Perceiving with the eyes and with the hands

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ABSTRACT

This article revolves around two experimental examples involving students with and without visual impairment in learning mathematics. The goal is to shed some light on the manner in which learning occurs and how concept formation is achieved within the students' available sensorial modalities. The examples are analyzed through a theoretical cultural-historical approach to teaching and learning—the theory of knowledge objectification. A key feature of the theory, which is sketched in the first part of the article, is the idea of sensuous cognition. Within this theory, human cognition is not considered as a simple natural or biological feature of living beings. Human cognition is rather considered as a culturally and historically constituted sentient form of creatively responding, acting, feeling, imaging, transforming, and making sense of the world. The classroom data illustrates the interplay of the various sensuous modalities in mathematical cognition in children with and without visual impairment and makes room for re-envisioning pedagogical actions in special mathematics education.

Keywords: Sensuous cognition, sensation, visual impairment, multimodality, gestures, objectification, semiotic node.

RESUMO

Este artigo gira em torno de dois exemplos experimentais envolvendo estudantes cegos e estudantes videntes em situação de aprendizagem matemática. O objetivo é avaliar como a aprendizagem ocorre e como a formação do conceito acontece por meio das modalidades sensoriais disponíveis dos alunos. Os exemplos são analisados por meio de uma abordagem histórico-cultural para o ensino e a aprendizagem – a teoria da objetificação do conhecimento. Uma característica fundamental da teoria, que é esboçada na primeira parte do artigo, é a ideia de cognição sensorial. Dentro desta teoria, a cognição humana não é considerada como um simples recurso natural ou biológico dos seres vivos. A cognição humana é considerada como uma forma perceptiva, constituída culturalmente e historicamente, de responder criativamente, atuar, sentir, imaginar, transformar, e dar sentido ao mundo. Dados coletados na sala de aula ilustram a inter-relação de várias modalidades sensoriais na cognição matemática de crianças cegas e videntes e abre espaço para revisitar ações pedagógicas na educação matemática especial.

Palavras-chave: Cognição sensual, sensação, deficiência visual, multimodalidade, gestos, objetivação.

Introduction

Vygotsky's psychological work is framed by an encompassing concept of development whose most striking characteristic is the assertion that children with sensorial impairment and children without sensorial impairment are both subjected to similar processes of development. Vygotsky tells us that we cannot presume the development of a child affected by some impairment is lesser than the development of his or her peers. Rather, it is "a child who has developed differently" (Vygotsky, 1993, p. 30).

Vygotsky complained that psychologists and pedagogues have usually conceived of physical impairments in a negative manner:

> A defect has been statically viewed as merely a defect, a minus . . . They [the psychologists and pedagogues] didn't understand that a handicap is not just an impoverished psychological state but also a source of wealth, not just a weakness but a strength. They thought that the development of a blind child centers on his blindness. As it turns out, his development strives to transcend blindness. The psychology of blindness is essentially the psychology of victory over blindness. (1993, p. 57)

However, Vygotsky was well aware that the question was not whether the lack of a sensory organ was automatically overcompensated by an increased sophistication of the other sensory organs. Such an account remains confined to *biology* and forgets that development is also a *social* phenomenon. Impairment has rather to do with its social conception:

> The blind do not directly sense their blindness, just as the deaf do not feel that they live in an oppressive silence . . . The psychological makeup of a blind person arises not *primarily* from the physical handicap itself, but *secondarily* as a result of those social consequences caused by the defect (Vygotsky, 1993, pp. 66-67)

There are nonetheless ruptures in the social systems that encompass the child's social behavior. This is why "the education of such a child amounts to rectifying completely these social ruptures" (1993, p. 66). Vygotsky goes on to say that "The main goal [of special education] is to correct the break in social interaction by using some other path" (p. 66).

Vygotsky's theoretical standpoint can only be understood within his ampler concept of development. For him, development in general unfolds as it is pulled by two main drivers. One such driver is social interaction. This is why, developmentally speaking, an impairment affects the child's social behavior and influences her cognitive and emotional development. However, Vygotsky firmly believed that an adequate structuration of the social environment could help the child to restore her links to her social world. Thus, drawing on ideas of A. V. Birilev, Vygotsky suggested that a blind person could use another person's eyes or experience as a vehicle of sight: "In this case, the other person's eyes assume the role of an instrument or vehicle, not unlike a microscope or a telescope" (Vygotsky, 1993, pp. 84-85). Social interaction, however, is not all. Development is also driven and shaped by our recourse to signs and cultural artifacts. In fact, Vygotsky and his collaborators considered that the first moment of cultural development was marked by the insertion of cultural artifacts in the activity of the child (Vygotsky & Luria, 1994).

Vygotsky's work has inspired a great deal of work in contemporary research on special education in particular (see, e.g., Fernandes, 2008; Fernandes & Healy, 2010, 2012; Gindis, 1999; Vygodskaya, 1999) and on education in general (Bartolini Bussi & Mariotti, 2008; Lerman, 1996; Roth & Radford, 2011). However, we still need to better understand the specific developmental paths in children with impairments. More specifically, we need to better know how learning occurs and how concept formation is achieved within the child's available sensorial modalities. Along this line of thought, in this article I present two examples of learning and concept formation in children without visual impairment and children with vision impairment. The examples seek to unveil differences and commonalities. My account of concept formation is framed by a Vygotskian cultural-historical approach to learning and development —the theory of knowledge objectification— that I sketch in the first part of the article. The theory assumes that, in its historical dimension, knowledge has been formed and refined in the course of centuries. Knowledge appears as a culturally and historically constituted way of thinking, reasoning, and doing with which the students become acquainted and gain fluency through the mediation of social practices (e.g., classroom activity). Acquaintance with cultural-historical knowledge occurs as the result not of a passive observation but of the concrete, practical, and sensuous reflective engagement in social practices and classroom activity. The theoretical and practical thematization of the student's involvement with historical ideal forms of knowing rests on a non-dualistic, non-representational, and non-computational view of the mind that puts forward the centrality of its multisensorial nature. Following Vygotsky's insight, we assume that learning and concept formation in children with and without sensorial impairment follow in principle the same developmental logic. That is, both types of children are subjected to the same developmental mechanisms (e.g., conceptual classification, symbolization, generalization, abstraction) the difference being that the array of sensorial channels is not the same. In the second part I present some experimental data. I conclude with a brief discussion of the implications for education and special education.

