History of Mathematics and History of Science: Some remarks concerning contextual framework[1]

História da Matemática e História da Ciência: Algumas considerações sobre o quadro contextual

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Abstract

This essay is devoted to the contextual methodology in History of Mathematics. The author discusses the contextual approach given by new trends in historiography in history of science and suggests that this could help to renew the contextual framework in history of mathematics. Here we base our study on primary-sources research and survey. Special attention is given to a set of documents concerning mathematical instruments which could convey a new appreciation of mathematical practices in the sixteenth and seventeenth centuries.
Keywords*: history of mathematics; contextual approach; Early Modern Mathematics.*

Resumo

Este ensaio é dedicado à metodologia contextual em História da Matemática. O autor discorre sobre a abordagem contextual das novas tendências historiográficas em História da Ciência, sugerindo que tal abordagem poderia auxiliar na renovação do quadro contextual em história da matemática. Este ensaio tem por base pesquisas e levantamento pautados em fontes primárias. Especial atenção é dada a um conjunto de documentos relativos a instrumentos matemáticos que oferecem uma nova visão das práticas matemáticas nos séculos XVI e XVII.
Palavras-chave*: história da matemática; abordagem contextual;matemática no início da era moderna.*

Introduction

New trends in historiography in History of Science have pointed to an approach in which Mathematics should be considered part of a broader contextual framework (MANN, 2011; ALEXANDER, 2002)[3]. Through this perspective Mathematics would not be different from Science in general, since neither the first, nor the latter have developed in a progressing way from Stone Age to Modern Era. Such approach proposes not only enhance the specificity of diverse fields of knowledge, but also take

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[3] On new trends in historiography in History of Science, see: Alfonso-Goldfarb e Ferraz (2009); Golinski (2005); Alfonso-Goldfarb e Beltran (2004); Gavrouglu, Christiandis e Nicolaidis

into account other aspects, such as its role in the process of constructing scientific knowledge and its impact on society. This can be observed in several technical and technological sectors in which mathematics was and is somehow inherently.

Bearing all this in mind we would like to stress here some points which have guided our investigations in history of mathematics. This essay is a result of discussions and studies concerning the project "História da Matemática e Ensino: As matemáticas nos séculos XVI e XVII" [History of Mathematics and Teaching: Mathematics in Sixteenth and Seventeenth Centuries] which focuses on sixteenth and seventeenth centuries mathematical practices[4] and teaching (SAITO, 2012).

Although we could find studies concerning this very subject, few of them have discussed the process of transmission and appropriation of mathematical knowledge. Indeed most of these studies have still focused their investigations on technical aspects of mathematical work without considering a contextual framework. As we shall see below, although some studies in history of mathematics have engaged in considering broader contextual aspects of mathematical work, the contextual approach has been criticized by some historians of science (MANN, 2011; BROMBERG; SAITO, 2010; ALEXANDER, 2002, 2006).

This leads us to account what should be taken as context and in which way we should orient our investigations in history of mathematics to contextualism. Thus the idea presented in this paper should not be taken as a prerogative, but rather as issues that should be discussed deeply by historians of mathematics. Our aim here is to present some issues which have emerged from a set of documents we have surveyed beforehand and which could be taken as possible sources for research in history of mathematics.

1. History of Science and History of Mathematics

History of Science and History of Mathematics are currently two subjects which are separated by independent practitioners, to some extent with different concerns and approaches, and relatively little interaction. However, recent works in history of science have cast new light on this topic, advocating new methods and contextual approach for

(1994); Alfonso-Goldfarb (1994a, p. 68-81).

[4] By this term we refer as P. Mancosu (1996, p. 4) states to: "mathematics as it was done in the past and not as it should be done according to preconceived philosophical point of view".

both disciplines. Since then, essays concerning contextual methodology have insisted upon the approach of history of mathematics and history of science.

One of these essays was published in 2006 in the Focus section entitled “Mathematical Stories”, in *Isis* Journal. In this work, A. R. Alexander stated that History of Mathematics had remained largely unaffected by new trends in historiography and suggested that a step toward a reunification of History of Science and History of Mathematics could help historians of mathematics to draw connections between technical mathematical work and its historical settings (ALEXANDER, 2006, p. 682).

