move to setting standards, national testing, international comparisons, inspection of schools and teacher education courses, and so on, are much more about Governments being seen to change what has gone before, setting targets for 'improvement' and being seen to have achieved those targets, as about the expression of the values of that party: getting re-elected is everything. One can look at Bush's "No Child Left Behind" as partly an expression of an attempt to regulate the schools – after all no teacher wants children to be left behind – and partly the kind of sound bite that makes a President popular.

What is much more important, though, is that these battles matter to young people for their future lives and therefore for society as a whole. Who succeeds and who fails in any particular version of school mathematics and why, are crucial research questions for us in mathematics education. When the same social groups are repeatedly failing, we might better say are being failed by schooling, a frustrated and angry underclass results. In the past the underachievement of girls has been studied extensively. Now that girls are performing better than boys in all subjects to the age of 16 in the UK, Australia and other countries, whilst the popular rhetoric calls for studies of the underachievement of boys (in many countries) more nuanced and sensitive research (see Zevenbergen, 2000, for example) reveals the interplay of gender, class and ethnicity in a complex picture of students' achievements.

I would argue that where our gaze is on what is produced in classrooms as appropriate mathematical thinking, on who succeeds and who fails, and on explanatory frameworks for these phenomena, rather than in a search for an *essence* of mathematical thinking, we lay the responsibility on politics, on school systems, on textbooks, on testing regimes and on ourselves as teachers (in schools, colleges, universities or wherever) and not on the cognitive failure of students. If we see any particular 'reform' as a set of choices based on a set of values, rather then a better representation of what mathematics *really is* we can perhaps welcome analyses of who is failing (e.g. Lubienski, 2001) so that we can also try and analyse why.

In the last 5 years or so I have been learning about and working with the theories of Basil Bernstein the British sociologist of education (e.g. Bernstein, 2000). I have found them particularly powerful in bringing together theories of the macro issues of changes in pedagogic forms (what brought about the shift from traditional to liberal-progressive pedagogies for example) with theories about the micro issues of what happens in the classroom and how those changes might affect different social groups in different ways. Following Bernstein, we could say that knowing school mathematics is being able to produce what is considered as a legitimate or approved text in the mathematics classroom. We therefore need to think about how explicit teachers are with students regarding what they need to produce there in order to succeed. In our (Morgan, Tsatsaroni and Lerman, 2002) study of teachers' assessment of students' written texts we saw indications of how teachers have different expectations of students in terms of what they require and value in their mathematics work. Some teachers looked for concise symbolic expression of the underlying pattern in a particular problem, others looked for discursive forms that could be read and understood by a naïve reader. I could put my own value judgement on what I would prefer to see students produce. For the purposes of this paper, however, what concerns me is whether the rules that teachers are drawing upon are explicit to students. Teachers have a crucial role to play here of course but so too do researchers. Cooper and Dunne's (1999) research has shown how everyday contexts can result in working class students not being able to demonstrate their mathematical knowledge, as the context distracts and positions those students within the everyday and not in the school-mathematical discourse. This is surprising to many researchers and teachers since we assume that everyday contexts will enable students to engage with the mathematical content and provide meaning that can help them produce correct answers.

In general, following Bernstein, we can say that the rules for producing texts that are appropriate in the mathematics classroom are invisible in reform or liberal-progressive classrooms and whilst middle class children are not disadvantaged by invisibility, working class children are disadvantaged. Although the rules in traditional classrooms are visible and therefore equally accessible to all in principle, clearly they have their own drawbacks too. As teachers we might want to stick to our liberal-progressive mode (or developments within this model towards ethnomathematics or criticalmathematics) but make the rules more explicit. As researchers (or teacher-researchers) we would want to research the effects of such 'pedagogic engineering' in an attempt to improve the opportunities for success of all our students.

I will describe here in more detail three studies in which I have been engaged recently, drawing on Bernstein's work.

Current research

In the first of these studies we (Morgan *et alii*, 2002) carried out a re-examination of Candia Morgan's (1998) research on how teachers assess students' coursework in mathematics when they produce written texts of investigations in high stakes situations. Morgan's study grouped teachers' responses into eight categories: examiner using externally determined criteria; examiner, setting and using her own criteria; teacher/advocate looking for opportunities to give credit to students; teacher/adviser, suggesting ways of meeting the criteria; teacher/pedagogue, suggesting ways students might improve their perceived levels of mathematical competence; imaginary naïve reader; interested mathematician; and interviewee.

