The constitution of mathematical science from a phenomenological perspective

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ABSTRACT

The objective of this paper is to present a phenomenological conception of mathematics initiated and developed by Edmund Husserl. This author left a vast collection of works regarding the interrogation that inspired him throughout his life: the objective nature of mathematics. With the aim of presenting only an introduction to the theme, we will highlight the following topics: biographical data about the author, the homeland in which the Husserlian vision bears fruit; aspects that constitute the phenomenological conception of knowledge and its production, including, therefore, those related to the process of the idealization of mathematical objects.

Keywords: Conception of Mathematics; Phenomenology; knowledge.

RESUMO

Este artigo tem como propósito apresentar uma concepção fenomenológica da matemática iniciada e desenvolvida por Edmund Husserl. Esse autor deixou uma vasta lista de títulos de trabalhos importantes, sempre tendo como norte a interrogação que o inspirou durante toda sua vida: a natureza objetiva da Matemática. Visando apenas a apresentar uma introdução ao tema, alguns tópicos organizam a trama do texto, tornando-o mais claro e coeso: dados biográficos sobre o autor; o solo em que a visão husserliana floresce; aspectos que constituem sua concepção fenomenológica de conhecimento e respectiva produção, incluindo, portanto, aqueles relacionados aos processos de idealização dos objetos matemáticos.

Palavras-chave: Concepção de matemática, fenomenologia, conhecimento.

Introduction

The objective of this paper is to present the phenomenological conception of mathematics initiated and developed by Edmund Husserl. This conception was initiated and developed by Edmund Husserl, who left a vast collection of works regarding the interrogation that inspired him throughout his life: the objective nature of mathematics. With the aim of presenting only an introduction to the theme, we will highlight the following topics: biographical data about the author, the homeland in which the Husserlian vision bears fruit; aspects that constitute the phenomenological conception of knowledge and its production, including, therefore, those related to the process of the idealization of mathematics.

The meaning of Mathematics is that worked on the context of the European Science.

Biographical information

Edmund Husserl was born on April 8, 1858, in Moravia, into a Jewish family. Unlike other Jewish children, he did not study in the Jewish technical school in the city, but was instead sent by his father, at the age of ten, to study in Vienna with the aim of initiating a classic Germanic education. One year later, he was transferred to a Staatsgymnasium in Olmütz, near his home. After graduating in 1876, he moved to Leipzig, where he studied mathematics, physics, and philosophy, and developed a particular interest in astronomy and optics. After two years, he moved to Berlin to continue his studies in mathematics. At the age of 24, he finished the work he had begun in Vienna, from 1881 to 1883, about *calculus of variations*, earning his doctorate. He remained for a short time in Berlin, where he took an academic position. In 1884, he returned to Vienna where he attended the conferences of Franz Brentano regarding philosophy. In 1886, he went to Halle where he studied psychology and wrote his *Habilitattionsschrif*, about *the concept of number*. This work was re-written in the first part of *Philosophie der Arithmetik*, published in 1891. In Halle, Husserl published this plus two volumes of *Philosophie der Arithmetik*, between 1900 and 1901. They are two important works, brought to light at the beginning of his long life as a researcher (Encyclopedia of Philosophy, 2004).

He went to the University of *Göttingen* in 1901, where he remained for sixteen years, teaching and conducting research. It was there that he elaborated the definitive formulas of phenomenology, presented in *Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie*, published in 1913 and translated into English with the title *Ideas Pertaining to a Pure Phenomenology and to a Phenomenological Philosophy*. In 1916, he left Göttingen for Freiburg to take a position from which he retired in 1928. Even following his retirement, he continued his investigations in phenomenology until his death in 1938.

The list of his publications, and the number of his class notes, are extensive. We will present Husserl’s greatest works, made public during his life. However, we would like to note that many works developed by Husserl were organized by his students and followers; according to the Encyclopedia of Philosophy (2004), there are approximately 45,000 pages of stenographic notes based on the classes he proffered and the research he conducted.

Observing that innumerable articles about themes that are central to phenomenology were made public in the Husserliana and in other specialized journals, we list below the principle works of Husserl, published while he was still alive, in the form of books.