Knowledge objectification

The theory of objectification starts from the idea that, at birth, we all arrive in a world that is already replete with concrete and ideal objects. The world in front of us is a historical world endowed with knowledge—aesthetic, ethical, political, mathematical, scientific, and so on. More specifically, knowledge is conceptualized in the theory as an ensemble of culturally and historically constituted embodied processes of reflection and action. To give but a few examples, arithmetic knowledge can be the processes of reflection and action about quantity mediated by *embodied forms* of representation, as in the case of the Oksapmin investigated by Saxe (1982), where the individuals count using body-parts in a 29-base system (see Figure 1). It can also be the reflective processes of counting through *material forms* of mediation, as in the medieval abacus, or the contemporary hands-on artifacts (e.g., blocks or cuisenaire rods). In the case of music, musical knowledge is the ensemble of processes of aesthetic and aural cultural expression mediated by instruments (e.g., archaic membranophones and tambourines, or modern violins, pianos, etc.).

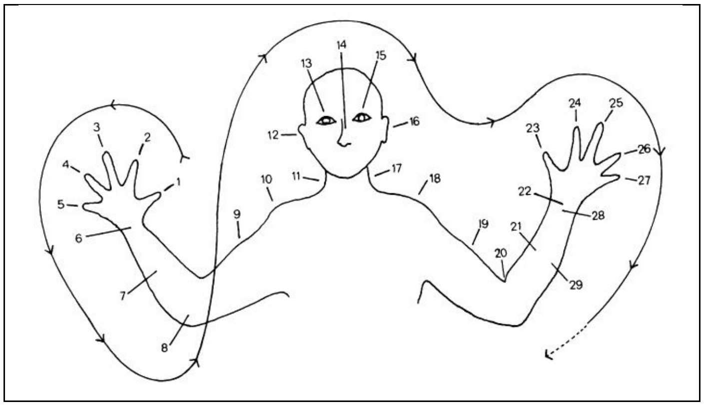


Figure 1. The Oksapmin 29-base system (From Saxe 1982, p. 585)

To conceive of knowledge in the previous sense is to subscribe to a precise ontological view. Although the ontological stance is important in the description of a theory of teaching and learning, the ontological stance is not enough in itself (Font, Godino, & Gallardo, 2012). It has to be supplemented by an epistemological view. In the *epistemology* that underpins the approach to education that I am describing, to *know* is to enact (through embodied and other types of signs and artifacts) cultural forms of action and reflection. To *know* is to make *present*, to expand, and to generalize them, and also to criticize and subvert them.

Now, how does *learning* enter into this picture? Learning appears as follows. The students cannot necessarily discern the historical-cultural forms of action, reflection, and expression that constitute, for instance, projective geometry or algebra. Let me refer here to an example that comes from a regular Grade 2 class of 7–8-year-old students. The example is part of a lesson whose goal was to get the students acquainted with a historical-cultural form of reflection and action that each of us, as competent adults, recognize as algebraic. This cultural form of thinking would easily lead us to generalize the sequence shown in Figure 2 in order to find, let's say, the number of rectangles in Term 100, as well as a formula for Term n.

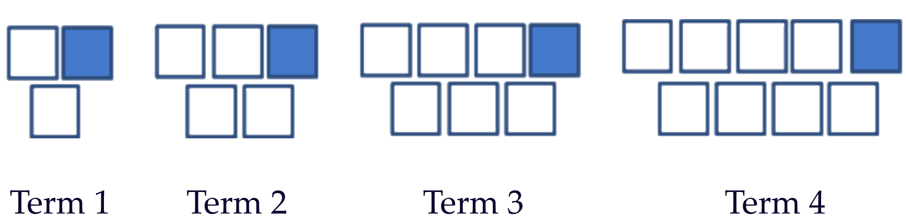


Figure 2. The first terms of a sequence that Grade 2 students investigated in an algebra lesson.

When asked, mathematicians—and even adolescents having some familiarity with algebra (Sabena, Radford, & Bardini, 2005)— often report that they "see" the figures as divided into two rows. Then, they generalize this property to all (visible and non-visible) figures of the sequence, and easily come up with both a formula to calculate the rectangles in remote terms, such as Term 100 (see Figure 3a), and, although not without difficulties in the case of adolescents (Radford, 2003, 2010a), another formula, such as 2n+1, to calculate the number of rectangles in Term n (Figure 3b).

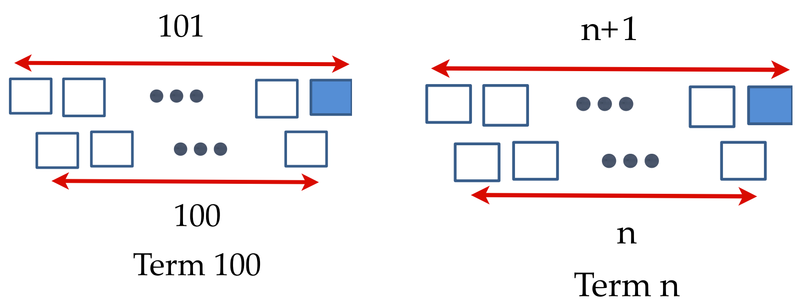


Fig. 3a Fig. 3b

Figure 3. For the trained eye, the terms are often reported as divided into two rows.