Drawing on and developing Bernstein's theories we were able to re-classify these into four positions based on whether teachers were orientated towards students or towards the mathematical texts and whether they drew on official (mainly advice from the examination board) or unofficial (teachers' own) discourses. This systematic, theory-driven re-analysis enabled us to make many observations that we could not make as a result of the former empirical classification, including the following:

> ... the approach that we have provided enables a conversation between the theoretical and empirical fields of the research focus, and allows us to understand teachers' relationships to the discourses at play in evaluation practices. Beyond assessment, the theoretical framework allows us to take account of social forces when studying teaching, teachers, and differences between teachers. (p. 459)

Furthermore,

> Bernstein's framework enables a more elaborated language for describing the mechanisms whereby social forces impact upon schooling. Without such a language, connections with the ideologies of social groups remain covert, hindering possibilities of resistance. (p. 459)

The model is currently being developed and applied to another study entitled "The production of theories of teaching and learning mathematics and their recontextualisation in teacher education and education research training" (see Lerman, Tsatsaroni and Xu, 2003; Tsatsaroni, Lerman and Xu, 2003). The aim of our research project[1] is to analyse the processes whereby mathematics educational 'theories' are produced and the circumstances whereby they become current in the mathematics education research field, are recontextualised, and are acquired by teachers and teacher educators. We are constructing a representation of the field of mathematics education research through which we are exploring the reproduction of identities, as positions, of researchers and teacher educators in the field, the recontextualisation of pedagogic knowledge and the reproduction of identities of mathematics teachers. We have explored, as sub-questions, who produces theories in mathematics education, with what methodologies and to what consequences for research and for school practice? Through examining the structure of the knowledge-discourse in its field of production we have explored the conditions and factors that affect the movements of the positions within the discourse thereby exploring questions such as the following: who are managed, whose identities are produced and who are the managers of these identities (e.g. the funding agents, journal publishers etc.). We are currently working on a model to talk about identities of academics, and changes in those identities over time and place. We are looking both at the intersections of the mathematics education research community with other research communities, such as science education research, educational research, psychologists, sociologists and mathematicians; and also with other 'stakeholders' of mathematics education research such as central and local education authorities concerned with education policy, parents, teachers and others.

Finally, in the project *Teaching and Learning – Mathematical Thinking*[2] we have been seeking to develop and integrate theoretical

1 The full text of the project proposal is at http://www.sbu.ac.uk/cme/ESRCProjectHOMEPAGE.html

2 *Teaching and Learning – Mathematical Thinking* has been supported by the Fundação para a Ciência e a Tecnologia, grant no. PRAXIS/P/CED/130135/98.

approaches to the study of aspects of school mathematics (see Carreira, Evans, Lerman and Morgan, 2002). The theoretical approaches we have utilised have in common a focus on the socially organised nature of thinking as it is embedded in social practices. An important part of the project has been the attempt to apply our theoretical concepts to study empirical data. This has raised a number of methodological issues.

At the beginning of the project, we chose a number of substantive aspects of school mathematics teaching and learning on which to apply the developing theory. In reflecting on the processes of research, the question is raised: how is the research object (e.g. transfer, assessment, emotion, mathematical thinking) identified? The 'problem' is present (that is, it is named) in the field of mathematics education research but it does not actually become a research object until we bring theory to bear on it. We must ask, therefore, what is the role of theory in constructing the object of research.

Given our focus on the social organisation of phenomena in mathematics education, it is only consistent to ask also about the ways in which specific practices of research in general, and of mathematics educational studies in particular, influence the construction of the research object and the ways in which the theoretical is linked with the empirical. In addressing this question it may be useful to categorise different kinds of theory, for example, metaphors, conceptual models, or what Maton (2000) calls theories orientated to knowledge or to the knower. This also raises the issue of the relevance of questions about the relationships between participants in research practices and the identification of the research object, that is, questions such as: Who identifies and defines the research object? To what extent is it the researcher's research object and how are other participants in the community (of mathematics education, education, research, etc.) present in its definition? How are the participants and the educational structures affected by the research and how is their voice heard?