According to Alexander (2006, p. 680), “the challenge facing historians of mathematics is to write a historical account of a field that appears to reside beyond the bounds of history”. This statement leads us to envision the most significant challenge which the historian of mathematics shall face in the coming years is to keep mathematical ideas in play, engaging in new approaches that deal with ideas which are not mathematical in its essence. Once these new approaches have focused on the process of constructing mathematical knowledge, the historian of mathematics shall consider a broad and profound knowledge of the contextual framework.

We can observe this is a very hard task for the historian of mathematics. It is certainly not a simple task to embrace the process of constructing knowledge, since mathematics was and is done for different reasons and ways. Moreover, mathematics was and is made in various contexts and settings in a dynamic way considering different purposes. In other words, the renewal of History of Mathematics seems to request more than mathematical knowledge and the historical context in which some mathematical ideas were evolved. Here we should redefine and revaluate what should be taken as contextualism, changing our tools of analysis, and therefore our methods of approaching to history of mathematics.

When historians of mathematics refer to a contextual approach they usually have in mind the idea of mathematics embedded within a broader social and cultural context. That is right. Nevertheless many studies in history of mathematics have focused on technical aspects of mathematical work themselves. Although such studies attempt to ground these very technical aspects in general social, economic, and political conditions, they only bring forward their development though time and space. Issues concerning the process of constructing mathematical knowledge in this way have usually been reduced to the history of the “progress” of the mathematical content or idea, including incremental and precipitous changes, such as “revolutions”, which would create new situations, questions and problems to mathematics themselves[5].

The concern of technical aspects of mathematical work by historians of mathematics is understandable if we take the audience into consideration. In 2011, in the *Isis* journal, once again in the Focus Section entitled “History of Science and History of Mathematics Reunited?”, J. Gray (2011) observed that the main reason for the apparent divergence between History of Science and History of Mathematics concerns their audience. Furthermore he also noted that the separation between historians of mathematics and of science became greater due to the activities of professional mathematicians in the nineteenth century onward.

In fact, in the second half of nineteenth century there was a widespread feeling that the building of modern science would be accomplished soon. At that time, scientists were no longer natural philosophers, but experts in specific and complex fields of knowledge. These very scientists felt prepared to deal skillfully with their own fields of expertise in order to ground science on solid foundations and to ensure the improvement and development of scientific knowledge. Under this light emerged a kind of “scientist-philosophers” who initiated a particular thought, philosophically and historically, of their field of expertise (ALFONSO-GOLDFARB, 1994a, p. 64-66).

Mathematics therefore could not be apart from this movement. When modern mathematics was finally settled as modern science, professional mathematicians endured in writing the history of their discipline emphasizing technical aspects of mathematical work. Following this line of thought one could say that History of Mathematics had then become a privileged and strictly mathematician field of investigation[6].

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[5] One should take into account that the very notion of “progress” cannot be unbound from the context in which it emerged, see: Rossi (2000) and Butterfield (2003, p. 187-205). It should be also noted that some trends in historiography which have endeavored to adopt a discontinuity approach, avoiding then the idea of accumulation and continuity in the process of development of scientific knowledge, have kept the very idea of “progress” in their analysis, see: Bachelard (1996) Kuhn (1997). On discussion on revolution in Mathematics, see: Gillies (1992).

[6] “Histories” of Mathematics have always been written. However the great books on history of mathematics began to be published from eighteenth century onward (BROMBERG; SAITO,

As Gray (2011) mentioned in his work, the audience of modern history of mathematics quite rightly is professional mathematicians and, inevitably, these people have different expectations and approaches that historians of science do not share.

Indeed, the main issue of History of Science here is to apprehend the process of constructing scientific knowledge in different times and places. Regarding this, historians of science seek to investigate not only technical features and contents of knowledge but also the circumstances by which this very knowledge was settled, taking into consideration a wide range of possibilities properly contextualized. Hence when historians of science write history of mathematics, they usually contextualize mathematical knowledge in a very particular way in order to reformulate old issues or questions (BROMBERG; SAITO, 2010).

However this does not mean that technical aspects of mathematical work should be left out. Instead, the relationships of these technical features of mathematical work to other aspects of the history remain a considerable principle. In other words, history of mathematics should not ignore mathematical contents but rather address them within the context of any other practice or process, including a study of the relevant social, political, and material circumstances. In this sense, updated trends in historiography in history of science has emphasized that a contextual methodology should take into account the understanding that mathematics and the process of constructing mathematical knowledge include a broad account deeply informed by specialized investigations based upon primary-source research, as we shall see below.

