**An overview of "bridging courses" from the ATD perspective**

**Una descripción general de los "cursos puente" desde la perspectiva de TAD**

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**Abstract**

The presence of bridging courses in the European university panorama has evolved from a simple spontaneous proposal to being part of a consolidated resource for new students in many universities. In Spain, the tendency points to the usual presence of these courses in almost all degrees containing mathematics in their first year. The analysis of different «bridging courses» led us to formulate the hypothesis that, due to the large number of mathematical praxeologies introduced and the type of didactic praxeologies used, they seem to contribute to increase the isolation and rigidity of mathematical praxeologies studied at secondary level (Serrano 2013). From the ATD, we have designed and experimented a course that tries to overcome this isolation by proposing connecting elements in the terms introduced by Fonseca (2004).

**Keywords**: Bridging courses, Elementary functions, Inequalities, Modelling, Mathematics for economics.

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**Resumen**

La presencia de los cursos puente en el panorama universitario europeo ha pasado de una simple propuesta espontánea a ser un recurso consolidado para los nuevos estudiantes que acceden a la universidad. En el caso de España, esta tendencia apunta a la presencia de estos cursos en casi todos los grados que integran cursos de matemáticas en su primer año. El análisis de diferentes cursos nos llevó a formular la hipótesis que, debido al gran número de praxeologías matemáticas introducidas y al tipo de praxeologías didácticas utilizadas, estos parecen contribuir al aislamiento y a la rigidez de las praxeologías matemáticas estudiadas en secundaria (Serrano 2013). Desde la TAD, hemos diseñado y experimentado un curso que intenta superar este aislamiento proponiendo conectar elementos en los términos introducidos por Fonseca (2004).

**Palabras clave**: Cursos puente, Funciones elementales, Desigualdades, Modelización, Matemáticas para la economía.

## An overview of "bridging courses" from the ATD perspective

The implantation of bridging courses in Spain goes back to the late 1990s. Among the several reasons for their appearance, we highlight on the one hand, the implementation of an educational reform in 1990, which modified the structure of secondary school, increasing compulsory secondary school to 16 years old and reducing non-compulsory secondary school to 2 years (16-18) instead of 4 (14-18). On the other hand, the new political scenario in Spain allowed the opening of the university to a large number of high school students. The distance between the new secondary school and the (old) university was perceived as so severe that most Spanish universities began to propose courses to facilitate the transition between the two institutions. The so-called "bridging courses" or "propaedeutic courses" are supposed to complete the mathematical contents of higher secondary school with those considered essential to attend the mathematics courses at the university.

Since 2005 we are analyzing these courses and their evolution in the Spanish universities. To delimit their status, we have been carrying out a detailed study through different instruments such as interviews with designers of the courses and with lecturers responsible for implementing, both "bridging" and "academic" courses; interviews with students; class observations; analysis of both teachers' and students' notes and materials. In this analysis, we have found several common characteristics. In most cases, the interviews with professors responsible for "bridging courses" revealed that most had not made the decision to implement them, and therefore, they were not very clear about their "raison d'être", understanding them as a mere review of secondary mathematical content.

This make us think that its implementation has become a fad imposed in each university to not be less than the others, without carrying out a prior analysis of its need, effectiveness, and adequacy of the design. Faced with the question of their relation to

university mathematics, they mostly say that despite not advancing the subject that will be seen in the university, they select those basic topics that will be developed throughout the course. In addition, as these courses are not always assessed, it is difficult to obtain data for their effectiveness.

The "bridging courses" are addressed to a wide range of students who will start different careers, where theoretically, mathematics also has different levels of complexity. This might suggest that the design and content of each of these courses would have to be fit to the peculiarities of the different university studies. Reality, however, belies this assumption. The main goal in most of these courses is to cover the maximum number of contents in the short time of the course, trying to make an exhaustive review of all secondary mathematics, regardless of its relation to the course or the type of students to whom they are addressed.

