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What knowledge do in-service teachers need to create SRPs?

De quelles connaissances les enseignants en exercice ont-ils besoin pour créer des SRP?

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Abstract

In this paper we present ideas regarding what elements of the anthropological theory of didactics in-service teachers need to design study and research paths and realise them in their classrooms. Based on preliminary experiences from other projects we try to identify crucial notions and how they can be transposed in order to support Danish in-service teachers, who has very diverse didactical knowledge.

Keyword: Anthropological Theory of the Didactics, professional development, didactic transposition, Study and Research Path.

Résumé

Dans cet article, nous présentons des idées sur les éléments de la théorie anthropologique du didactique en cours qui ont besoin pour concevoir des parcours d'étude et de recherche et les réaliser. Sur la base des expériences préliminaires d'autres projets, nous essayons d'identifier les notions cruciales et comment elles peuvent être transposées afin de soutenir les enseignants danois, qui possède des savoir-faire didactiques très diverses.

Mots-clés : Théorie anthropologique de la didactique, développement professionnel, transposition didactique, parcours d'étude et de recherche.

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During the last 10 years emphasis has been put on inquiry based teaching methods in mathematics and science education. In this paper we will discuss how Study and Research Paths (SRP) can support in-service teachers engagement in realising inquiry based teaching in their own classrooms.

A report by Rocard and colleagues (2007) concluded that much teaching in science and mathematics in Europe was taught as transmission of knowledge, even though the disciplines encompass a dimension of inquiry as one way to knowledge construction. Therefore, it was suggested that teaching should reflect the inquiry approach as well. EU has funded several projects to promote this. One of those projects was Primas, which offered a guide for professional development providers. It argued that teachers have difficulties changing the teaching paradigm if no further structures of action research are offered to support this change (PRIMAS, 2013). But change to what? Artigue and Blomhøj (2013) have made an effort to conceptualise what is meant by inquiry based teaching in mathematics. They argue that the inquiry approach to mathematics is part of how to do mathematics, referring to Polya (1945) and his characteristic of problem solving processes. From a generic perspective Dewey (1938) have promoted an approach to teaching, which was based on students own actions. Artigue and Blomhøj list a number of theoretical approaches to the teaching of mathematics capturing those ideas; including ATD and SRP.

Throughout the last decade SRPs have been realized at different levels of school systems in several countries. These realizations have often been developed and conducted by researchers in their own classrooms (e.g. BARQUERO, BOSCH and GASCÓN, 2013; FLORENSA, BOSCH and GASCÓN, 2016 and OTAKI, MIYAKAWA and HAMANAKA, 2016). But also constraints and conditions for the implementation of

SRPs have been studied (BARQUERO et al., 2013; BARQUERO, BOSCH and ROMO, 2015; RASMUSSEN, 2016). Rasmussen (2016) and Barquero et al. (2015) both study challenges of engaging pre-service and in-service teachers respectively in SRP based teaching. However, the presented experiments often draw on ATD theory in great detail, something which is not always realistic for in-service teachers to engage in, having no interest in particular didactic research and only limited time available. This leads to the research question of this paper:

What didactic transposition of ATD notions is needed to engage in-service teachers in creating and realising SRP based teaching?

In this paper, we will consider what notions, tools and concepts are needed for the average teachers to engage in SRP-based teaching. Do they need to be able to create praxeological analyses to design and realize a SRP? Is the idea of and dynamics in a question-answer map sufficient for the teachers to navigate in this kind of teaching? We will discuss and present our considerations during the design process of a course for in-service teachers having very different educational backgrounds and didactical knowledge. We focus on teachers of secondary education, but we regard the considerations as having wider generality when disseminating ATD knowledge to teachers of all age-levels.