- Über den Begriff der Zahk. Psychologische Analysen, Habilitationsschrift. 1987.
- Philosophie der Arithmetik. Psychologische und logische Untersuchungen, 1891.
- Logische Untersuchungen. Erst Teil: Prolegomena zur reinen Logik,1900.
- Logische Untersuchungen. Zweite Teil: Untersuchungen zur Phänomenologie und Theorie der Erkenntnis, 1901; segunda edição 1913 (para a primeira parte); Segunda edição em 1921 (para a segunda parte).
- Philosophie als strenge Wissenschaft,” *Logos* 1 (1910-1911) 289-341.
- Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie. Erstes Buch: Allgemeine Einführung in die reine Phänomenologie, 1913.

- “Vorlesungen zur transzendentale Logik. Versuch einer Kritik der logischen Versunft,” *Jahrbuch für Philosophie und phänomenologische Forschung* 10 (1928) 367-498.
- “Formale und transzendentale Logik. Versuch einer Kritik der logischen Vernunft,” *Jahrbuch für Philosophie und phänomelogische Forschung* 10 (1929) 1-298.
- Méditations cartèsiennes, 1931.
- “Die Krisis der europäischen Wissenschaften und transzentale Phänomelogie:Eine Einleitung in die phänomenologische Philosophie,” *Philosophia* 1,(1936) 77-176.

Some critics have developed arguments regarding the work of Husserl, seeking to characterize it in three phases, according to the universities where he was working: Halle, Göttingen and Freiburg. However, this procedure, as we understand it, does not express the process of Husserl’s production of thought that we found in his works: constantly re-visiting themes, to which he returned reiteratively, in an attitude of interminable analysis and reflection on the writing, explicitly showing rigorous self-criticism.

It is true that the initial works have characteristics, which reveal conceptions of a more psychological nature, and in a sense, realist with respect to the production of knowledge. However there is a continuity in the themes and in the self-criticism, a constant returning to take up points considered to be unclear or not well-elaborated, or that had been the subject of criticisms and observations of his companions, students, or other scholars.

Although we did not find a biography of Husserl, in the true sense of the word, there are retrospective texts. In the introduction that Husserl himself makes to the English translation of the first book of the *Ideen*, dated 1931, he sketched a self-criticism of his work, describing himself using the metaphor of an explorer who opens roads into new territory. It is a spatial and geographical metaphor in which there is a reference to a *new land*, which was a pioneering adventure to unveil, requiring courage, and that he himself pre-view. The new land is phenomenology, referring here to the conceptions of knowledge and reality that come to light when one assumes the *phenomenological attitude*. Husserl expresses himself as follows:

> The author sees the infinite open country of the true philosophy, the “promised land” on which himself will never set foot. This confidence may wake a smile, but let each see for himself whether it has not some ground in the fragments laid before him as phenomenology in its beginnings. Gladly would he hope that those who come after will take up these first ventures, carry them steadily forward, yes, and improve also their great deficiencies, defects of incompleteness which cannot indeed be avoided in the beginnings of scientific work (Husserl, 1972b, p.22).

The homeland in which Husserlian work bears fruit

Husserl was educated in philosophy and mathematics during the second half of the 19th Century, in the principle European centers of science. Thus, he witnessed the advances in formalization, generalization and abstraction that became central to the history of mathematics at that moment in time. Conceptions assumed by different mathematicians regarding science, such as Platonism, nominalism, Hilbertian formalism, pragmatism, and conventionalism sustained ways of explaining mathematical reality and fed controversies that pointed to the inherent difficulties of these conceptions.

One of the greatest difficulties of *Platonism*, for example, has always been to explain how it is possible to have knowledge about immutable, independent, abstract entities such as numbers, sets, and functions. The epistemological link between the knowing subject and these entities, in the Platonic view, remains a mystery.