For young students, however, discerning what we, as competent adults, could easily discern as algebraic is not necessarily easy. In fact, in our Grade 2 class, when the students extended the sequence and drew Terms 5 and 6, they produced answers like those shown in Figure 4.

Figure 4. To the left, a student finishing drawing Term 6. In the middle, Terms 5 and 6. To the right, Term 5 according to another Grade 2 student.

This is where learning comes into the picture. Learning, I suggest, is the social, sensuous, and sign-mediated process of creatively and critically discerning and getting acquainted and conversant with historical, cultural forms of expression, action, thinking, and reflection. Those creative and active encounters with historically constituted knowledge occur in what I call processes of *objectification* (Radford, 2002, 2008).

The notion of objectification conveyed in this article is hence related to the social processes in which individuals engage creatively in order to make sense and become conscious of historically and culturally constituted forms of doing and thinking (e.g., forms of doing and thinking mathematically, artistically, ethically, etc.). These social processes unfold gradually—hence the methodological interest to investigate them ontogenetically, that is to say, as they occur temporally in the course of learning. Let me notice that processes of objectification are not merely mental processes: they are mediated by language, signs, artifacts, and embodied actions (e.g., gestures, motor movements, tactility, and perception). Processes of objectification are not simply replications of something already there, in culture: through them the historicity of knowledge is made apparent in a form of appearance that is always new and that opens up a space for novelty and creation.

In the next section I present an example with students without sensorial impairment to illustrate learning and concept formation. Then, I shall comment on an example involving a student with a visual impairment. What I want to discuss is the manner in which, in both cases, through objectification processes, the students' sensuous practical activity is transformed into a theoretical cultural form of sensing required in concept formation.

The eye as a theoretician

The first example comes from an ongoing longitudinal classroom study involving a Grade 2 class (7-8-year-old students without visual impairment). I shall focus on two passages from a pattern generalizing activity.

Intention and Perception

At the beginning of a five-day activity, the students and the teacher explored some sequences together. They started with the figural sequence shown in Figure 2 above. The first question of the mathematical activity consisted in extending the terms of the sequence up to Term 6. Then, in questions 2 and 3, the students were asked to find out the number of squares in terms 12 and 25. In question 4, the students were given a term that looked like Term 8 of the sequence (see Figure 5). They were told that this term was drawn by Monique (an imaginary Grade 2 student) and encouraged to discuss in small groups, in order to decide whether or not Monique's term was Term 8. The trained eye would not have difficulties in noticing the missing white square on the top row. The untrained eye, by contrast, may be satisfied with the apparent spatial resemblance of these terms with the other terms of the sequence and might consequently fail to note the missing square.

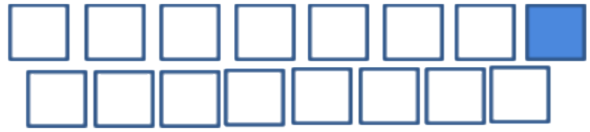


Fig. 5. The students were requested to discuss whether Monique's term is Term 8 of the given sequence. Monique's term appeared on the second page of the activity sheet.

In dealing with problems 1 and 2, some students focused on the numerical relationship between consecutive figures, noticing that there were two more squares between one figure and the next. They did not take advantage of the spatial clues suggested by the arrangements of the squares in each figure and produced answers like those shown in Figure 4 above, where Figures 5 and 6 of the sequence were drawn as having a *single* row each. In other cases, the figures were drawn as having *more than* two rows (see Figure 4, right). Why? Contrary to what empiricist psychology has claimed, the image of an object in consciousness is not the simple mapping of the object. What is grasped of an object in a perceiving act is not the object in its totality. This is true even of "simple" objects, for they present to us many attributes (color, shape, weight, odor, and so on). As Levinas (1989) remarked, the essential character of perception is *inadequate*. This is why it is not enough for the students to have the figures before their eyes. Instead of being complete, human perception is selective or, as Husserl said, *intentional*. In perception, Husserl argued,

> I am turned towards the object, to the paper, for instance… Around and about the paper lie books, pencils, ink-wells, and so forth … but whilst I was turned towards the paper there was no turning in their direction, nor any apprehending of them, not even in a secondary sense. (Husserl, 1931, p. 117)

To apprehend the paper, an intentional act (a form of *intuiting* it) has to come to the fore. In the same way, to apprehend the figures as divided into two rows, a specific intentional act has to lead perception, for "'intuiting' [an object] already includes the state of being turned towards" it (Husserl, 1931, p. 117). The problem for the students, then, is to attend to the figures in a certain intentional way. They have to go beyond the intentional stance focused on numerosity, which makes the figures appear in a certain way in consciousness, and take a different stance, based on rows.

Now, how do the students move from a 'mundane' or every-day phenomenological apprehension of the figures to a more sophisticated theoretical one? To move from a mundane to a theoretical form of perceiving the figures in the sequence, the students have to undergo a cultural transformation in the perceptual manner of attending things. The students have to undergo a process that can be termed "the domestication of the eye."

The domestication of the eye

Naturally, to the mathematician's eyes, perceiving the figures as divided into two rows may seem a trivial endeavour. And surely it is. But it is so only to the extent that the mathematicians' eyes have been culturally educated to organize the perception of things in particular *rational* ways. The mathematician's eyes have undergone a lengthy process of domestication. The domestication of the eye is the process in the course of which we come to see and recognize things according to "efficient" cultural means. It is the process that converts the eye (and other human senses) into a sophisticated intellectual organ — a "theoretician" as Marx put it (Marx, 1998).