According to Serrano (2013) we classify the courses observed in two large groups, considering the kind of *didactic moments* (Chevallard, 1999) that appear more directly addressed. Let us remind that the theory of didactic moments provides a frame to describe study processes based on the praxeological structure of the knowledge to be learned: a *first encounter* with the *question* or *type of tasks* to be solved; the *exploratory moment* of the type of tasks till a germ of technique emerges (or is introduced); *the moment of the technical work* where the variations of the technique and its scope are examined; the *technological-theoretical moment* when questions about the description and justification of the techniques used and the delimitation of the types of tasks are addressed ; the *evaluation moment* to assess the value and robustness of the praxeological elements obtained ; and the institutional moment, which provides a public version of the work mobilised and relates it to the outside world.

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The first group of the observed "bridging courses", these were based on a list of problems organized by thematic areas but not too articulated among themselves. The study of this list of different problems led the students to live a kind *of eternal first encounter and exploratory moments* that, due to lack of time, was not necessarily productive. It was then the teacher who ended up with a continuous "bombardment" of reminders of (supposedly known) techniques and technologies in a very opportunistic way, following the needs of each case. The students were thus faced with a large number of specific and isolated mathematical praxeologies, starting from their practical block (types of problems and techniques). Despite the appearance of an "active" methodology focused on solving problems, classroom work was done mainly by the lecturer. This strategy prevailed in the case of courses offered in scientific careers.

The second large group consisted of what we call "classical style" courses characterized by a large number of mathematical praxeologies introduced throughout a series of topics fixed *a priori*, where the only common thread between them, if any, was situated at the level of the theoretical discourse, far from the mathematical responsibility of the students, and exclusively assumed by the teacher. In the end, the student also ended up working with a large number of specific and isolated mathematical praxeologies, although the teacher's speech appeared more connected and structured in local or regional mathematical praxeologies. This structure mainly corresponds to the courses offered in social careers. The theoretical block of the praxeologies was here the main element of the structure presented. In this case, the *moment of the first encounter* was strongly linked to the *technological-theoretical moment*.

In the observations made, the "bridging courses" were (implicitly) based on didactic models that, at the most, take into consideration two of the didactic moments or dimensions of the study process. They also appeared to be based on epistemological

mathematics models essentially "conceptualist" and "cumulative" – that is, interpreting mathematical knowledge as a network of concepts that can be expanded by the accumulation of new concepts and new relationships between them. As a consequence, these courses tend to reinforce two moments of the study process to the detriment of the others, thus favoring the isolation and disarticulation of the mathematical praxeologies that are studied in secondary (Fonseca, 2004).

We can connect these observations with the proposal made by Biehler and Hochmunth (2016) when they distinguish four types of bridging courses according to the elements of the praxeologies to be taught that are taken into account:

- Improving skills in applying techniques stemming from current or past school mathematics (development of the practical block);

- Improving technical skills and technological competences in school mathematical contexts (development of the theoretical block);

- Introducing theoretical and technological aspects of university mathematical practice within topics from school mathematics (questioning the practical block);

- Reflecting relations between school and university mathematics (questioning the whole praxeology).

In these types of strategies, all praxeologies (those already available from secondary level and those newly introduced) are supposed to be previously established and the focus is put in the way to prepare the transition from the previously studied to the new ones to come.

## A "bridging course" based on the ATD

The previous analyses led us to propose an alternative "bridging course" that, in a certain way, goes on the opposite direction of the observed ones. If, as we have argued,

the main handicap of students arriving at the university is their inability to articulate theoretical and practical knowledge to go beyond the simple application of specific techniques to isolated problems, then we believe that the main goal of a Secondary-University transition course would have to facilitate this work. Instead of a quick and necessarily superficial review of an important part of the knowledge that the students have just studied (or should have studied), we propose to carry out a detailed and in-depth study of a few problematic questions related to the university degree students are about to start (Barquero, Bosch & Gascón 2009, Bosch & Gascón 2007). This study is aimed at letting them connect different praxeologies, to question the techniques learned at secondary school and to develop them until endowing them with more scope and validity (Ruiz-Munzón & Serrano 2011).