To confront this problem posed by Platonism, some mathematicians assume a *nominalistic* view, for which the question of explaining knowledge of universal and abstract entities does not exist, since they claim that only particular spatial-temporal entities exist, which can be known in the world itself in which causal relations form the basis for the construction of knowledge. Upon reducing mathematical language to the language of particular and concrete entities, the *nominalists* can not account for characteristics of mathematical objects, whose nucleus of invariants continue to support concepts with specific denominations, such as number, for example, and that are assumed, in the production and application of mathematical knowledge, as being an identity in different spaces and times.

> A major barrier to a nominalistic treatment of mathematics lies in the fact that many mathematical propositions are about infinite sets of objects, like the set of natural numbers, but it is difficult to see how such propositions could be reduced to a language in which only spatio-temporal particulars are mentioned. (Tieszen, 1996, p.439)

An attempt to affirm the consistency of the foundations of mathematics can be found in *Hilbertian formalism,* which is supported by the resource of consistent proofs for mathematical theories. Traditional formalism emphasizes the role of syntax in these proofs, and does not consider questions concerning the analysis of the meanings of symbols that express mathematical statements. This is an obstacle raised with respect to this theory. An exemplary opposition is presented by Gödel's theorem of incompleteness, which shows that:

> We will not be able to have consistency proofs for interesting parts of mathematics, even for elementary number theory, if these proofs are to be based, as Hilbert wished, on only the immediate intuition of concrete, "meaningless" finite signs configurations. (Tieszen, 1996, p.440)

Tiezen[1] comments that consistent proofs for interesting mathematical theories would have to involve analysis and reflection regarding the meanings, or would have to introduce more abstract elements, and the traditional formalism of Hilbert leaves no room for these considerations.

Another question posed regarding this way of conceiving of the foundations of mathematics is related to the possibility of applying this science to practical questions, since this theory distances mathematics from the reality in which the relations between science and technique are established.

[1] Tiezen, Richard, from United States, San Jose University, developed important studies on Phenomenology, Logic and Philosophy of Mathematics. His work on Gödel incompleteness theorems looked through phenomenological focuses opens ways to understand the complexity of the mathematical science constitution.

This problem faced by Hilbertian formalism does not pose itself for the pragmatist view of mathematics, as this conception judges the validity and credibility of mathematical statements on the basis of their applicability; or, said in another way, based on their utility. For Quine for example, the axioms of mathematical theory are justifiable and supported by their statements only to the degree to which they are part of the best scientific theories. Two arguments against this theory are presented: How to explain the parts of mathematical knowledge that are not applied? And the argument proposed by Charles Parsosons, how to explain the obviousness of elementary mathematics? One way to get around questions that follow from the pragmatic conception is to assume the concept of *informal rigor*, as Georg Kriegel called it, for mathematical concepts that are under investigation and appear to be promising. *One of the best recent examples of such informal but rigorous concept analyses can be found in Zermelo's description of an iterative concept of set (Tieszen, 1996, p.441).*

Another relevant example can be found in the description of lawless sequences used by Troelstra and VAN DALEN to obtain axioms for the intuitionist theory of these objects.

Husserl lived in the environment in which the construction of Twentieth Century mathematics was being realized, and witnessed the clash regarding the production and credibility of what was produced mathematically, and respective questions about its applicability, universality, and identity, as well as the language used as the vehicle for its ideas. His intentionality was directed to the objective nature of mathematics - the relations between the psychological or subjective aspects of our mathematical experience, and those that are logical, or objective, and historical. This is what perplexed him. This is what he worked on throughout his life, and his history reveals comings and goings, re-taking up of the criticisms and self-criticisms, and the gradual construction of the layers of meaning of his ideas.