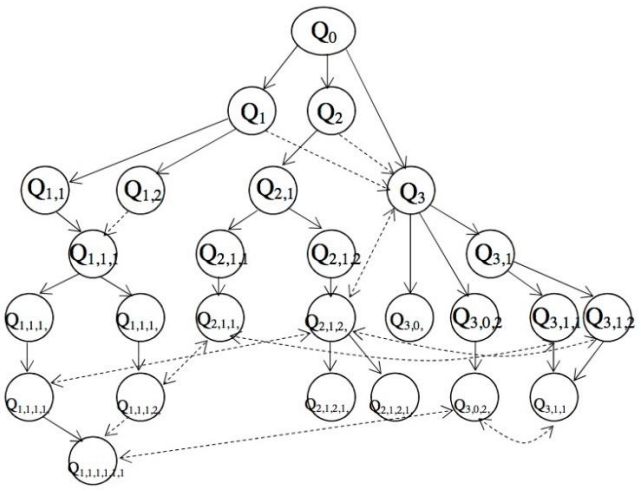
Theory and background

Study and research Paths (SRP) is the design tool for teaching proposed by ATD, initially as a way to design cross disciplinary student work, which was a new requirement in French Secondary school (Chevallard, 2004). Furthermore, SRP is a way to realise the teaching paradigm of "questioning the world", which is an ATD alternative to transmission of knowledge. Chevallard (2015) argues that learners should develop a questioning attitude to the world:

> "[…] this receptive attitude towards yet unanswered questions and unsolved problem, which is normally the scientist's attitude in his field of research and should become the citizen's attitude in every domain of activity." (CHEVALLARD, 2015, p. 177).

This attitude can be said to capture elements of the inquiry process which Rocard and colleagues (2007) suggested to be the point of reference of teaching. A SRP is initiated by a generating question, Q0, posed by the teacher, which in turn lead students to pose derived questions, Qi,j, addressing elements of the original question they need to explore further. The generating question, Q0, should be formulated so students understand the question, but the answer requires that students engage in study and research processes. The study process is when students study media to acquire knowledge regarding a certain topic. Media can be textbooks, webpages, online video, data materials or other resources from which students draw knowledge. The research process is when students gather the new and already acquired knowledge in the development of partial answers to the generating questions. These processes has been characterised as the de- and reconstruction of knowledge (BARQUERO et al., 2013, p. 334) and describes the essential idea of learning processes in a SRP. Students are expected to continue questioning the knowledge at stake leading to further derived questions. Those questions and their relations can be depicted in tree diagrams as figure 1.

Figure 1
Tree diagram (Jessen, 2017, p. 18)



It is evident that an open generating question might lead to very different answers – even within a particular class having similar prerequisites if no other scaffolding of the study and research process is provided. Different ideas have been experimented such as letting SRP be a workshop that runs parallel with another mathematics course (BARQUERO et al., 2013), rubrics for formative assessment known by the students (FLORENSA et al., 2016, side questions posed to the teachers (Rasmussen 2016), teachers poses the first derived questions (JESSEN, 2014) and to each Q0 a list of media was suggested to the students (JESSEN, 2017).

Jessen (2017) further experimented with Q0's aiming at elements from curriculum, called study and research activities (SRA). Chevallard (2004, p. 6) argues that there is a risk of SRA not supporting students' development of the rationale behind the mathematics they use to develop answers to questions. Instead SRA perpetuate the unhelpful transmission of knowledge. However, Barquero, Serrano & Ruiz-Munzón (2016) try to categorise different kinds of SRA ranging from those with no potential of study and research processes to those being small versions of SRP, i.e. a branch of the tree diagram. Jessen (2017, p. 98) argues that SRA have the potential of engaging students in study and research processes if the generating question of each SRA fulfils the criteria of being an open question, that require students to engage in both study and research processes. The potential of the generating questions in this regard can be visualised in an a priori analysis of possible derived questions depicted in a tree diagram showing possible paths and their relations.

Detected challenges regarding the implementation of SRPs might hinder teacher engagement with SRP based teaching. Barquero et al. (2013) discuss the ecology of SRPs at tertiary education. Jessen (2017, p. 156) argues that the back wash effect of exit examinations might hinder its realisation in upper secondary education. Furthermore,

teachers' lack of tradition for collaboration regarding their professional development hinders their adaptation to SRP based teaching (Jessen, 2017, p. 182). Otero, Llanos, Parra, and Sureda (2014, p. 24) points out, that students attitude towards mathematics teaching as being something where the teacher pose the questions and they answer, represent another challenge in realising SRP based teaching.

Who do we want to engage in SRP based teaching?

Secondary education is in Denmark divided between two different institutions and carried out by teachers educated at two separate institutions. Primary and Lower Secondary teachers study at university colleges for 4 years, where they are specifically prepared to become teachers. That is; the whole pre-service education is geared towards them becoming part of the teacher profession. Upper secondary teachers, on the other hand, study two disciplines at universities for 5 years, which do not put them on track for a career as teachers. The upper secondary teachers are expected to study a major and a minor in order to teach both disciplines. During the first year of employment after the master degree the candidates are supposed to complete a practicum; teaching half the time and study learning theories and pedagogy the other half. The practicum part has had minor structural changes through out the years and the theoretical part takes up more time than earlier.