2. Some brief remarks on documents and sources for the History of Mathematics

The choice of documents and sources is a very important issue in any work of history since different set of documents and sources move the historian point of view to other features of mathematical and scientific knowledge. However these very documents and sources can also raise controversy if they are not properly contextualized and analyzed with discretion.

Usually traditional history of mathematics is written in such way that each mathematical development is succeeded by another. Each mathematical idea or discovery is usually

2010, p. 47-48).

attributed to a single mathematician and the testimony of such discovery is always a document (a treatise, a book, a letter or an article, etc.).

This way we can say documents are reduced to a mere testimony of the past for each discovery is usually linked to another in a sequence of events which reach us today. In this light these events give us the false idea that there was only a single path in which mathematics could tread from the past to the present day. We can say that these events are connected to one another by considering mathematics from the point of view of the present, giving us the impression that the path followed by mathematics was natural. In other words, written according to the point of view of modern mathematics, this kind of history pursues the “precursors” of some mathematical idea, notion, concept, and so on. Then once based on some content of modern mathematics, the historian goes back to the past in order to find the “precursors” of this very content, rowing them artificially from the past to the present day. In this sense, each mathematical idea is a link in the reconstruction of its history and would be attached to one another forming then a long chain of ideas. The connection of these links is not done arbitrarily by historians, but so done by establishing a logical connection between them and then privileging the internal dimension of mathematics in linear, continuous and progressive way.

This trend of writing history has basically two folds: on the one hand, it maintains the idea that there is a logical connection between mathematical ideas without connection with other non-mathematical ideas. On the other hand, when some of these very non-mathematical ideas are considered, this trend has retrained them to the assumption that this movement follows a logical connection, that is, that of the formal logic.

In recent decades, new trends in history of mathematics has emphasized that mathematical ideas cannot be fully understood without a consideration of the contextual framework which has a historical dimension. These trends have insisted that historians of mathematics should move away from a mode of history of ideas and see mathematics as an activity (ROUX, 2012; BENNETT, 1991). This means that historians should reconsider their assumptions concerning their sources in order to make new questions to the past without considering them a mere testimony of the past.

The issue that a document (whatever its nature) should not be considered merely as a record or a testimony of a past event was widely discussed (FOUCAULT, 2000; VEYNE, 1987). Since history has been understood as a field of knowledge which purposes are not to give a mere narrative of events, new trends in historiography of History of Science, for example, has stressed that a document *per se* is only a fragment in a set of ideas and practices in vogue at a given time. A document provides evidence not only of what is explicitly stated in it, but also of the different features of "knowing and doing" Science in a given time and place in the past (SAITO, 2011).

However a simple survey of documents is no longer enough without considering the context which these documents are grounded (ALFONSO-GOLDFARB, 2008). The analysis of these documents requires special attention. They should be read in the light of other contemporary documents and other sources which can help to illuminate the ideas therein. This is so because mathematics was defined in different ways and done for different purposes in different times.

The conception of what mathematics was is defined by its *episteme*, i.e. a set of epistemological relationships on which knowledge in a given time is based and grounded, representing this way the conditions of discursive possibilities which constitute an epistemology (FOUCAULT, 1999, 2000). Therefore by redefining our perspective and considering other directions which is allowed by this very *episteme* we can move from modern mathematics toward the past where we might find other mathematical connections in the historical context. In such procedure a new relationship between mathematics, its object, its purposes, and context is revealed without making us lose the focus of the many interrelated features between mathematics and other fields of knowledge.

The traditional trends in historiography seem not to cope with this, since mathematics is defined according to modern mathematics and when its objects are considered historically they are only removed from the present and allocated in the past. Rather if we consider the document in its past and the past in the past, this very document can allow us to envision the plot given by its *episteme* in which it belonged. The document can reveal to us the connections which give meaning to its existence in that historical context allowing us to ask new question and raise new issues.

But anachronisms raised from some epistemological proposals which were developed later should be considered. Since epistemology is historical, it would be artificial to analyze documents of the fifteenth century, for example, in the light of modern epistemologies. Also it would be artificial the attempt to find in these documents epistemological obstacle, paradigms, logic of discovery, which concepts are very common in philosophy of science, and evidences related to labels that claim to be self-explanatory such as Platonism, Kantianism, idealism, romanticism, rationalism, etc.