In his work, Husserl responds to the more significant questions posed to mathematics, and to the ontological and epistemological aspects concerning its way of being. The arguments raised about Platonic theory regarding the reality of mathematical objects present the conceptions of idealization that involve the ideality of these objects, as well as their aspects of abstraction and immutability, whose process of cognitive production is rooted in the real world of the subject's experiences (transcendental) or of his companions. With this, the obstacles to understanding and explaining the link between idealities and cognitive processes, with communication between subjects and with permanence, no longer find a place. Regarding the arguments that are critical of nominalism, the Husserlian conception considers them to the degree to which they highlight questions of language, which is also useful to express the generality and unity of variations experienced in the everyday world that, by way of intentional acts, can, through the unfolding of cognitive acts, constitute the noemetic nucleus. The objections to the traditional formalism of Hilbert assume the importance of formalization in the constitution of mathematical theories, but also assume the need to seek the meanings of formal language, through the reactivation of the original intuitions that constitute the roots on which the acts of signification are founded, making it possible for them to be comprehended through analysis and reflection. Regarding the objections to pragmatism, they are diluted in Husserlian phenomenology in that this phenomenology is conceived of as a science based on itself, firmly founded on its bases. These bases are those that are grounded in the life-world experience of the subjects and co-subjects, which sustain the process and production of mathematical theories as well as their application. This conception also responds to the objections regarding Hilbertian formalism, concerning the separation between mathematical theories and reality, in such a way that the possible applications of these theories are compromised.

This said, we continue our line of thought, discussing important aspects of Transcendental Phenomenology, to then focus on the phenomenological conception of mathematics, which makes sense in this context.

Transcendental Phenomenology

Many of the critics of Husserl, and of phenomenology in general, who speak from academic circles today, do not base their criticisms on a responsible analysis of his work, but rather limit themselves to one book or another, referring, in general, to the work published in the first phase of his academic life. One criticism that remains, placing all of transcendental phenomenology under suspicion, concerns the solipsism of the transcendental "I". Desanti[2], for example, states that he abandoned the foundations of phenomenology in his work *Les Idealités Mathematiques (*Desanti,1976) when he had to assume the conception of the transcendent *subject* or *constitutive subject*, having used in his work only Husserlian forms of analysis.

This objection aligns itself with the criticisms regarding the impossibility of a phenomenology, for understanding it as realist and given to psychologism, or for being Platonic, to the degree to which it works with reality, or for being transcendental and founded in the pure "I", in a solipsistic attitude, and so on.

In the *Preface* that he wrote to the first English edition of *Ideas (1972b)*, published in 1931, HUSSERL stated that he makes advances in the book with respect to the elaboration of a transcendental phenomenological idealism, as opposed to a form of psychological idealism. However, according to him, the thoughts expressed in paragraph 33 of the book are incomplete. They lack proper consideration of the problem of transcendental inter-subjectivity, of the essential relation of the objective world that is valid for me and for other transcendental co-subjects. According to Husserl, *Ideas* opens roads for the essential question that drives his work - the radical philosophizing. That is, philosophizing that does not support itself on presumptions or *a priori* beyond its own process of philosophizing, but rather is autonomous, founded on its own bases. Philosophy that follows this way of thinking is a first philosophy, a philosophy of beginning, based on its *origin*. This is extended to the eidetic sciences in general, with mathematics as an example, when seeking the origin of the founding idea.

At this point, a clarification is needed regarding the conception assumed by Husserl to be essential to the comprehension of his total phenomenology, termed *origin*. As happens with language, *origin* is a word with multiple meanings. It can signify a *search for principles* that generate or began an existing system or being, as in the case of pre-Socratic philosophy which sought the origin of the universe, focusing on its generating principle. Thus, for a philosopher from this period, it may be the air, for example; for another, fire, and for water. Another meaning derives from the search for

[2] Desanti , Jean-Troussaint (1914-2003) is a philosopher from Sorbonne University. He studied Husserlian Phenomenology aiming to explain the constitution of mathematical ideality. He translated to French the Husserl's work on the Origen of Geometry and made a meaningful introduction to the French edition of this work, clearing phenomenological concepts in relation to the mathematical idealities.

the name of origin, or as Desanti says, *the inauguration of a style without which mathematics would not be able to admit the object (1976, p.110).* Another way of understanding *origin,* assumed by Husserl, is to seek the originating insight, the first perception, the original essence that lies in the constitution of the formations-meanings in modes of validity of existence, perpetually new and self-organizing. In the system of intentional actions, where existential objects of varying modalities ascend to the level of object, the original essence occurs in/for the Transcendental Ego, as the occasion demands. In the words of Desanti,

> Yet another way to seek the "origin" would consist of excavating in the locale, in the hope of finding the mathematical sub-soil. For example, below mathematics is nature. Also, one would not look for the birth date, but the natural terrain of mathematics. But up to here, hope is in vain, and the investigation, illusory. There are only supposed sub-soils of a mathematics in movement. It would be naive to believe that collections of rocks are at the origin of the idea of numbers".(1976, p.111). (author's translation).