Sociology and Psychology

I have found sociological theory to be very fruitful and indeed powerful in these studies. For studying teaching and learning, however, they are not enough. What we require, in my view, are a *process* and a *mechanism* of learning. Sociological theories could be said to provide us with a way of understanding how society produces and reproduces identities. Forms of pedagogy provide the power and control mechanisms whereby the social relations of the mathematics classroom, for instance, position students (and teacher) such that certain social groups are advantaged and others disadvantaged. What is needed is a *complementary* psychology to enable us to articulate how individuals are regulated and how learning comes about. These are not addressed in sociology (Daniels, 1993). Complementarity is not just an issue of inserting a theory into a space where theory is needed: the manner in which different theories are claimed to be complementary needs to be examined and justified. I say this because it is quite common within the mathematics education research community to find researchers taking an individualistic psychology and 'complementing' it with elements of the social situation of the classroom. Without care, theories that are largely incompatible, perhaps contradictory, may be put together under the umbrella term of complementarity. I would argue that compatibility needs to be sought between discourses that are predominantly independent, such as sociology and psychology, not assumed. Sfard (2001) indicates just such a need when suggesting that traditional approaches to teaching, drawing on an acquisition metaphor, can be thought of as complementary to thinking-as-communicating, drawing on a participation metaphor. Sfard offers two analogies, that of Euclidean and non-Euclidean geometries and the incommensurable theories of light as wave and as corpuscle. The former seemingly incompatible theories are brought together through perceiving of them from the perspective of axiomatics: the choice of different sets of axioms results in different geometries. The latter may be brought together if physics ever arrives at a unified field theory. How do these help us as exemplars for theory building in mathematics education research? I am not sure that they do. We can describe psychology and sociology (and philosophy and…) as parallel knowledge-discourses which may address the same objects (learning for example) in quite different ways and with quite different tools. They have a horizontal, rather than hierarchical, relationship to each other; there cannot be a subsuming discourse. Thus the first analogy doesn't model the situation. Educational knowledge production needs to draw on several intellectual fields and needs to look to practice as well. For this reason Bernstein (2000, p. 52) calls education a region rather than a field, and likens it to medicine. If we are to draw on psychology and sociology we should look, perhaps, for a common metaphor or common foundations to ensure that we are not trying to put together incompatible theories. Examples I offer of incompatibility are: 'learning leads development' and 'development leads learning'; "learning without being taught" (Papert, 1980, p. 7) and "instruction... determines the fate of (the child's) total mental development" (Vygotsky, 1986, p. 157). Thus, as I have argued at length (Lerman, 2000b) to put together a Piagetian constructivism (the first part of each of these dichotomous pairs) with extracts of Vygotskian cultural psychology (the second parts) are highly suspect. We can and should avoid incompatible and incommensurable theories if the results of our research are to be coherent.

I have suggested elsewhere that Vygotsky's psychology is a good candidate for a complementarity with Bernstein's sociology (see Daniels, 1993). Both look to Marx for the foundations of their theories; they regard consciousness as social products. Where Bernstein argues that different forms of consciousness are produced according to one's relation to the means of production, in this case of symbolic production rather than material production, Vygotsky provides both the mechanism and the process whereby consciousness is a socio/historico/cultural production. The *mechanism* is the adult or more informed peer in the zone of proximal development; it can also be "the student's imaginative play and the child's solving a problem at home relying on a model that has been shown in class" (Gredler and Shields, 2003). The *process* is internalisation and in particular the development of higher mental functions. Thus Vygotsky's cultural psychology enables us to examine in microgenetic detail the effects of the regulation of social practices described by Bernstein in such locations as the mathematics classroom.