That is the reason why we have considered in our investigation that the past of a science should not be confused with that of science in its past (CANGUILHEM, 1977, p. 15). In other words, the past of Mathematics should not be confused with that of mathematics in its past. In this sense we have to assume that Mathematics has not been a single body of knowledge definitely established ever since. Indeed new approaches in History of Science have shown that mathematics was a complex field of knowledge which has included various domains from its very beginning and it has kept developing new domains throughout history (POPPER, 2006; AXWORTHY, 2009; BESSE, 2009; ROUX, 2010).

Actually Mathematics as we know today is a relative new field of knowledge which found its foundations in nineteenth century. Thus in the same way that there were not "scientists", there were not "mathematicians", as they are recognized today by the academic community in general, before that time.

It should be taken into account that the word "science" in the modern sense and its adjective form "scientist" were forged in nineteenth century. The ancient Latin term *scientia* which could be translated as "science" meant until then what was known as natural philosophy (ALFONSO-GOLDFARB, 1994a, 1994b). Moreover Mathematics was usually associated to different fields of knowledge since antiquity such as optics, pictorial arts, hydrostatics, pneumatics, mechanics and so on (KUSUKAWA; MACLEAN, 2006; CUOMO, 2001; CROSBY, 1999; TAYLOR, 1954).

Although we could not refer to the professional mathematicians before nineteenth century, it does not mean that scholars like Blaise Pascal (1623-1662) and René Descartes (1596-1650), for instance, could not be recognized as "mathematicians". This is so because when we submit these scholars to a historically situated and empirical definition of mathematics at that time, we find them recognized themselves as mathematicians. The point at issue here is that we should figure out what mathematics was considering what people, like Pascal or Descartes, were doing at that time as mathematicians. As S. Roux stated "what should be called 'mathematics' is the activities of those who called themselves or were called by others 'mathematicians'" (ROUX, 2010, p. 325).

The issue here is that it is obviously meaningless to refer to the professional mathematicians as we know today before Mathematics became a unified and specialized field of knowledge which would occur from eighteenth century onward. As we mentioned above, it was only in nineteenth century when Mathematics acquired the form of the diverse disciplines and fields of knowledge which are known today we can consider a mathematician as an expertise.

This means that a document which is focused on the exposure of a subject cannot be completely reduced to its disciplinary content. It is anachronistic to refer to the fields of knowledge prior to the eighteenth century as specific disciplines. Actually, recent studies in History of Science have pointed out a vast and complex web in which different fields of knowledge interweaved as a plot of a woven (ALFONSO-GOLDFARB; BELTRAN, 2002; ALFONSO-GOLDFARB, 2003; SAITO, 2011). When a part of this web is cut off, its plot reveals some features of the structure of different fields of knowledge enabling us to hold the process of constructing the very mathematical and scientific knowledge.

Thus instead of adopting a normative and philosophical perspective, we choose here a descriptive and historical point of view giving special attention to the historical dimension of the documents in their past. However, we are aware that these historical surroundings (whatever their nature may be) are not sufficient to characterize and embrace the conditions and circumstances under which these very documents were part. That is the reason why we should assume that the context is raised from the analysis of the document itself in order to replace it into the historic tissue which it belongs. In other words, we have assumed that contextualizing does not merely mean to allocate the document at a certain time because it may be dated from that time. This requires a survey of other sources linked to this document in order to bring to light the very context in which it is part. This way, the context shall be raised from the intersection of different documents and sources enabling us then to hold a multifaceted mathematics which will establish relations with different scientific and social practices at a time.

With all this in mind we have considered in our analysis documents and sources which do not seem to belong to the History of Mathematics in general. In order to supplement documents usually considered by historian of mathematics, we have focused our investigation on a set of documents which were left out by the traditional trends of History of Mathematics. These documents and sources have revealed some features of mathematical practices, helping us to fill some gaps in the process of the evolvement of mathematical knowledge, as we would argue next.