Focusing specifically on mathematics, Husserl's interrogation is posed clearly in *the Origin of Geometry* (Husserl, 1970c), and inquires about the possibility of something that originates in the intuitive act, entrusted to the subjectivity of a subject, coming to constitute itself in an objective way, historically stable, albeit bearing in its essence the mobility of advances and regressions, able to be taken up and re-activated by subjects from other epochs and cultures, as well as applied to a variety of situations.
Viewed in the context of his work, his inquiry regarding mathematical objects makes sense when one understands his conception of Transcendental Idealism, or Phenomenological Idealism.

Transcendental Phenomenology, as Husserl makes explicit in the *Preface* to *Idea* (HUSSERL, 1972b), is not a theory seen as a response to the historical problem of Idealism, which affirms itself in presumptions external to its system of proposition, and always came up against mystery, or at least difficulty in making clear the link between ontological and epistemological reality.

Phenomenological Idealism does not negate the positive existence of the real world and of nature, as if it were an illusion, clamoring for validity obtained in the light of epistemology, such as it is explained in Cartesian philosophy. This is not to say that it goes to the other extreme, that of naive realism and its implicit pragmatism, as if reality were objectively given in such a way that the propositions of science show only a correspondence of the perfect accommodation between the comprehended and the stated regarding this reality.

The task of Phenomenological Idealism is to clarify the meaning of this world and this reality, the sense in which anyone will accept it as existent, with undeniable certainty. The clarification of the way of being of the world and, eidetically, of a real world in general, carries with it, as an unfolding, the acceptance that only *transcendental subjectivity* signifies, ontologically, *Absolute Being.* This is to say that the essence of the world, that is, of its *eideticity*, is relative to transcendental subjectivity; the subjectivity that already effected the reduction of its meaning to the transcendental Ego and involves, in its constitution, its experience and that of its co-subjects, companions in experiencing the world.

The starting point for Phenomenological Reduction is the non-sophisticated starting point of mundane reality, taken in its positiveness and held to be undeniable by those who experience it. As Husserl said, the objective world is there for you and me, intentionally placing me in *epoché,* movement made possible by reflection, accepting me as a human being in *own body* and capable of realizing spiritual activities, I reflect on this reality and its characteristics, beyond what is shown in its manifolds. This self-reflection, when it advances in the direction of more elaborated analyses, reveals that the Absolute Ego, or Absolute Subjectivity, contains the expressions of absolute subjectivity of other Egos, co-subjects that experience this world, thus constituting a zone of inter-subjectivity capable of validating the eidetic objectivity of the world. Here we have a difference between the reality of the world empirically validated directly and naively, and eidetic reality, validated by transcendental phenomenology. The work of this phenomenology is characteristically philosophical. It is intentional. It pursues the paths of experience that lead to the radical origin that reveals the self-evident of the comprehended.

It is maintained, with this explanation, that the real world exists, without being subject to suspicion. However, regarding its essence, it is relative to transcendental subjectivity in such a way that its meaning is explained as a reality existing only as intentional product-meaning of transcendental subjectivity. This transcendental subjectivity, in turn, is already constituted by the transcendental subjectivity of the other co-subjects that constitute themselves with it and are constituted by it.

Transcendental Philosophy, or Philosophical Phenomenology, is rooted in a radical reflection about the meaning and possibility of its own way of bringing about the consciousness act. Its work consists of taking possession of the absolute homeland of the pre-conceptual experience that constitutes its own reserve. The work is the fruit of eidetic reduction, generated by the intentionality of the Transcendental I, which creates original concepts that make sense on their homeland and upon proceeding in this way, is apt to expose its method in a transparent manner. Its procedures reveal a movement that frees its foundations from all presumptions, revealing an absolute base; absolute in the sense of free from presuppositions, and rooted in its own homeland of experiences and in the evidence of original acts.