Of course, I am not saying that the students did not see two rows. They surely did. But they did not deem it important to recognize the figures as being divided into two rows.

Geometric clues were relegated to the background of attention in order to yield space to numerical matters. The capacity to perceive certain things in certain ways, the capacity to intuit and attend to them in certain manners rather than others, belongs to those *sensibilities* that students develop as they engage in processes of objectification.

Let me now get back to the students. As usual, the students worked in small groups of 2 or 3. When the teacher came to see the work of James, Sandra and Carla, the students had worked for about 31:50 minutes together. They had finished drawing Terms 5 and 6, answered the question about Monique's Term 8 (which they considered to be Term 8 of the sequence) and tried (unsuccessfully) to find the number of squares in Term 12. Noticing that the students were dealing with the sequences by adding two rectangles each time, the teacher engaged in collaborative actions to create the conditions of possibility for the students to perceive a general structure behind the sequence. The teacher referred to the first four terms of the sequence that were drawn on the first page of the activity sheet (Monique's term, which the students examined previously, was drawn on the second page of the activity sheet and was hence not in the students' perceptual field in the first turns of the following episode):

1. Teacher: Ok. … We are going to look at the squares at the bottom… just the squares at the bottom… (*Emphasizing the word bottom and slowly moving her finger three times horizontally from Term 1 to Term 4, the teacher points to the bottom rows of the figures; see Figure 6, picture 1*), not those that are on the top. (*Pointing to the bottom row of Term 1*) In Term 1, how many…?
2. Students: 1!
3. Teacher: (*Pointing to the bottom row of Term 2; see Fig. 6, Picture 2*) Term 2?
4. Students: 2!
5. Teacher: (*Continuing to point and speak in a rhythmic manner, as she will do in the next interventions, she points to the bottom row of Term 3*) Term 3?
6. Students: 3!
7. Teacher: (*Pointing to the bottom row of Term 4*) Term 4?
8. Students: 4!
9. Teacher: (*Moving her hand to an empty space after Term 4, the space where Term 5 is expected to be, she points to the imagined bottom row of Term 5*) Term 5?
10. Students: 5!
11. Teacher: (*Moving her hand again to another space, she points to the imagined bottom row of Term 6*) Term 6?
12. Students: 6!
13. Teacher: (*Similarly, pointing to the imagined bottom row of Term 7*) Term 7?
14. Students: 7!
15. Teacher : (*Similarly, pointing to the imagined bottom row of Term 8; see Fig. 6, Picture 3*) Term 8?

16. Students: 8!

17. Sandra: There should be 8 on the bottom!

18. Teacher: Excellent! Let's see if she [Monique] has 8 [squares] on the bottom [of her figure]. (*The teacher turns the page and the students can see Monique's term*).

19. Sandra: (*Counting the squares on Monique's term*) 1, 2, 3, 4, 5, 6, 7, 8! Yes, she has 8!

20. Teacher: Very well. Now we are going to check the top *(she twice makes a slow gesture to indicate the top rows of the figures)*. We'll look at the top.

Fig. 6. Left, the teacher points slowly to the bottom rows of the terms. Middle, the teacher and James point together to the bottom row of Term 2, while Carla (left) and Sandra (right) look attentively. Right, the teacher helps the student imagine Term 8 as being present in the sequence and points to its imaginary position.

In Line 1, the teacher makes three sliding gestures to emphasize the fact that they will count the bottom row of the four given terms. Through an intense sequence of gestures the teacher suggests a cultural form of perceiving the terms of the sequence—one in which the mathematical ideas of variable and relationship between variables are emphasized. Now, the teacher does not gesture silently. Gestures are coordinated with various sensorial channels and different semiotic registers. In this case, the teacher coordinates eye, hand, and speech through a series of organized simultaneous actions that orient the students' perception and emergent understanding of the target mathematical ideas. Generally speaking, in traditional educational research, gestures and other corporeal aspects of the teacher and the students' activity are not taken into account.[1] However, gestures, body posture, kinesthetic actions, artifacts, and signs in general are a fruitful array of resources to be taken into account in order to investigate how students learn and how teachers teach (Arzarello, 2006; Bautista & Roth, 2012; Edwards, Radford, & Arzarello, 2009; Radford, 2009a). Instead of being mere epiphenomena, a surplus of teaching and learning, these resources, as our previous excerpt intimates, mediate the teacher's and the students' classroom activity in substantial manners.

In our previous work we have termed this complex coordination of various sensorial and semiotic registers a *semiotic node* (Radford, 2009b; Radford, Demers, Guzmán, & Cerulli, 2003). The investigation of semiotic nodes in classroom activity is a crucial point

[1] A couple of notable exceptions are Alibali and Nathan (2007) and Goldin-Meadow, Nusbaum, Kelly, & Wagner (2001).

in understanding the students' learning processes. From a methodological viewpoint, the problem is to understand how the diverse sensorial channels and semiotic signs (linguistic, written symbols, diagrams, etc.) are *related*, *coordinated*, and *subsumed* into a new thinking or psychic *unity* (Radford, 2012). Such a methodological problem makes sense only against the background of a conception of the mind as *embodied* through and through. Such a conception leads to a non-dualistic view of the mind and a concomitant concept of cognition that I have termed in previous work as *sensuous cognition* (Radford, 2009a). In dualistic accounts, the mind is conceived of as operating through two distinctive planes, one internal and one external. The internal plane is usually considered to include consciousness, thought, ideas, intentions, etc. The external plane refers to the material world—which includes concrete objects, our body, its movements, and so on. In opposition to this dualistic view, I adopt a monistic position according to which mind is a property of matter. More specifically, mind is conceptualized as a feature of living material bodies characterized by a capacity for *responsive sensation* that, in humans, evolves—both at the phylogenetic and ontogenetic levels—intertwined with the material culture in which individuals live and grow. The general problem is to understand how, given an array of available sensorial channels, the students (both without and with specific sensorial impairments, e.g., visual) cope with the mathematical tasks at hand and engage in processes of objectification in teaching-learning activities.