3. History of Mathematics in the Sixteenth and Seventeenth Centuries

The study of mathematics gained momentum between the sixteenth and the seventeenth centuries when the quick growth of mathematical activities and the production of mathematical research became notorious. From the seventeenth through the eighteenth century, those fields of knowledge related to mathematics, such as arithmetic, geometry, trigonometry, algebra and calculus, among others, have acquired similar features to those disciplines we know today. Indeed Mathematics as a field of knowledge began to acquire a more defined shape from the eighteenth century onward producing gradually exclusively mathematical knowledge. From the eighteenth through the nineteenth century, the number of journals devoted exclusively to mathematics researches increased significantly. At that time Mathematics had just become a specialized field of knowledge and other disciplines, such as Astronomy and Music, for example, which were once mathematics realm, moved to other fields of knowledge (BOULIER, 2010, MARONNE, 2010; MANCOSU, 1996).

The growing interest in mathematics in the sixteenth century led to impressive new developments not only in the study of nature, but also in the fields of mathematics itself. The works on algebra by Niccolo Tartaglia (1500-1557), Girolamo Cardano (1501-1576) and François Viète (1540-1603) and others such as the logarithms by Jean Napier (1550-1617) and infinitesimal calculus by Gottfried Leibniz (1646-1716) and Isaac Newton (1643-1727) were some of outcomes of such developments.

Several factors contributed to the rise of interest in mathematics by scholars at that time. One of them certainly is the recovery of classical works. Two other significant factors are related to the study of the movement which took place particularly in universities such as Oxford and Paris in the fourteenth century (SYLLA, 2010; PALMERINO, 2010; CROMBIE, 1995, v.2, p. 97-174; HALL, 1988, p. 65-108), and the revival of Platonic, Neoplatonic and Pythagorean doctrines in the fifteenth century (COPENHAVER, 1990, 1992; YATES, 1995, p.13-215). However the key factor is probably the association between mathematics and the mechanical arts and other technologies that have developed from sixteenth century onward, particularly those related to measurement and calculation (COHEN, 2005; CROSBY, 1999; ROSSI, 1989, 1970).

As it is well known by the historians of mathematics, the ability to measure and calculate gained great importance at that time because of the new challenges proposed by the expansion of trade and navigation. Not only scholars, but also nobles, merchants, navigators and all sort of craftsmen engaged themselves to mathematics in order to supply their activities which required more sophisticated calculations by innovative resources and methods (CONNER, 2005, p. 190-247; CUOMO, 1998; CHAPMAN, 1998; WATERS, 1983; HALL, 1983).

However this appreciation of mathematics should be seen as a wide range of possibilities rather than a strictly mathematical view. Such procedures led to the capturing of some aspects of mathematics in the fashion it was used to be pursued in various fields of knowledge at that time.

Regarding this we started surveying documents and other sources which dealt with mathematics contents along with sources commonly considered by the traditional historical sets. This survey has revealed to us a complex web of relationships where some manipulative aspects of knowledge were not disconnected from the other theoretical and operational features[7]. We refer here to the works written by all sorts of craftsmen and other people usually named “mathematical practitioners” (HIGTON, 2001; TAYLOR, 1954), especially works dealing with “mathematical instruments” (BENNETT, 1991, 1998, 2003).

We can say that knowledge shared by these mathematical practitioners in general was more empirical than theoretical since this knowledge was applied to the real world. This is quite reasonable if one considers the growing demand for instruction in practical geometry and arithmetic which were needed for the development of new technologies for navigation, surveying, horography, cartography, artillery, fortification and so on at that time (MARR, 2009; CORMACK, 2006; HEILBRON, 2001; CAMEROTA, 1998;

[7] The list is relatively long and includes treatises of optics, perspective, architecture, surveying, navigation, horography, instruments of different sorts, cartography, geography, cosmography, medicine, law, music, pneumatics, hydrostatics, among many others which deal with mathematics in a certain way.

BENNETT, 1998; TURNER, 1998).

These mathematical practitioners usually referred themselves as “professors” in the sense that they “professed” the mathematical art. Most of them did not have university background and were often members of a guild or worked in a workshop. Thus it was very common to see these “professionals” developing their own instruments and publishing treatises about them (CIOCCI, 2009, p. 59-71; BIAGIOLI, 1989).

We should take into account that these treatises were not mere manuals by which one could build and use instruments, since instructions on “how to make” and “how to use” them were not simple to be followed. Preliminary analysis of these documents has revealed that they were aimed at a very skilled audience who knew not only of mathematical knowledge embodied in these instruments but also the skills to craft them (SAITO; DIAS, 2011).