Considering the results on the mathematical and didactic discontinuities between the secondary and the university presented by Fonseca (2004), we designed a course that tries to overcome such discontinuities. In other words, facing the phenomenon of the atomization and rigidity of school mathematical praxeologies, the didactic problem that we pose is to design a process of study that makes it possible to integrate certain specific praxeologies into a relatively complete local praxeology.

The course that we propose fulfills the following characteristics. First, the general goal is the articulation of a few specific or local mathematical praxeologies studied in secondary, which present a low degree of "completeness". These are praxeologies that the students are not able to connect spontaneously and that they are part of a central mathematical praxeology in the curriculum of the mathematics course that students will begin at university. Second, the articulation of these specific mathematical praxeologies should be done in response to an initial problematic question that has interest and "meaning" for the students. It is intended that the articulation of specific mathematical

praxeologies has a mathematical functionality and does not respond (only) to pressures of the didactic contract. And third, the reconstruction of the local mathematical praxeology from the specific mathematical praxeologies "belonging" to students, must cause that the different moments of the study process arise and are integrated in a functional way without the predominance of a few to the detriment of the others.

The initial question $Q_0$ that we have chosen as the starting point of the didactic process can be condensed in the study of inequalities of the type $f(x) \geq g(x)$ where $f$ and $g$ are elementary functions of real variable. The origin of this question is linked to a problem of comparison of economic magnitudes:

> $Q_0$: Determine for what sales company's incomes are greater than costs (or unit prices higher than average costs, or profits greater than a given value, or costs less than a given value, etc.).

If we model incomes, costs, unit prices, average costs or profits of a company through functions that depend on a single variable –sales– then, the question raised leads to the study of an inequality of the type considered. We will suppose that in all cases the mathematical model $f(x) \geq g(x)$ is already given, that is, we will not ask the students to determine the functions from certain information about the company, and the problem lies in determine the values of x for which the inequality is satisfied.

The mathematical praxeology that will allow to answer the initial question $Q_0$ can be considered as an articulation of three specific mathematical praxeologies that are studied at secondary level:

- The *resolution of equations* (linear and quadratic, cubic with integers or rational roots, logarithms or very simple exponentials), which we will call $P_{EQ}$;

- The *resolution of algebraic inequalities* (first and second grade basically), very incipient at secondary level and that will be designated by $P_{IN}$;

- The *graphical representation of elementary functions* (polynomials of degree $\leq 3$, exponential functions and rational functions), that we will denominate by $P_G$.

For students who finish high school, these three praxeologies appear poorly connected to each other. The study of equations is part of the algebraic work with expressions of first and second degree that the students study before the introduction of functions. The resolution of the other types of equations (cubic or logarithmic and elementary exponentials) appears in different subjects of the curriculum, usually linked to the study of functions, but not very linked to their graphic representation. Although the resolution of equations can appear as a sub-technique for the graphical representation of functions, the graphs are never used as a tool to solve equations or inequalities. In fact, within the study of functions, graphs always appear as the end product of a very standardized technique that begins with the determination of the domain, the study of the limits of the function in the border points of the domain, the determination of points cutting with the axes, calculation of the derivative, the study of monotony, concavity and convexity, etc. In this sense, the graph is always the sole and final objective of the process, it is never considered as a model of the relationship between two magnitudes that can be useful to solve problems. Students learn to represent functions, but they do not learn to do anything with these representations. For this reason, each function, whether elementary or not, is considered as an isolated element, almost as a pretext to implement a technique whose steps are specified previously (domain, limits, etc.) and that leads to the construction of the graph. In particular, families of elementary functions are not always studied as such, neither the relationships between the values of the parameters that determine a function of the family and the form of the function graph.

We assume that praxeologies $P_{EQ}$, $P_{IN}$ and $P_G$ are available for students who finish secondary school. Thus, we present as a *reference epistemological model* the way in which these three praxeologies provide the needed ingredients to produce an answer to $Q_0$, allowing the articulation of those ingredients and inserting them in a more complex and wide local praxeology. For this, we need to start from a powerful questioning that guides and at the same time justifies the process of study that has to be taken. In our case, we proposed the problematic question:

$Q_0$; What values of $x$ satisfy $f(x) \geq g(x)$ if $f$ and $g$ are any function?