Unit of analysis

My attention was first drawn to the issue of a suitable unit of analysis by Vygotsky's call for the bringing together of affect and cognition and the identification of a unit of analysis for this. In general, research is framed by what one chooses to look at, and necessarily by default what one does not look at. If in principle one wants to work from within a sociocultural perspective the issue of the choice of object of research and the unit of analysis is crucial. In Lerman (2001a, p. 98), seeking to develop a unit to take into account Bernstein's work, I wrote the following:

> The title of Vygotsky's book *Mind in Society* captures that unit, and it is also expressed by Lave and colleagues as person-in-practice and by Wertsch as person-acting with mediational means (Wertsch, 1991, p. 12). We could extend that unit further by taking account of the discussion of the regulating features of social, discursive practices. As a person steps into a new practice, in social situations, in schooling, in the workplace, or other practices, the regulating effects of that practice begin, positioning the person in that practice. Goals and needs are modified by the desire to participate, the desire not to participate, or the many other possible positions. Even if a person withdraws from a practice after a short time, she or he has been changed by that participation. We might therefore talk of practice-in-person to capture the regulative effects of participation. Combining these, we might talk of a unit of analysis of person-in-practice-in-person, or mind-in-society-in-mind. (Slonimsky, 1999)

In that paper I attempted to outline a toolkit for research to work with the unit of analysis person-in-practice-in-person. I also attempted to bring together analyses of details in a classroom at the finest level and macro-issues of social forces on education through the metaphor of a zoom lens. Elsewhere I wrote the following:

> I want to suggest, though, that psychology can be seen as a moment in socio-cultural studies, as a particular focusing of a lens, as a gaze which is as much aware of what is not being looked at, as of what is. This is an adaptation of Rogoff's planes of analysis, into a dynamic metaphor in which one might envisage a researcher choosing what to focus on in research through zooming in and out in a classroom, as with a video or still camera, and selecting a place to stop... A discursive, cultural psychology locates its

> interpretation of the individual at the intersection of overlapping language games in which the person has developed and thus is necessarily rooted in the study of cultures and histories. Draw back in the zoom, and the researcher looks at education in a particular society, at whole schools, or whole classrooms; zoom back in and one focuses on some children, or some interactions. The point is that research must find a way to take account of the other elements that come into focus throughout the zoom, wherever one chooses to stop. (Lerman, 2001b, p. 4)

I want here to try to operationalise the unit of analysis in another way. As I have already mentioned, there is much talk in the mathematics education community of the need to merge the social with the cognitive. I will interpret this to mean that there is a need, when working from a sociocultural perspective, to be able to explain what happens in the individual's mind and to answer the question of what constitutes the individual's knowing. Billet (2002) proposes a three-part analysis, the sociocultural, the situational and the ontogenetic. The sociocultural carries the history of the practice whilst the situational carries the context specific, regulatory aspects of the practice at the local level. These together, constitute the sociogenetic. The ontogenetic conveys the personal histories of the individuals, although these too have social origins. The 'cognitive' is interpreted as what ends up in the goals/actions/procedures of the actor.

Therefore, considering goal-directed activities within a cultural practice provides a basis from which to understand the relations between sociogenetic sources and ontogenies (i.e. the relations between social and cognitive experience) through an examination of the inter-psychological processes that comprise the enactment of these activities. (Billet, 2002, p. 139)

Billet's study is in a vocational setting, that of hair-dressing, and he examines a range of hair-dressing salons in Australia and abroad in order to identify (a) what is in common across salons and in the practice of hair-dressing, the sociocultural, (b) what is unique to a particular salon, the situational, and (c) what is idiosyncratic to an individual hair-dresser, the ontogenetic. He develops criteria by which to determine the particular character of actions or utterances.

I want to discuss here how we might apply his model to the mathematics classroom. I will do this by presenting some data that I have analysed previously by looking for the emergence, or otherwise, of zones of proximal development in mathematical activity of a pair of students (Lerman, 2001c). Here I will propose an extension of the data collection to enable an analysis using Billet's approach and I will then conjecture what might be gained by such an analysis.

The extract I chose from amongst many hours of video recordings of a particular 8th Australian grade class and their teacher was of an episode of instruction of a sub-group of the class, followed by the work of a pair of students. The teacher set some ratio questions in what she called a 'ratio pep test' to all the students in the class, telling them to cross out the ones which contained algebraic terms. She then called several of the students to the front of the class, the ones who elsewhere she referred to as 'those who like working ahead'. She gave these students some extra instructions on cancelling algebraic terms in fractions and ratios, which later she called an 'algebra trick', so that they could also answer the crossed-out parts of the question. At the end of the transcript her "Bye" (utterance 15) sent them back to their desks to work on all the ratio questions.