Let me come back again to the Grade 2 students with no sensorial impairment (I shall deal later with an example of visual impairment). As mentioned previously, Term 8 of the sequence was not materially drawn on the first page of the activity. In the previous excerpt, the teacher *pretends* that Term 8 is on the empty space of the sheet, somewhere to the right of Term 4. She points to the empty space, as she pointed to the other terms, to help the students *imagine* the term under consideration.

To explore the top row of the terms, the teacher came back to the first page and repeated the same set of rhythmic pointing gestures that she used when exploring the bottom row, and engaged in the same format of questions. The students responded to the consecutive questions and figured out that there must be 9 squares in Term 8. Then, the teacher invited the students to verify Monique's drawing. They turned the page and Monique's term was there, in front of them. The teacher pointed one after the other to the squares in the top row of Monique's term while Sandra counted in a rhythmic way: "1, 2, 3, 4, 5, 6, 7, 8…!?".

The students were perplexed to see that contrary to what they believed, Monique's Term 8 did not fit into the sequence. Here their activity reached a tension. Figure 7 shows Sandra's surprise. Sandra and the teacher remained silent for 2.5 seconds, that is to say, for a lapse of time that was 21 times longer than the average elapsed time between uttered words that proceeded the moment of surprise (for details, see Radford, 2010b).

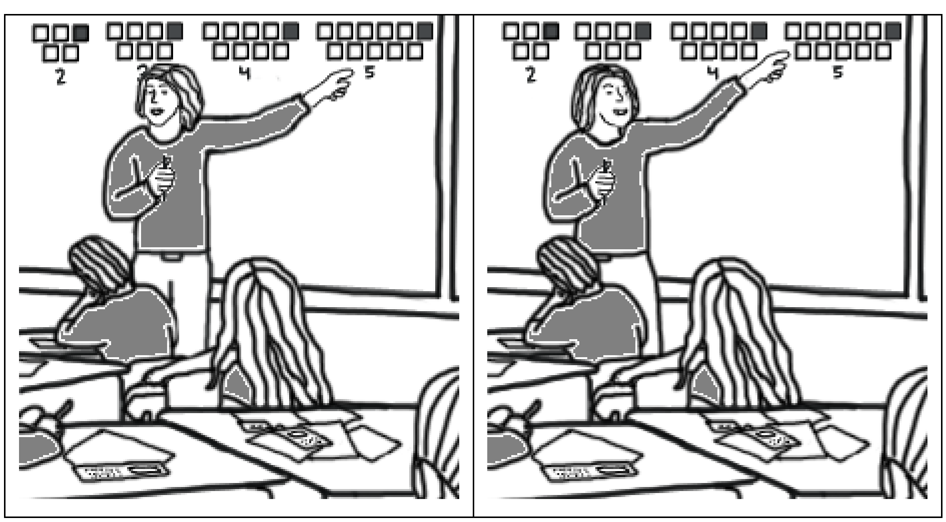
Fig. 7. Left, the teacher points to the last square in the top row of Monique's figure, while Sandra says "8". Right, astonished by the result, Sandra stays still and silently looks at the teacher for 2.5 seconds.

The previous episode took place towards the end of the math class. The next day, the Grade 2 teacher started the math lesson with a general discussion. She drew the terms on the blackboard and discussed with the class a counting method similar to the one used in Sandra's group at the end of the previous day. One month earlier, during the design of the activities with the teacher, we decided that it would be important to encourage several ways of perceiving the mathematical structure behind the sequence. With this idea in mind, the teacher appealed to a method that, she said, was devised by another group of students. The method consisted of conceiving of the terms as being divided into two rows, and counting separately the dark square.

As on the previous day, the teacher illustrated the method through a complex use of gestures, words, and rhythm:

21. Teacher: (*pointing to the number of the term*) Term 1, (*pointing to the bottom line*) one on the bottom, (*pointing to the top*) one on top, (*pointing to the dark square*) plus one.

Joined by the students, she counted in the same rhythmic way the other terms up to Term 5 (see Fig. 8)



Pic. 1 | Pic. 2

Pic. 3 | Pic. 4

Fig. 8. The teacher and the students counting rhythmically say (see Picture 1) “Term 5”, (Picture 2) “5 on the bottom”, (Picture 3) “5 on top”, (Picture 4) “plus 1.”

The students were now able to tackle the activity’s subsequent questions. Among these questions the students had to find out the number of squares in figures that were not perceptually accessible, such as Term 12 and Term 25. Here is an excerpt from the dialogue of Sandra’s group as they discuss without the teacher:

22. Sandra: (*Referring to Term 12*) 12 plus 12, plus 1.
23. Carla: (*Using a calculator*) 12 plus 12… plus 1 equal to…
24. James: (*Interrupting*) 25.
25. Sandra: Yeah!
26. Carla: (*Looking at the calculator*) 25!

Then, a few minutes later, dealing with Term 25, Carla quickly says: “25+25+1 equals 51.”