Primary and Lower secondary teachers' education has been through five major reforms during the last 30 years. Consequently teachers currently employed have very different educational prerequisites for carrying out their job. The most senior primary and lower secondary teachers gained quite extensive disciplinary knowledge, but their education had little didactic depth. Teachers more recently educated obtain "only" a fair knowledge of three disciplines but more substantial didactic knowledge is integrated. Teachers of the more recent variety specialize in teaching age groups, grade 1-6 or grade

4-10, while the "older" have formal competence in all age levels. Consequently, the group of teachers we are considering in this paragraph are the ones engaged at teaching grades 7-10, which correspond most closely to lower secondary education.

Given this background, our primary concern is that the average teacher might not have sufficiently depth, command and overview of mathematics and its didactics to be comfortable letting their students embark on the exploratory voyage of SRP.

What Danish experiences have been accumulated so far?

In the Danish context SRP has been experimented and reported in master theses addressing upper secondary teaching, after being introduced in a master course. Recently, a development project, using ATD theory, and aiming at easing the transition in mathematics from lower to upper secondary education was conducted. (Jessen & Winsløw, 2017). The course introduced in-service lower secondary teachers to SRP as modelling activities. The teachers were provided with two generating questions and asked to try to develop an a priori analysis formed as a mind map, where they used the idea of technique to formulate all possible strategies students would take. Eventually, the teachers were provided with tree diagrams and suggested derived questions relevant for the Q_0's. After the course the teachers should realise the activities in their own classrooms.

The experimentation proved to be difficult because the teachers felt insecure with the openness of the generating questions and wanted to make sure that all students got on "the right path". Hence, they needed further ideas on how to manage the SRP and scaffold the students learning during SRPs. Most teachers ended up altering the design into a less inquiry based method. Similar findings were reported by Barquero, Bosch & Romo (2015, p. 813) regarding an online in-service teacher course on SRP based teaching.

SRPs have also been experimented in pre-service teacher education for lower secondary teacher education within the interdisciplinary ASTE project (Rasmussen, 2016). Nevertheless it has not been a major feature in the overall education, and it remains to be seen whether the now graduated teachers attempt to implement SRP in their own practice. The ASTE students were not taught ATD theory, but course elements were taught through SRPs.

Proposal for knowledge to be taught

Barquero, Bosch & Romo (2015) present in-service teachers at their online course for ATD notions as praxeology, mathematical organisations, SRP, tree diagrams, "role play" where teachers imitate students and finally create a lesson plan for the SRP teaching. The activity they engage the participants in, is called SRP-TE – study and research paths for teacher education. Due to the Danish teachers' educational backgrounds we consider this model out of reach for most Danish teachers given the constraints of most in-service courses: limited time and the fact that many expect to receive several "ready to use" teaching proposals from the in-service training. The experience from Jessen & Winsløw (2017), reported above, made us consider to employ a rather transposed version of the ATD theory.

We have an in-service teachers course for upper secondary teachers at the University of Copenhagen. The course runs from mid August 2017 until mid March 2018. The teachers meet 7 times of 4 hours during the period. There were no requirements with respect to the teachers' didactical prerequisite and we have enrolled 47 teachers with very different backgrounds. One teacher has 20 years work experience from private companies but no teaching experience, another one has an additional university education as mathematics counsellor (see further in JANKVIST & NISS, 2016). The aim of the course is to teach the participants how to meet new requirements from the 2017 curriculum for

upper secondary mathematics. In the didactical principles, it is stated that parts of the teaching should be inquiry based and compel "students autonomously discovery of 'new' mathematical theorems […] to pose mathematical questions […] that can be answered (solved) by students" (MINISTRY OF EDUCATION, 2017, p. 21). Hence, the participants where taught to develop, experiment and evaluate SRP based teaching.

Transposed knowledge on SRPs

The first two course sessions were devoted to introduce the notions of SRP and SRA as well as a special form of lesson plan.