1.	T:	Just working ahead a little bit?... OK. Now, I'm going to think of three numbers, right?, x is going to be 7, y is going to be 9, and uh, m is going to be 3. OK? Now I'm going to multiply x by 5. I would write it as 5x. OK? I'm going to multiply y by 5, how would I write it?
2.	Ss:	5y.
3.	T:	OK. And I'm going to multiply m by 8.
4.	Ss:	8m.
5.	T:	All right. Now, I'm now going to divide x by 5. Now what's going to happen if I do that?
6.	Ss:	Be the same number.
7.	T:	Ah. It's going to go back to the same number. All right. I'm now going to take this, um, I multiplied m by 8, I'm now going to divide it by m. What am I going to be left with?
8.	S:	Eight.
9.	T:	Um. Is it?
10.	S:	Yes.

11. T: Right, m is 3, 8 times 3 is 24, divided by 3, brings it back to 8. Do you notice that this one you told me brought it back to 7. Seven times – 5 times 7 is 35, divided by 5 is 7. Good. And this one here you told me went to 8. Now can you see a pattern?
12. S: Yup.
13. T: Right, if anything's on the top I'm multiplying, if anything's underneath I'm dividing. So this is actually a multiply by, and this is actually a divide by. Can you see how they cancel each other out?
14. Ss: Yeah.
15. T: So really you say 5 into 5 goes once, and 5 into 5 goes once, so really I've got 1x over 1, which is just x. And this one here is I've got n, which is a number and I'm going to divide it by itself. They cancel out and give me 1, so I've just got 8... Bye.

The extract below is the work which the pair, named here D_ and M_, undertook when they sat down after the extra instruction. There were a few interchanges in which they confirmed what they were supposed to do. The conversation begins immediately after those interchanges when they began work on the task "Simplify ab:ab".

1. M: What? Equals ab? [pause, D looks on M's page] Equals ab?
2. D: Yeah.
3. M: No, it equals one.
4. D: Wait a second...
5. M: 'Cause one, [punching calculator buttons] twelve times tw... no. One, look, look, look. One times two, divide one times two...it shouldn't equal four. [M appears to be substituting the values one and two for a and b]
6. D: [laughs]
7. M: Um, yeah, it's, 'cause I'm doing [punching buttons] one times two, divide one times two, equals one.
8. D: So that's cancelled. The two b's are cancelled out.
9. M: Equals one.
10. D: Right? The two b's are cancelled out.
11. M: Hey, where'd my pen go? No come on, look, look, look, look. You've got to do BODMAS. Watch, watch, watch, watch. [punching buttons] One times two, divide one...come on, one times two. That's stuffed up. [with emphasis] One.
12. D: ... I'm going to ... this is ... better ....

13. M: Look, look, look, look at this one, look at this one.
14. D: ... Hang on ...
15. M: Divide.
16. D: ... I'm going to do these, this one first.
17. M: Equals 1, it does equal 1. I've got to do this first.
18. D: Takes [inaudible]. [pause] Two.
19. M: Oh, reduce to the simplest terms. Oh, OK, um. [pause, punches calculator] All right. Shit. [pause, both work]
20. D: [inaudible] here and speed.
21. M: One point 1? [inaudible]
22. [T talks with nearby student]
23. M: Mrs B_? Mrs B_, do we simplify that as well? Question one [?]?

24. T: Yes, you do.
25. M: OK.
26. T: And you're right.
27. M: OK.
28. T: [looking at M's work] So you got one right and you got two right.
29. M: I know.
30. T: Yep. Ah no because it's a ratio. Oh, I guess you could say yes, I'll accept that. Yep.
31. M: Thank you. [T goes to A and L's desks]
32. D: We don't go on to that yet.
33. M: Yes, we do. [with emphasis]
34. D: Are we supposed to do that?
35. M: Yes. [with emphasis]
36. D: Yeah, that [points with pencil on M's paper], that's not, that equals ab, doesn't it?