To sum up, in this section I presented an example that illustrates a classroom process of objectification related to the students’ encounter with a historically and culturally constituted algebraic way of thinking. In the course of this process, the students achieved a generalization expressed through a practical schema that allowed them to find the number of squares in any particular figure of the sequence.[2] The objectification process was decisively mediated by a complex and coordinated multi-sensuous activity that included gestures, body posture, kinesthetic actions, and signs. As mentioned in a previous section, the learning and concept formation of children with some kind of impairment are not qualitatively different from those of children without impairment. The processes of objectification are roughly the same. The presence of an impairment does

[2] We could symbolize such a generalizing schema through the formula “x + x +1,” where x is a *specific* number. Of course, the students did not use notations; yet, they knew now how to apply this practical, embodied schema to any particular term of the sequence.

not inhibit the objectification process; what it does is modify its actual course. It is expected that the multimodal activity reconfigures itself differently. This is what I would like to explore in the next section.

Tactile perception

In this section I draw on the interesting work of Agnese del Zorro (2010) and Elisa Cortesi (2010) on children with impaired vision. Cortesi suggests that touch can be considered as a form of vision and vision as a form of touch. Vision can be considered as a form of touch at distance. In the case of individuals with total loss of sight, touch can be considered as a form of vision without sight. Naturally, as we can see in our previous classroom example, the other senses and our body in general also participate in the relationship between vision and touch. Aurality participates through language; the body participates through its relative position to the other individuals and the objects in the surroundings; gestures and kinesthetic actions participate through the muscular efforts that are made to intuit, signify, and intend something while we interact with others. The resulting complex sensuous experience of the world still needs to be synthesized, without which the world would appear chaotic. The sensuous elements of experience need to be integrated in a meaningful way:

> It is necessary to integrate all perceptions obtained through the vicarious senses of sight: touch, hearing, smell, and taste, but also thermal sense, the sense of moving air (anemestic sense), kinesthesia, muscular and plantar sensitivity, muscle memory, the associative skill and imagination properly formed for extending the concept of space. It is due to the synthesis of all these perceptions that a blind person stops before hitting an obstacle without even touching it. (Cortesi, 2010, p. 16)

In children with visual impairment the hand undergoes a process of domestication, much as the eye does in the case of sighted persons. The child learns to palpate objects; gripping, sliding, opening, closing, covering artifacts or parts of them become skilled actions. The *coordination* of the hands and the specialization of the fingers are a vital aspect of practical and theoretical investigation of the world. One of the hands becomes the reference or dominant hand; it serves as an anchoring spatial reference to explore the object under scrutiny. Thus, to explore a box, the child uses a hand (or fingers of it) to support it. The other hand (or fingers of it) moves in relation to the first one to explore the object, its edges, faces, and so on. Naturally, the distance between the hands cannot be perceptually gauged. A length is rather kinesthetically *felt* through the *sensed* separation of hands or fingers. Figure 8 (left) shows a typical exploration of parallelism. The episode comes from Cortesi's investigation conducted with Marco, an adolescent who lost his sight and who does not have a visual memory. Marco puts two fingers (the index and the middle) over the lines; then, he moves the fingers along the lines and *senses* whether or not the fingers' separation remains constant.

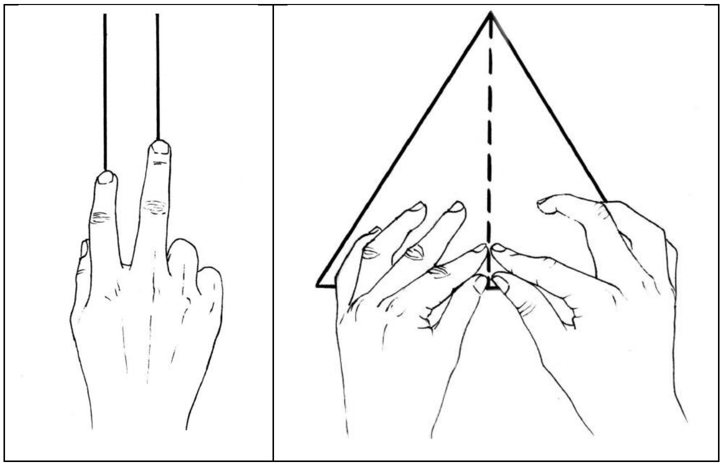


Figure 8. Left, an exploration of parallelism. Right, an exploration of the height of a triangle. (From Cortesi, 2010, pp. 22 and 24)

A more sophisticated mathematical sensing process is involved in gauging areas, perimeters and volumes. Usually, "the thumb offers the reference point to assess the dimension of an object" (Cortesi, 2010, p. 17). Figure 8 (right) illustrates the tactile process through which Marco explores the height of a triangle. The thumbs are placed around the intersection of the segments, on the horizontal one, while the index fingers are placed on the other segment to explore its verticality through the sensuous assessment of the approximate size of the angle.

Geometrical concepts bear in their emergence the sensuous dimension of their genesis. Thus, Anna, an adolescent student with a progressive loss of sight since childhood, who since the beginning of her adolescence can only see shadows and light, defines the concepts of vertex, edge, and face as follows: "the vertex is where it 'pricks,' the edge is where 'you can pinch yourself' and the face is 'where you can place your fingers and exercise circular movements without departing from the face itself'" (Cortesi, 2010, p. 52).

Following a sequence of teaching-learning sessions, Anna was led to distinguish and classify—in the sense of historically constituted geometric knowledge—3D geometrical forms of different sizes made in cardboard and other materials: cubes, pyramids with square base, prisms with triangular base, a cylinder, etc. (for a sample of polyhedrons, see Figure 9).