For the purpose of clarity, it is perhaps important to say at this point that original acts do not close on themselves like a cocoon, but rather open as they unfold in the form of understandings and interpretations, and in this movement, constitute layers of meanings that carry with them the meaning that the world makes for those that interrogate it, and the modes of expression and their respective forms of production and applications, thus carrying thus the meanings present and transported in the tradition transmitted by the world of language.

The view of the mathematical object

The task of Phenomenological Idealism, in relation to mathematical objects, is to clarify the meaning of these objects, and of the reality where they encounter their homeland and origin. It is to accept as existing, with undeniable certainty, mathematics as it is present in the world, through its means of expression. In the context of the mathematics of the Western world, it is to go to the homeland in which the essential origin occurs, seeking to uncover the layers in which mathematical ideas - the statutes of these objects - are enfolded, as well as understand the modes of existence of their theories, production, and practical application, and the way in which it preserves stability, even as it shows its essential mobility.

It can be stated outright that, for Husserl, mathematical objects are abstract and ideal. They are not understood in the Platonic conception of ideality, as they have no ontological *a priori* that provide the foundation for them. The Husserlian ideality concerns that which is constituted in the intentionality of transcendental subjectivity, in the homeland where mathematical experiences occur and make sense, for the subject as well as the community of co-subjects that are present together in time and space, or that are culturally and temporally distant. They are abstract objects whose referentials are terms such as number, function, etc. The intentionality of the transcendental subject is directed at these objects, to the degree to which they behave as contents of acts or states of occurrence. In HUSSERL's most recent works, these contents, in the noesis-noema (act –object) relation, are denominated the noematic nucleus.

According to Tieszen (1996), it is curious that while the intentionality of cognition is recognized and discussed in other areas of philosophy, it has been set aside in the philosophy of mathematics. To assume intentionality, which is a characteristic of the way of being of consciousness, is the turning point of the natural attitude, which takes mathematical objects as objectively given in the natural world in the form of an abstraction that is named and defined by properties that characterize it, for the phenomenological attitude. This attitude treats them as constituted based on the noematic nucleus, contents that enfold themselves with other contents, modes of expression, inter-subjective and cultural-historical understandings, as they occur in the homeland where the experiences of a mathematical nature are lived by the subject, and are traditionally conducted by the modes of expression that take form and structure themselves in the field of language.

According to Husserl, language is meaningful only when it expresses the subject's intentional acts, or cognitive acts effected in the movement of the consciousness when facing ... a mathematical object: contents that say of the meaning of numbers or sets, property *of* or relations *between* these objects. In *Ideas* (Husserl, 1972b), the philosopher states that we can think about the contents of an act as the meaning of the act as states of occurrences. Here we encounter, therefore, a convergence between intentionality and attribution of meanings.

The appropriate way to outline the general structure of intentionality, focusing on mathematical objects, is presented by Tieszen (1996) as follows:

Act (content) ----- [object]

where the object is placed between brackets to indicate that one cannot assume that the subject of an act always exists. In his analysis, he determines the contents of acts considering the clause *that,* as in the following examples: *Mathematician M believes that 7+5=12; M knows that there is no prime number greater than* . . . In these propositions, certain acts characteristic of the contents of the consciousness are indicated, such as to believe, to know.

This procedure shows the possibility of working phenomenologically to clarify the content of acts of consciousness without having to be concerned about whether the mathematical objects, or the referentials of these objects, exist or not. In this case, the content or meaning of the act of consciousness can be clarified through genetic analysis or analysis of the *origin* of the essential nucleus, or even through a procedure that he called *the free variation of the imagination*. The latter procedure consists of imagining the possible variations of the object, analyzing what remains invariable in these variations, and intuiting what is essential to it.