Here the unit of analysis is not a particular conversation, in fact, but the social practice as a whole and its effects on the activity of the pair since one is examining the three aspects: the sociocultural, the situational and the ontogenetic. The first element in Billet's model is the sociocultural, the history of the activity. It consists of school mathematics as it is practised in the state and the school as expressed by a curriculum and perhaps guidance or requirements of forms of pedagogy (this describes the situation in Australia – in other countries it may be as practised in the form of pedagogy of a whole country or of a school district, and it may be by fiat or by tradition). One could access the soiocultural both through looking at curriculum documents used by the state/school and by looking across at other classrooms covering the same content. The sociocultural would be what is in common across those classrooms and 'rules' for the identification of commonalities would need to be developed in interaction with the data. Bernstein's pedagogic device (2000, pp. 27-37) could be drawn upon to provide an appropriate set of tools for such an analysis.

Second, the situational would focus on what is specific to this teacher and her classroom and how she sets up the possibilities for students' mathematical activity. I will list and number these here and then make direct references to the transcripts. In the extract above, in terms of mathematical strategies, she offers (1) both cancelling of common factors in denominators and numerators and substitution of particular numbers in place of letters to gain a sense of what is meant by an expression. We can also note that she does not distinguish between fractions and ratios (2). The teacher has positioned some students as more able (3) and students are made aware explicitly who is considered to be in this group and, by default, those who are less able. The teacher expects students to work in pairs and sets up the classroom so as to enable shared work. She is not averse to noise, nor to students talking beyond their pairs when comparing answers and methods (4). The use of calculators is encouraged (5).

Evidence of the effects of the situational can be seen in in relation to these 5 points:

1. M_ works with substitution, at least in the attempted demonstration of the correctness of his answer of 1 as can be seen in utterance 5 and beyond. D_, on utterance 8, is drawing on rules for cancelling to begin his independent work on the task.

2. M_ comes up with an answer of 1, rather than the ratio 1:1, in utterance 3 and the teacher accepts that answer on utterance 30.

3. Of the pair of students D_ and M_, the latter is always one of those called out. In this instance one of the 'able' students chose not to join that group. D_ asked if he could join the group for this occasion and the teacher agreed. In general however D_ was positioned as less able than M_. In a subsequent interview the teacher said, on looking at the video of the two boys working, "And the fact that he's helping D_ with this is fantastic because it'll help M_ in the process... And D_ I think is just keeping it there at the same pace....".

In fact D_ seems not to follow M_'s explanations and turns away to work alone, as seen in utterances 4, 8, 10, 12, 14 and 16. In the video D_ appears to be somewhat despondent. When the work was handed in D_ had crossed out his own answer and written "1".

4. Although not obvious from the transcript, the video shows students interacting, although 'off-task', between pairs.

5. The free use of a calculator has certainly helped to structure the interchange between the two students and M_'s failure to get the calculator to produce the answer he wants is a factor in the lack of useful communication between the students.

As observers we might want to note that D_'s work using cancelling was quite correct and more general in its applicability than M_'s substitution. D_ did not pursue his method, however, and M_ might only have been using cancelling to demonstrate the correctness of his solution. In general, to distinguish between what might be idiosyncratic to either or both of these students rather than the effects of the situational we would need to look at the work of several pairs of the students in this class working on that task at that time who were in the group of 'those who like working ahead'. I have attempted to demonstrate, however, the kind of analysis that might take place when identifying the situational.

Third, looking at the ontogenetic would require further data, such as interviewing the students as they work individually on similar problems as soon as possible after the lesson and at least before further instruction.

Concluding remarks

Finally then, what might be gained by using Billet's analysis in this way? We are able to look at the cognitive as it is regulated and produced in the sociogenetic circumstances. We are able to look, also, at what students take from the learning activity, again enabling an examination of the individual, ontogenetic activity arising from the inter-subjective. At the same time we gain a perspective on the sociogenetic itself and we are able to distinguish the features of the practice that are characteristic of that particular classroom, set against the context of the school and the curriculum more generally. The unit person-in-practice-in-person can be usefully examined from Billet's framework to provide insights that draw on sociological and the psychological discourses in a complementary way.

To return to where I began the paper and the arguments set out here, Billet's analysis, and indeed the Vygotskian one I gave in Lerman (2001c), are focused on looking at how mathematical thinking and mathematical competence are produced in mathematics classrooms. I have chosen not to continue and contribute to epistemological analyses but instead to work with sociological theories, complemented by what I have argued are compatible psychological theories. In placing notions of what counts as successful mathematical activity within classroom interactions locates the burden of achieving the success of as many of our students as possible in the social context and not in the individual's ability or its lack.

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