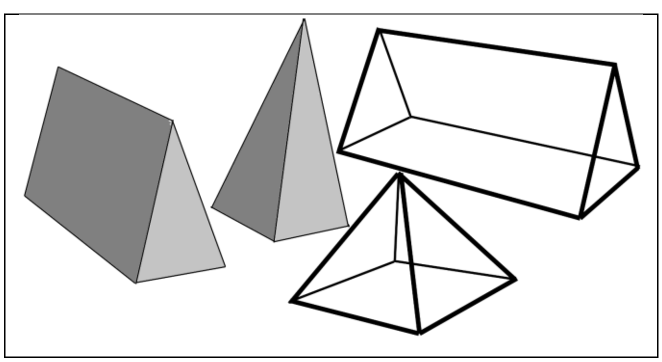


Figure 9. Some polyhedrons explored by Anna. (From del Zozzo's (2010) research).

Anna did not have problems in recognizing the cubes as belonging to the same category. Nor did she have problems recognizing the pyramids. Yet, at the beginning she found similar a cylinder and a prism with hexagonal base: "they are similar in the form and the dimensions" (Cortesi, p. 2010, 72). After tactually exploring the solids a bit further, she realized an important difference: the presence of edges in the prism.

Figure 10 shows Anna's classification of solids. Discriminating between solids and finding similarities between them require, of course, attending to what makes solids mathematically equal and different. This theoretical theme of distinctions and similarities is, as Otte (1998) suggests, the essential question of epistemology. Drawing on Anna's classification, the teacher and the student discussed further the difference between the cylinder and the prism with hexagonal base; such a difference was formalized by asserting that the cylinder, as opposed to the other explored solids, is not a polyhedron. Here language comes to objectify at a more general level what was objectified with the hands. By introducing the term "polyhedron," the teacher accomplishes something similar to what the Grade 2 teacher accomplished in the previous section by introducing the deictics "top" and "bottom" row. In both cases a distinction is made. This distinction allows seeing things both as different and similar: rows are *distinguished* in terms of their spatial position. But they are also *similar* in that the distinction applies to other terms as well. Students can indeed predicate about top and bottom rows in *any* term of the sequence. In the geometry lesson, the term polyhedron serves to *distinguish* cylinders from other solids; at the same time, cubes and pyramids, for instance, can now be considered *similar* in a new sense.

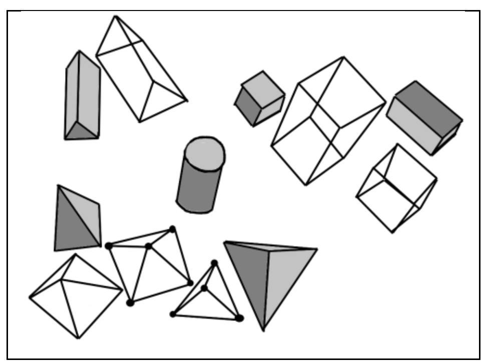


Figure 10. Anna's categories of geometric solids.

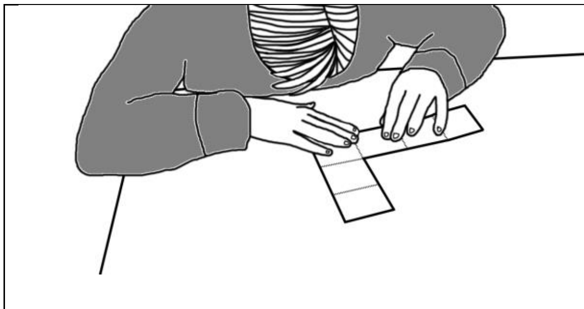
Anna's encounter with a cultural way of thinking about shapes and solids continued with a discussion about the developments of a cube and other solids. In the first part of the activity, a cube in cardboard was "opened" and displayed on a table, according to a cross-shape. Anna explored the development with her hands and made a drawing of it on a drawing rubber surface. Later on, she worked on a "T" shape development. At the end of the teaching-learning sequence, Anna was required to produce as many developments of the cube as possible; she produced four of them. Then she was confronted with a new problem: given a planar shape, she was asked to assert whether the shape was the development of a solid.

A "trompe la main" problem

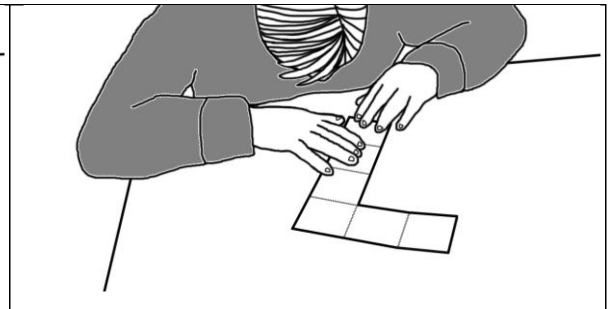
In one case, the teacher gave her an "L" shape composed of six squares. The shape was chosen in such a way that it did not correspond to the development of any solid. This problem is similar to the one our Grade 2 students tackled when they were working on Monique's term. Monique's term is a *trompe l'oeil* problem. That is, the term *looks* like Term 8 of the sequence, but a mathematical scrutiny reveals the flaw of the naive visual impression. The six-square "L" shape is, for Anna, a deceptive shape that, from a tactile naïve experience, may "*look*" like a development of the cube. It is a *trompe la main* problem.

Anna started exploring the "L" shape in a global manner. She positioned the L shape in such a way that it appeared to be rotated 90 degrees counterclockwise (see Figure 11). First, keeping her hands facing down, she left her left hand on the horizontal part of the rotated L shape, and swiped her right hand diagonally from the bottom to the top, as to roughly get acquainted with the shape. She repeated the same hand exploration but quicker. Then, she located the upper square by sensing the traces of the folds on the shape. To explore the square, she kept her index and middle left hand fingers still on the bottom left corner of the upper square, while she moved her index and middle right hand fingers around the square systematically, from left to right (in Figure 11. pic 1, she has finished moving her right hand fingers around the border of the upper square). Switching the role of the hands, she made a quick verification: the index and middle right hand fingers stayed on the right bottom corner of the square. In the meantime, she explored the contour of the square with her index and middle left hand fingers. Right after, she started exploring the two squares below. Like in the verification process, Anna used her right hand as a reference point while she explored the squares with her left hand (see Figure 11, pic 2). She resorted to a similar exploratory pattern to deal with the squares of the bottom part of the shape (see Figure 11, pics 3 and 4).