Advancing in the explicitation of mathematics, Husserl emphasizes the importance of the formalization of mathematical theories. However, for him, this formalization is a question of seeing clearly what is being done, as the efforts to develop this formalization should make evident the intentionality of expressing what is essential to the form of the acts of the *noematic nucleus*, considering the various suppositions of the homeland of experiences from which it originated. With this approach, he distances himself from the structuralist and functionalist approaches to mathematics, which remain tied to questions of the application of mathematics.

Essential nuclei of the mathematical object: abstraction, ideality, and exactitude.

These three characteristics are central to Husserl's thinking and are at the core of the way he understands and explains the production of mathematical knowledge and the constitution of mathematics itself. The manner in which these characteristics are linked constitutes the Husserlian conception of ideality, revealing that this conception is basically different from the Platonic. In his work, he seeks incessantly to show that mathematical abstractions, idealizations, and formalizations are not cut off from their origins, which are in our everyday experience of the world. However, according to him (Husserl, 1970c), a rupture can occur between this origin and mathematics, as is propagated in academia and applied mathematics, when the search for comprehension and interpretation of this science remains intentionally at the level of repetition of its formulas and their applications.

The root of the constitution of the mathematical object is found, therefore, in our mundane experience, in which and by which the cognitive process, from the perspective of Transcendental Phenomenology, should be understood in terms of its task of perceiving the *invariants* or *identities* that emerge in the variation of this experience, whose flow of mobility is constant.

Tieszen (1996) makes an appropriate synthesis regarding modes of perceiving the invariants in experiences with physical objects, with empirical facts, and with mathematical objects. Physical objects, as he explains, are identities that emerge for us through diverse sensorial experiences, or through diverse observations of these experiences. The perception of identities among diverse facts about objects established through empirical induction, and expressed by empirical laws, allows the emergence of invariants that form part of the constitution of the essential nucleus of these facts. Invariants or regularities in mathematical experience are also detected, although the facts have not been established in this ontological region, speaking of the written mode, through empirical induction, but require proofs of their veracity or existence.

The meaning of invariants that is found at the root of the constitution of mathematical objects is related to the aspects of *abstract* and *ideality* of these objects.

The *Abstract Being* character of mathematical objects, in the Husserlian conception, signifies that these objects are immutable, a-temporal, without causality, and are independent and transcendent of the transcendental subject in some respects. To be immutable, atemporal, and without causality are meanings understood in the context of the affirmation that mathematics presents stability and essential mobility.

That is, on the one hand, it shows itself to be closed – for example, the system of its axioms – but, on the other hand, it always opens to the chains of properties that the position of its objects demands. For this we must keep in mind its relative stability and its essential mobility.(DESANTI, 1976, p.108).

In addition to these meanings present in the conceptions of abstraction and ideality, there is also that which refers to the constitution of the essential nucleus that enfolds itself in other perceptions, interpretations and expressions, in such a way that there is a variation in its character of immutability. With respect to the meaning of being atemporal, this conception refers to their being mathematical objects which can be comprehended by other mathematicians who re-activate the *origin* of the constitution of this object, when the subject who intuited the invariability and expressed it is no longer present, as well as the co-subjects who went through the experiences of their homeland with him. They are without causality as they are not submitted to empirical induction. At the same time, they are free idealities, as they are independent, in their stability, of the intentional act that first constituted them. They transcend this consciousness, as they are not what it would like them to be, nor are they only what it intuited. They carry with them possibilities of completing, of application, of mobility in the chain of is articulations.

Exact objectivity and idealization

In *Objectivity and the World of Experience,* HUSSERL states that:

…exact objectivity is a cognitive accomplishment which 1) presupposes a method of systematic and determined idealization creating a world of ideals which can be produced determinately and constructed systematically in infinitum and 2) makes self- evident the applicability of these constructible idealities to the world of experience. (1970b, p. 348)

In this text, Husserl explains that objectification, or the process of an idea becoming objective, is a question of method, based on pre-scientific data of the experience. He clarifies that, beginning with the intuitive comprehension of the transcendental subject, the mathematical method makes possible the construction of ideal objects, and this method also teaches how to deal with these objects in an operative and systematic way. This exposition is linked with the author's understanding that the originating homeland of scientific knowledge is in the world of real experience, the world of sensibility, despite showing, in its immense variety of occurrences and possible experiences to be lived by humans, individually and collectively, that it is invariant in its structure and generality. It is in this variant homeland of diverse and mutable experiences that objectivity is rooted.