Given the impossibility of giving a unique and definite answer to this question, it is decided to consider particular cases of it, taking as a criterion the families of elementary functions $f$ and $g$: linear, quadratic, cubic, hyperbolic and exponential. The introduction of new families of functions will modify the techniques used to solve inequalities. Thus, the variable *family of functions* is the motor of the study dynamics. When enlarging the kind of functions considered, the limitations of the algebraic techniques are soon made visible, and the graphic techniques, that seem initially more complex, appear as more efficient and economical, although they only provide, in general, approximate answers that must be "finalized" by some numerical method. This work leads to new problematic questions to be considered. For example, how do solutions vary if one of the functions is fixed and not the other?

In a second part of the course, we proposed a new type of problems generated by the study of an economic question where a cost function is available and the income function has to be constructed given some constraints. The question is how to choose the appropriate unit price for the company to always have benefits from a certain sales value. This kind of problems reinforces, in a certain sense, the work with the graphical technique

and the interest of graphically representing the two functions $f(x)$ and $g(x)$ instead of the difference $f - g$.

The course here presented has been addressed to students who started a Business degree at IQS School of Management of Ramon Llull University in Barcelona (Serrano, 2013). The course had a duration of 30 hours distributed in 10 sessions, and the number of participants varied between 40 and 50 per session. The course was structured in two distinct blocks. The first, dedicated to work with inequalities between elementary functions: "Income and Costs", and the second to elementary modeling with parameters: "T-shirt buying and selling" (Ruiz-Munzón 2006). At the end of these two blocks a final test was performed.

Even if closely related, this strategy does not seem to correspond to any of the four types of courses proposed by Biehler and Hochmunth (2016). It coincides with the two first ones in the fact that, as a bridging course, it is part of a remedial strategy based on "completing" the mathematical education students receive at secondary level – a strategy that can only fail since it pretends do in three weeks something that has not been done in two or more years…

## Conclusions

The appearance of bridging courses on the university scene seems closer to a provisional remedy proposed by universities than to a really support of the secondary-university transition. Therefore, the only proposal is a short reinforce of this preparation in a 30 hours course. This strategy appears as a "coup de force" from the university institution in front of secondary education that marks a strict institutional hierarchy. In other words, what would be a problem of university education – where students begin to fail more than what is considered "tolerable"– is transformed into a problem of secondary

education. Little is questioned of the university pedagogy. And little is made to elaborate a joint solution in a collaboration between secondary school and university.

In our analysis, we have seen that the university spontaneous response materializes in bridging courses with a vast program, seems to reinforce some of the university pedagogy pitfalls, instead of overcoming them. Students are proposed to "visit" a large number of mathematical praxeologies in a very short period of time and through a poor didactic praxeology regarding the different moments of study they prioritize: the exploratory moment when considering the *topos* of the student; a double institutionalization and theoretical moment when considering the *topos* of the teacher. This situation leads us to postulate that, apart from other possible benefits, which undoubtedly contribute, this type of "bridging courses" aggravate the disarticulation and rigidity of the mathematical praxeologies that make up the students' background and, therefore, the mathematical and didactic discontinuities between the secondary and university educational levels.

Our proposal based on the ATD is to design a course that give students the chance to implement experimental *research and studies courses or activities* that, starting from an initial question with a strong generating level, allow them to articulate some of the praxologies that are studied in high school and which, for different reasons (the university access exam among others) cannot be inter-connected. Two important differences between the bridging courses observed and our proposal are, on the one hand, the number of "concepts" that are considered during these 30 hours (much lower in our case) and, on the other hand, the fact that the activities proposed in our course are guided by an economic question (the study income and costs) that structures and gives a rationale to the study process.

In any case, we postulate that their incidence remains very limited in relation to the enormous problem of the discontinuities mentioned. Bridging courses are located in a "limbo" between secondary education and the university. They appear more as a way to avoid the discontinuity problem than a strategy to address it in all its complexity.

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