Connecting Factors for the Application of a Digital Algebra Learning System: A Study with Textbook Author

Facteurs de connexion pour l'application d'un système d'apprentissage d'algèbre numérique: une étude avec un auteur de manuels

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Résumé

Le projet MAL (Multimodal Algebra Learning) développe un système numérique d'apprentissage de l'algèbre. Un problème central est son applicabilité dans les écoles et les manuels scolaires. Cet article part d'une étude partielle du projet qui se propose de découvrir des facteurs de rattachement qui permettraient au système de s'adapter aux praxéologies de l'école et des manuels. Nous présentons ici l'étude de départ et les résultats préliminaires de notre recueil de données.

Mots-clés : TAD, Apprentissage numérique, Manuels, Apprentissage de l'algèbre.

Abstract

The MAL-project (Multimodal Algebra Learning) develops a digital system for learning algebra. A key issue is its applicability in schools and textbooks. The paper reports about a partial study of the project which aims at uncovering connecting factors that allow adapting the system to praxeologies in schools and textbooks. In the paper, the study and preliminary results are presented.

Keywords: ATD; Digital Learning, Textbooks, Algebra Learning.

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The project presented in this paper is part of the MAL-project (*Multimodal Algebra Learning*) (Janßen et al., 2017). Its aim is to develop an interactive digital algebra learning environment (MAL-system) enhanced with tangible user interfaces, so called smart objects (O'Malley, Fraser & Danae, 2004). These smart objects combine virtual and haptic features making the students interact multimodally with the system and receive automated feedback on their actions.

The tangibles are designed in the style of Algebra Tiles (Dietiker, Kysh, Sallee & Hoey, 2010). In the project they are supposed to cover at least three thematic branches: solving linear equations, working on quadratic expression, and algebraic patterns. The basic idea of Algebra Tiles is to represent integers and variables according to the shape and colour of a rectangle. There are three different types of shapes: a small square with side length one representing the number one; a rectangle with side length one and side length x representing x; and a large square with side length x representing x^2. Laid out on a surface the tiles are added and represent an algebraic expression. The determination of a left and right side allows stating an equation. Equations can be solved by laying out additional tiles or taking away the same tiles on both sides.

There are two possibilities to express subtraction with Algebra Tiles. One is to introduce a subtraction zone, where all the tiles within this zone are supposed to be subtracted. The other is to introduce negative tiles. Negative tiles have the same shape as the positive ones, but they have a different colour, usually red. There are more features about Algebra Tiles that are not mentioned at this place, because it is sufficient to know the basics in order to follow this paper.

From the beginning of the development of the MAL-system, its applicability in schools is a central issue. Therefore a specific partial project investigates the institutional

conditions of teaching and learning algebra in schools and textbooks, conceptualised as praxeologies (Bosch & Gascón, 2014).

Textbooks as well as teachers have their own praxeologies. These praxeologies evolve into school praxeologies when they are put to use in class. In contrast, the MAL-system will also bring into play its own praxeologies shaped by the researchers. Hence to ensure the applicability of the MAL-system, it must be adaptable to the prevailing praxeologies in schools and the textbooks used. Based on that the research question is formulated:

What are the connecting factors within a conflicting situation of different praxeologies, especially when the MAL-system is supposed to be used in class or adapted to textbooks?

Methodology

The specific methodological approach is to reconstruct these connecting factors of the praxeologies within groups of textbook authors. These groups represent the praxeologies used in textbooks or are at least familiar with them, as they are the developers of the textbooks. Furthermore they are in-service teachers. Thus they are also familiar with central praxeologies within schools, which use the same book.

In order to find these connecting factors, the author groups were asked to take part in a group discussion. Group discussion is a familiar practice to the authors, as this is what they do when developing the textbooks. Within two different groups the topics algebra learning, digital learning, gamified learning, and the MAL-system were discussed. The authors were given input on material, applications and conceptions, which are unknown to them. In this way not only the praxeological elements of the textbooks become visible but also possible connecting factors for the presented input within the team. In addition the constraints, obstacles and challenges when introducing the new

system in school will become apparent. Thus it will be possible to find favourable conditions for the MAL-system and to indicate where resistance can be expected.