This movement is that in which the link between opinion and knowledge, or *doxa* and *epistime*, is established. This movement is, for Husserl, that of realization of exact objectivity, understood as the execution of method, practiced by humans in the world of experience as a praxis in which the imperfect determinants of the things perceived constitute the material the method works with. In this process, which is that of the realizations of the cognitive acts carried out intentionally, experience of an individual thing, of an occurrence, etc., reveals what is being experienced by way of a representation, as imperfect. This imperfection is comprehensible and accepted in the context of the variability of the real world, and its perceptions are carried like gradations that move in a limited series of modifications, in terms of the experience of the fact, as well as the experience of intuition. However, concomitantly with the unfolding of cognitive acts, an *empty anticipation* of something more perfect occurs, instituted like a continuous series of these anticipations. According to HUSSERL, here begins the process of idealization, or constituting idealities. He refers to the repetition of what has been carried out that occurs in the direction of that empty anticipation of the *more perfect* of the series of acts and that fills this empty intention. This filling occurs when the intended object is really experienced or intuited or perceived. There is still the case of voids which could be filled, when the experience of the real is foreseen as possible. In this process, the series in which the *most perfect voids* install themselves in a continuum is continued. This process is one of iteration in the sphere of the new and newly unconditional of that which can be renewed with ideal liberty. [It makes evident the thing which is perceived as an unconditional generality, act in which the properties of this that is experienced are intuited. In this process, the ideal property emerges as a unit in the infinity conceived in the sphere of the thinkable and exact. In this passage or modification of the way of perceiving the individual no longer as an example, but rather as generator of possibilities of intuiting the ideal property, origin is given to the knowledge of idealities in which the thing is not only real, but brings the germ of ideally possible experiences.

Thus, the *idealizing* thought conquers the infinity of the experiential world, as a world, which can be constructed by continuous thought, and with a tendency to perfection of the experience of the mundane real.

According to Husserl (1970b), mathematics, which, with its method, constructs ideal objects and teaches how to deal with them operatively and systematically, conceives an ideal of perfection based on a conception of infinity of imperfection that can tend toward perfection. It idealizes the properties of perceived things and, with this, idealizes correlatively, its possible identity. On the other hand, it also idealizes the possibility of perfection of the imperfect experience, according to what our direct experience perceives of things. This science, however, does not produce things from other things, in a manual manner, but produces ideas and constructs idealities.

To summarize, this process consists of: the execution of the idealization together with the exactly identifiable ideas that it can produce, such as, for example, mental structures based on the manifolds of appearances that are placed in suspension in the relativity of the world such as they can be experienced; operative construction of the ideal structures based on pre-given ideas, or pre-perceived in the real world. The interconnection of these steps forms an objective scientific mind that includes both infinities: that of manifolds of appearance in one and the same thing that is exhibited, and the manifold of the things themselves.

This is the process of cognitive realization that constitutes exact objectivity, scientific knowledge, and idealization. However, the historical construction of this knowledge, of the exactitude and the idealities, count on the communication of these ideas through language, and on the comprehension and interpretation of what was communicated by other co-subjects. Propositional language carries with it a logical structure and semantic meanings that give form to what is being expressed, at the same time they provide an historic *a priori* of other idealities and conceptions that enfold themselves in the nuclei that give origin to idealities. In addition to these constitutive aspects of idealities, there is also the process of mathematical formalization, whose properties, operations, and *logic of the predicated* allow the mathematical object to be constituted as existing. Silva thus expresses himself as follows with respect to the existence of the object:

> As Husserl told his audience in Göttingen, a formal manifold is the collection of all formal objects that a formal theory somehow identifies. If a formal theory proves that a unique object exists satisfying a certain property, then a formal object exists in the formal manifold determined by this theory satisfying this property, and conversely. (Silva, 2000, p.51).

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