When introducing SRP and SRA the participants were presented a generating question and the idea of study and research processes. Instead of using the notion of media, it was discussed which resources could support the study process. The term 'media' is colloquially associated with newspapers, television programs etc. We wanted to emphasise of the wide ATD meaning of media. Instead of presenting tree diagrams as results of a priori analyses presenting an epistemological reference model, we posed the following generating question:

> Q_0: Look at the arbitrary triangle with side lengths a, b and c. If the sides are enlarged with a certain amount, how much bigger area will the triangle cover?

The teachers were asked to produce a mind map similar to figure 1, which they had been introduced to. They were asked what solution strategies they could find based on their mathematical knowledge and possible resources. Also, they should describe strategies and questions, which they could imagine their students would use:

> "What does enlargement mean?", "How can we determine the area of a triangle if we do not know the height?", "What does $\sin(A)$ mean, if A denotes one of the angles of the initial triangle?", "What can we say if we look at right angled triangles?" and "What does Heron's formula tell us?"

These questions were posed as derived questions. The participants created the maps in groups and they were shared and discussed in plenum. After this, the maps were named knowledge maps and presented as the main guidance tool for the teachers when realising a SRP. Next, an experimentation with SRA on exponential functions (Jessen, 2017) were undertaken with its sharing sessions and media. After this the participants were asked to create their own knowledge map and plan how to teach linear functions based on the following question:

> Q_0: In a lower secondary class three friends chooses different career paths. One do not wish to attend school anymore, another wants to become a nurse and the last wants to be a upper secondary school teacher. How old are the students when the nurse has had a total income larger than the friend who does not pursue an education? How old are the friends when the teacher has had a total income larger than the others?

Below you find a scheme where you can see years of education and which average annual income it gives rise to.

Table 1:

The relation between years of education and average annual income in Danish currency.

Years of education	9	12	14	16	18	21
Income (dkr)	210.000	310.000	365.000	370.000	490.000	530.000

The participants engaged nicely in the development of knowledge maps presenting a variety of ideas including strategies using ICT. However, the tendency of expecting students to develop particular answers from research processes alone made the participants doubt in the viability of the design. They had difficulties imagining how to navigate in the knowledge map: When should they directly answer students' questions? What questions were they "allowed" to answer and which ones should the students

answer? If students should be assisted to discover "more than one" paths in the knowledge map, how can the teacher then interact with the students?

At the second course session, some teachers remained insecure with respect to the SRP in linear functions, others had tried it in their own classes before the second session, and with positive experiences. Others had tried to design their own SRPs or SRA. One group had been curios about prerequisites of students coming fresh from lower secondary. Hence they had designed and run a SRA starting from: The teachers observed through several different approaches, the students realised that both graphs intersect the x-axis in the same value, though not all could explain why. The teachers thus gained valuable insightof the students prerequisites. This insight was later used in the design of a SRP regarding piecewise linear functions.

Lesson plans and navigation

Our inspiration for the special form lesson plan stems from Østergaard (2016) who deal with the issue of how to relate the theoretical and the practical part of teacher training at university colleges. The participants of the in-service course were introduced to preparing a plan in which they were to write down information regarding: Where the teaching was planned to be realised, title of the lesson, concrete learning goals, broader goals, students expected prerequisites, Q_0, media suggested to students, additional media which students might find on their own; and a time schedule indicating teacher activities (including questions they might ask the students) and expected students activities (e.i. derived questions, partial answers etc.). As appendices additional work sheets, datatables and the knowledge map should be fashioned. The time schedule was created together with the knowledge map and referred to the map with respect to students' expected strategies and derived questions. In the time schedule a column had to be reserved for observation notes, which should be used during the evaluation and improvement of the lesson.

Filling in the time schedule based on the knowledge map support the teachers' ideas of how to navigate in the classroom without ruining the potentials of the generating question. Based on the knowledge map teachers formulate questions, which can be characterised as generating questions that support the development of a certain path in the map, or the question initiate a particular SRA, if the teacher considers it needed.

At the time of writing, we have no record on the realisation yet. The following course sessions the participants are developing SRPs on the introduction of vectors in two dimensions, probability theory and discrete mathematics according to the Danish curriculum.

Concluding remarks

What is worth noticing from the Danish experiences, though they are limited, is that upper secondary teachers relatively fast got the courage to realise SRPs, once the theoretical terminology was downplayed and transposed (c.f. 3.1 and 3.2). On the other hand; lower secondary teachers were somewhat more reluctant, though they had more time and theory introduced (c.f. 2.2).