These documents have allowed us to approach and understand the different mathematical practices, raising new issues on the relationship between theory and practice concerning the application of mathematics. Moreover they have provided us with new clues by which we can approach mathematical knowledge to natural philosophy. The notions of operative knowledge and of “knowing by doing” provided by these very documents have allowed us to renovate the discussion of the reality of mathematical entities and their effectiveness on the world experience.

However one should consider that the instruments described in these treatises should not be considered only as a result of mathematical development which was already known. Doubtless we can say that these instruments have embodied a very well established mathematical knowledge. Nevertheless the procedures of their construction and use reveal more than a simple application of this very mathematical knowledge. These treatises and the instruments described wherein bring to light significant aspects of the practice of mathematics and science of that time. In short they allow us to discuss them in the growing interaction between two domains of knowledge: “natural philosophy” and “mixed-mathematics” (GABBEY, 1997; DEAR, 1995; WARNER, 1990, 1994).

A preliminary analysis of these treatises, even if superficial, has revealed that these mathematical instruments were more than mere artifacts. Such instruments were results of rearrangement of handling material and ideas (SAITO, 2011; HINDLE, 1981).

Actually they unveil not only the articulation between knowing and doing mathematics but also reveal them as part of a complex mathematical economy with many layers and interconnections.

Thus in spite of some historians of science such as Bennett (2003) has characterized such instruments belonging to the mathematics domain, since they are not addressed to deal with natural philosophy, they embody some features that refer to the knowledge of the material and causal structure of nature. There is no doubt that these instruments do not reveal or detect new natural phenomena in order to increase the knowledge of natural world. However, even they did not allow someone to discover things, they point out some features of mathematical practice, especially that of mixed-mathematics.

We here refer to a sort of subjects like astronomy, music, optics, and so on which were defined since antiquity, notoriously by Aristotle, as those concerning to natural phenomena and quantitative aspects. This kind of knowledge acquired such importance in the sixteenth century that we should not leave out it from other deployments which had taken place at that time[8]. This means that we should not make a sharp distinction between those activities which concern the representation of heavens and Earth, embraced by those mathematical practitioners, for instance, from those which concern theoretical aspects of heavens and Earth, owned by astronomers and geographers. This split would sound artificial because these mathematical instruments were conceived and framed along the lines of a scientific theory in particular. Furthermore these treatises dealing with these instruments published by mathematical practitioners were quite considered by natural philosophers who were interested in mathematics.

While scholars devoted themselves to mathematics and natural philosophy, other practitioners dedicated to the studying and publishing this kind of treatises. Along with all sorts of craftsmen and artisans, people like Egnazio Danti (1536-1586), Cosimo Bartoli (1503-1572), Gemma Frisius (1508-1555), Oronce Finé (1494-1555) and many others were involved in this endeavor. Drawing on their own spheres of influence, each of these scholars and practitioners pulled in others who were working in several related fields of knowledge.

We might say that this interchange provided by these diverse documents and sources

[8] Indeed, since antiquity, there have always been mixed-sciences. See: Roux (2010); Nascimento (1998); Vescovini (1969); Gagné (1969).

reveals many nuances of the process of transmitting mathematical knowledge. Also they shed light to the relationship between mathematics and diverse fields of knowledge, propitiating then a better understanding on how mathematics appropriated from knowledge of different realms in the past. This context ongoing, in which several realms of knowledge intersect with one another, enables us to hold the process of constructing mathematical knowledge in a broader historical context.

Some final remarks

We have stressed four points on which we briefly refer to here. The first is that contextualizing requires a deep understanding of the circumstances in which some mathematical ideas emerged. This means that technical aspects of mathematical work and its contents of knowledge should be investigated along with the surroundings by which this very knowledge was settled. Regarding this, a contextual methodology should take into consideration a broad account deeply informed by specialized investigations based upon primary-source research.

However, analysis of these documents requires special attention, and here we see a second point: such documents should be read in the light of other contemporary documents and other sources which can help to illuminate the ideas therein. This means that a simple survey of documents and sources is no longer enough without considering them properly contextualized. In other words, we should consider such documents and their sources in its past and the past in the past, moving away from a mode of history of ideas and visualizing mathematics as an activity.

The third point is that anachronisms raised from some epistemological proposals which were developed later should be considered. This does not mean that epistemological aspects should be overlooked, but rather that they should be properly contextualized. The epistemological features of a document should arise from the past along with its social, economic, and political conditions. This procedure enables us to hold the historical dimension of a document as well as its epistemological framework. However we here should take into consideration that a document which is focused on the exposure of a subject cannot be completely reduced to its disciplinary contents, and here we face a fourth point: the past of Mathematics should not be confused with that of mathematics in its past.