Although the textbook praxeologies are dominant in the discussion, the authors also take into account their individual and school praxeologies. These will be the focus in a second round of data collection. In this round, compressed results of a first analysis of the group discussions are given back to the participants individually. The authors are asked to comment on the results and to complete a questionnaire which still needs to be constructed. In the style of Delphi methods (Linstone & Turoff, 1975) there will be several iterations of this procedure.

Preliminary Results

The data collection is still work in progress. Therefore, just preliminary results of the two discussions are presented. In group A, working on a book for comprehensive schools - Maths Alpha - twelve authors participated in the discussion; group B counts five participants and is working on a book for low achieving students, called Maths Beta.

From each discussion, a short section shows, the authors comparing Algebra Tiles with the balance model, which is widespread in Germany. Using a balance with two pans, additive equations with positive integers can be modelled. Therefore, the numbers in the equation are represented by blocks with weights according to the number and the variable is represented by blocks with unknown weight. In both sections the authors came up with the balance model on their own, without any intervention from the researcher.

The preliminary analysis of the data is a praxeological analysis. Starting from the mathematic praxeologies the aim is to recognize parts of didactic praxeologies. In the chosen sections the mathematical praxeologies are made up of tasks where pupils are asked to solve linear equations. These tasks are not explicit in the discussion, but as the

authors are talking about tools to help students learn and understand how to solve equations, this is part of the mathematical praxeology.

For solving these tasks, the authors consider two different techniques. The first one is using the scale model and the second one is using Algebra Tiles. Since the authors' interest focusses on the didactic praxeologies, I skip the explanation of the logos part of the mathematics praxeologies.

The first section displayed here is taken from the discussion in the Maths Alpha group:

> Expert1: I don't think that it is suitable for solving equations. I rather take my balance model that I can stick on the board. This is easy for the kids, easier to understand. And they know the balance. They know what it means if the same is done on both sides. For me this [Algebra Tiles] is too abstract.
> Expert2: Obviously the problem of the balance model is that I cannot express negative numbers with it.
> Expert3: Yes, well, negative doesn't work.
> Expert2: This somehow makes it [Algebra Tiles] quite attractive I think. That if you have something like 5x+7-8x+6 the one can slide together the xs, that cancel out each other. Well, this also always helps and with the balance this not possible. To that extend it is […]
> The first expert offers a didactic technology and points at a theory supporting the

mathematical technique of using the scale model and rejects the use of Algebra Tiles. The technology "the scale model is easier to understand for kids" is underlined by the statement "they know what it means if the same is done on both sides". This shows aspects of a theory that backs up the mentioned technology.

Then Expert2 comes up with a technology supporting the mathematical technique of using Algebra Tiles. She[2] recognizes that the limits of the scale model concerning negative numbers can be evaded when using Algebra Tiles. This technology is related to the scale model but in the further course she detaches it when saying that something where positive and negative variables can be cancelled out is helpful. The theory in this case is

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[2] The gender is not taken into account so I always use the feminine pronoun, however both genders were represented in both groups.

hidden. To find out which theory Expert2 has in mind, the question why this cancelling out is helpful has to be answered. The answer could be either an individual empirical theory or an elaborated didactical theory.

A similar development can be identified in the discussion of the Maths Beta authors:

> Researcher: […] Can you imagine that it makes sense to use Algebra Tiles in class?
> Expert1: Honestly, I cannot imagine that. I think that the balance model is much easier.
> Expert2: More illustrative as one has the pans, well the balance in mind. That has to be balanced all the time. One has to introduce these [Algebra Tiles] first and this concept why is this x, why is this one and th- (sighs). It is so, one has to, well this is again a new introduction that the kids must keep in mind. So for our clients I think it is very abstract.
> Expert3: Uhm I find it uhm not so not so bad (laughing). I only had uhm myself uhm with this laying and the subtraction zone, minus three, why do I place it there and sure they lay out like this. But this is, this needs to be told to the pupils, why uhm they are placed in the subtraction zone or wheresoever. But if this is clear, then everything else is clear. I balance it and take it from one side. Uhm pff the balance only helps by imagination but it does not help in real life. Because we cannot place the tiles there. And here I can do it, so I find it
> Expert4: I also think, well I have uhm (incomprehensible) tell pupils when one also tells them that the equals sign effectively stands for the balance. It must be the same amount on both sides. I think in this way the equals sign should be, well not only as a sign leading to the result but one says it has the meaning that on both sides must be the same. This could be introduced with the balance. Then I think one could get to a representation like that. I mean, the subtraction zone surely is something abstract. Well to say here is something that actually is something that is to be taken away. This is a demanding conception. Basically I think this laying out on both sides and maybe in the beginning only using additive elements. I can imagine that.

In this section the researcher implicitly gave the didactical task to integrate Algebra Tiles in classroom practice. The openness of the question provides space to rejecting and integrating the tiles. It allows finding obstacles and constraints that are in the authors' minds and gives space to a process of approaching the integration of Algebra Tiles. This process is highly interesting as it directly reveals connecting factors.

At first Expert4 rejects the Algebra Tiles using the same technology as Expert1 in the Maths Alpha discussion. She only states that the balance model is easier. Expert5 then

goes more into detail outlining parts of a theory, "the balance has to be balanced all the time", which is again similar to the theory of Expert1 (lines 4-5, Maths Alpha). In addition Expert5 gives a technology that explicitly rejects the use of Algebra Tiles, saying that it "is again a new introduction that the kids must keep in mind". This statement refers to low achieving students. The implicit theory in this case could be that learning new things is hard for those low achieving students.

Expert6 then brings in a new point of view. She first argues on the level of a didactic technique. For her it is necessary to give the pupils an explanation of the mathematic techniques for the use of Algebra Tiles. Having done this Expert6 does not see any problems in working with Algebra Tiles. Similar to Expert2 from the Maths Alpha discussion she sees an advantage over the balance model. She reasons that the *concrete action*, which can be performed with the tiles, is more helpful than having *the balance in mind*. This is a technology she presents to justify the mathematic technique of solving equations using Algebra Tiles. The theory again is hidden. It would be the answer to the question why the concrete action is more helpful in this case.

Conclusion and Outlook

The two sections presented above offer an insight in the collected data. Especially similarities within the groups are shown here, however, also many differences can be found throughout the discussions. In both cases one of the experts detects a need (the ability to express negative numbers, and the concrete action with the tiles) that cannot be satisfied with the elaborated balance model. This is the starting point for the consideration of using Algebra Tiles. It shows that new material can be better accepted if it fills a gap of the familiar material. This provides an idea of how connecting factors can be identified in the discussion.

The next step after the preliminary praxeological analysis is a discourse analysis. The aim is to see how the discussed praxeologies change and develop during the discussion. The second step in the data collection will be the organized feedback with questionnaires addressing the praxeologies of schools and detecting differences between the praxeologies identified in the author groups, and the individual ones, which are clearly influenced by schools.

Acknowledgement

References

Bosch, M., & Gascón, J. Introduction to the Anthropological Theory of the Didactic (ATD). In: *Prediger, Networking of Theories as a Research Practice in Mathematics Education,* SpringerInternational Publishing Switzerland, S. 67-83, 2014.

Dietiker, L., Kysh, J., Sallee, T., & Hoey, B. *Making Connections: Foundations for Algebra, Course 1*. Sacramento, CA: CPM Educational Program, 2010.

Janßen, T., Reid, D., Bikner-Ahsbahs, A., Reinschlüssel, A., Döring, T., Alexandrowsky, D., et al. *Using tangible technology to multimodally support algebra learning: The MAL project*. Poster Presentation at CERME10. Dublin, 2017.

Linstone, H. A., & Turoff, M. (Eds.). *The Delphi Method: Techniques and Applications.* Reading, MA: Addison-Wesley, 1975.

O'malley, C., Fraser, S., & Danae. Literature Review in Learning with Tangible Technologies. *nesta Futurelab Series*, 2004.