This suggest that for in-service teachers, who are not accustomed to extensive usage of theoretical notions, we should consider the transposition of ATD notions and language even more.

It is striking how both groups of in-service teachers have focus on the research process and have difficulties in designing and exploiting the study process. This is despite the fact that inquiry based teaching and modelling has been elements of curriculum for both educations for almost a decade. But if we look at the other approaches to inquiry based teaching presented by e.g. Artigue and Blomhøj, the emphasis of the study process is unique for the ATD approach and therefor in-service courses might need to enlighten this aspect in particular.

References

Artigue, M. & Blomhøj, M. Conceptualizing inquiry-based education in mathematics. ZDM, 45, p. 797-810, 2013.

Barquero, B., Bosch, M., & Romo, A. A study and research path on mathematical modelling for teacher education. In: Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education, p. 809-815, 2015.

Barquero, B., Bosch, M. & Gascón, J. The ecological dimension in the teaching of mathematical modelling at university. Recherches en Didactique des Mathématiques, 33 (3), p. 307-338, 2013.

Barquero, B., Serrano, L., & Ruiz-Munzon, N. A bridge between inquiry and transmission: The study and research paths at university level. Paper presented at the First conference of International Network for Didactic Research in University Mathematics, Montpellier, France., 2016. https://hal.archives-ouvertes.fr/hal-01337885

Chevallard, Y. Vers une didactique de la codisciplinarité. Notes sur une nouvelle épistémologie scolaire. Journées de didactique comparée Lyon, 2004.

http://yves.chevallard.free.fr /spip/spip/article.php3?id_article=45

Chevallard, Y. Teaching Mathematics in Tomorrow's Society: A Case for an Oncoming Counter Paradigm. In: The Proceedings of the 12th International Congress on Mathematical Education: Intellectual and attitudinal challenges, Cham: Springer International Publishing, p. 173-187, 2015.

Dewey, J. Logic: The theory of inquiry. New York: Henry Holt and Company, Inc., 1938

Florensa, I., Bosch, M., Gascón, J. & Mata, M. SRP design in an elasticity course: the role of mathematical modelling. In: Proceedings of the first conference of International Network for Didactic Research in University Mathematics, Montpellier, p. 191-200, 2016.

Jankvist, U. T., & Niss, M. A framework for designing a research-based "maths counsellor" teacher programme. Educational Studies in Mathematics, 90(3), p. 259-284, 2015.

Jessen, B. E., & Winsløw, C. Matematikbroen: brobygning for elever gennem efteruddannelse for lærere. Mona, 3, p. 39-59, 2017.

Jessen, B. E. Study and Research Paths at Upper Secondary Mathematics Education – a Praxeological and Explorative Study. PhD thesis, University of Copenhagen., 2017

Jessen, B. E. How can study and research path contribute to the teaching of mathematics in an interdisciplinary setting? Annales de didactique et des sciences cognitives, 19, p. 199-224, 2014.

Ministry of Education. Matematik A/B/C, stx - Vejledning. Copenhagen, Denmark, 2017.

Otaki, K., Miyakawa, T. & Hamanaka, H. Proving activities in inquiries using the internet. In: Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education, 2016.

Otero, M. R., Llanos, V. C., Parra, V., & Sureda, P. Pedagogy of research and questioning the world: teaching through research and study paths (rsp) in secondary school. Review of Science, Mathematics and ICT Education, 8(1), p. 7-32, 2014.

Polya, G. How to solve it? Princeton, NJ: Princeton University Press, 1945.

PRIMAS Guide for professional development providers, 2013.

http://www.primas-project.eu/artikel/en/1300/professional-development/view.do

Rasmussen, K. The direction and autonomy of interdisciplinary study and research paths in teacher education. Journal of Research in Mathematics Education, 5(2), p. 158-179, 2016.

Rocard, M, Csermely, P., Jorde, D., Lenzen, D., Walberg-Henriksson, H. & Hemmo, V. L'enseignement scientique aujourd'hui: une pédagogie renouvelé pour l'avenir de l'Europe. Commission Européenne, Direction générale de la recherche, Science, économie et société, 2007.

Østergaard, K. Teori-praksis-problematikken i matematiklæreruddannelse: belyst gennem lektionsstudier. PhD thesis, Roskilde University, 2016. http://forskning.ruc.dk/site/files/5