This means that we should submit mathematics to a historically situated condition since it was a complex field of knowledge which has included various domains from its very beginning and it has kept developing new domains throughout history. Instead of adopting a normative and philosophical perspective, we should take into account a descriptive and historical point of view, considering the several mathematical practices. In other words, we should consider mathematics as it was done in the past and not as it should be done according to preconceived philosophical point of view. However this procedure requires a detailed knowledge of mathematical literature of the past which include not only mathematical contents in its essence, but also which deal with mathematics in a certain way. Regarding this we should face a set of documents which were left out by the traditional trends of History of Mathematics.

We can say that research focused on particular groups of documents, such as mathematical instruments, typically does not yield large amounts of information about discoveries in mathematics. However they contain evidences concerning ongoing mathematics and mathematical practices which could help us to hold the process of constructing mathematical knowledge.

A preliminary analysis on documents dealing with mathematical instruments has revealed that instrumentation was ongoing issue at that time. Moreover it has raised new questions concerning the practical aspects of mathematics, not in the sense of the application of theoretical mathematics, but as a manipulative knowledge which may or may not encourage new theoretical ideas that may or may not answer a practical need.

We have endeavored to analyze such documents focusing on the processes and not on their results. In other words, that is not quite the mathematics (or science) that determines how an instrument must be designed. Although its performance as an instrument is based upon mathematical (or scientific) ideas, their components and parts are thought and arranged in such way focusing on practices and other modalities of investigation which were ongoing at a certain time and place. This set of documents thus seems to allow us to hold the context in which mathematics had developed and convey a new appreciation of mathematical practice.

Appendix: Primary-sources on mathematical instruments

Here follows some treatises which deal with mathematical instruments we have considered to compose this essay:

BARTOLI, C. (1564). *Cosimo Bartoli Gentil'huomo, et accademico Fiorentino, Del modo di misurare le distantie, le superficie, i corpi, le piante, le province, le prospettive, & tutte le altre cose terrene, che possono occorrere agli homini, Secondo le vere regole d'Euclide, & de gli altri piu lodati scrittori*. Venetia: Francesco Franceschi Sanese.

BABINGTON, J. (1635). *A short treatise of Geometrie: in which are contained sundry Definitions and Problems, for the mensuration of Superficies and Solids, as also the use of the Quadrant and Quadrant in measuring of Altitudes, Latitudes, and Profundities, with sundry Mechanicall wayes for performing the same. Unto which are adjoyned certaine Tables wherein the Square Root is extracted to 25000, and the Cubick root to 10000 Latus, only by Ocular Inspection.* London: Thomar Harper for Ralph Mab.

BEDWELL, W. (1639). *Mesolabium architectonicum that is, a most rare, and singular instrument, for the easie, speedy, and most certaine measuring of plaines and solids by the foote: necessary to be knowne of all men whatsoever, who would not in this case be notably defrauded: invented long since by Mr. Thomas Bedwell Esquire, and now published and the use thereof declared by Wilhelm Bedwell, his nephew, Vicar of Tottenham*. London, I[ohn] N[Orton].

BISHOP, G. (1597). *The Navigators supply: Conteining many things of principall importance belonging to Navigation, with the description and use of diverse Instruments framed chiefly for that purpose; but serving also for sundry other of Cosmography in general: the particular instruments are specified on the next page.* London: by G. Bishop, R. Newbery and R. Baker.

BLAGRAVE, J. (1590). *Baculum familliare. A booke of the making and use of a staff, newly invented by the author, called familiar staff. As well for that it may be made usually and familiarlie to wlake with, as for that it performeth the Geometrical mensurations of All Altitudes, Longitudes, Latitudes, Distances and Profundities: as many myles of, as the eye may well see and discerne: most speedily, exactly and familiarly without any maner of Arithmetical calculation easily to be learned and practiced, even by the unlettered. Newlie compiled, and at this time in great Ordinance, and other millitarie services, and may as well be imployed and by the ingenious, for measuring of land, and to a number of other good purposes, both Geometricall and Astronomicall*. London: Hugh Iackson.

BROWNE, J. (1671). *[A description of a mathematical instrument] made by John Browne*. London: [s.ed.].

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