	
Pic.1	Pic. 2



Pic. 3



Pic. 4

Figure 11. Anna exploring a *trompe la main* shape in a task about solid development concept.

After the tactile exploration, she suspected that if the shape was to be the development of a solid, it should be a cube. But she dismissed the idea: “I can't understand what it could become...Maybe a cube, but it doesn't close. I think that it cannot become a cube” (del Zozzo, 2010, p. 116).

To reach such a conclusion, Anna displayed a sophisticated tactile investigation of the shape. The hands collaborate with each other, changing roles to explore contours, and to gauge distance and the relative position of the different parts of the figure. Anna’s hands and fingers, the “L” shape, the traces of the folds, the angles, and edges, constituted the semiotic means of objectification. The semiotic means of objectification were organized in a complex semiotic node through which Anna came to realize that the shape could not become a solid. The semiotic node consisted of the sequence of coordinated hand movements, the evolving relative position of the hands, the meticulous palpation of the borders of the shape and the folds of its parts, the ensuing muscular sensitivity, the rhythmic manner in which the tactile exploration is organized, the entanglement of the material shape and Anna’s fingers. From this semiotic node emerges a visual tactile image in Anna’s consciousness, allowing her to assert that the shape *seems* to be the development of a cube, but it does not close, and hence cannot really be one.

Indeed, Anna did not fold the shape to verify her conclusion. Her conclusion was based instead on a sensuously derived image that is both ideal and material at the same time. As del Zozzo suggests, talking about ideal (or mental) images in general in children with vision impairment,

> mental images are not connected to a single specific sensory modality . . . Conceiving mental images as simple reproductions of perceptive data seems to be a limited view of a process that actually involves a series of perceptual, attentional and mnestic [i.e., memory-related] mechanisms. (del Zozzo, 2010, p. 26)

Anna’s hands have been transformed into theoreticians organs to explore the environment, much as the Grade 2 students’ eyes have become theoreticians to perceive shapes in the environment. In both cases, such a cultural transformation—the domestication of the senses—has resulted from processes of objectifications mediated by an intense linkage of sensuous modalities.

Synthesis and Concluding Remarks

In the first part of this article, I sketched a theoretical approach to teaching and learning that highlights the role of sensation as the substrate of mind and all psychic activity (for a more precise account, see Radford, 2002, 2008). The role of sensation in human cognition has been a recurrent theme in philosophical inquiries since the pre-Socratics. Since Plato and in fact since the Eleatic thinkers sensation was nevertheless understood in negative terms—as something that hinders the road to knowledge (see e.g., Radford, Edwards, & Arzarello, 2009). This is the sense with which rationalist epistemologies of the 17th and 18th centuries up to the present have endowed sensation. For some contemporary rationalists, sensation does play a cognitive and epistemic role. Yet, our sensing organs are considered to have little (if any) relation with culture and history. Their only history is one of biology and natural development. Piaget's genetic epistemology is not, of course, the only example of such a theoretical stance. By contrast, within the theoretical approach here sketched sensation is not merely part of our bodily and biological constitution. Sensation is rather conceived of as a culturally transformed sensing form of action and reflection interwoven with cultural artifacts (language, signs, diagrams, shapes, etc.) and material culture more generally. The theory of objectification calls into question the usual divide between mind and matter and offers a perspective in which to cast the role of the body and artifacts in knowing processes.

In the second part of the article, I presented two short examples whose goal was to shed some light on the manner in which objectification unfolds in teaching-learning activity in children with and without sensorial impairment. Following Vygotsky's insight, it was argued that learning and concept formation in children without sensorial impairment and children with sensorial impairment follow in principle the same developmental logic. That is, both types of children are subjected to the same developmental mechanisms, the difference being that the array of sensorial channels is not the same. The experimental data presented in this article lends support to such a claim, and is consonant with recent results obtained by other researchers in the field (e.g., Fernandes, 2008; Fernandes & Healy, 2010, 2012; Rosich Sala, Núñez Espallargas, & Fernández del Campo, 2000). In the Grade 2 example we noticed a complex perceptual transformation mediated by the teacher's and the students' intercorporeal activity. This complex perceptual transformation gave rise to a new form of attending sequences. As a result, the students were able to quickly tackle questions about remote terms (like Term 25), that is, terms beyond the material perceptual realm. The second example revolved around the processes of objectification in an adolescent with visual impairment. The analysis presented suggests that, like in the previous example, the objectification of knowledge follows a similar logic. Anna's practical-theoretical investigation of solids was underpinned by a complex semiotic node where sensuous activity and material culture became intimately entangled. Through her hands Anna explored the cultural objects (the cylinder, the polyhedrons) and, in the course of the palpating actions, her hands became apt to recognize the shapes in their materiality and cultural conceptuality and to make distinctions between them— the essential question of epistemology (Otte, 1998). In the same way as the Grade 2 students' eyes were culturally shaped to recognize the ideality of the material objects in the world, Anna's hands were shaped to recognize and distinguish the material forms they touched and the cultural ideality that they conveyed.

These examples, taken together, are a plea for a reconceptualization of the education of children with sensorial impairment along the lines of the famous examples of A. Meshcheryakov (1979) (see also Bakhurst and Padden, 1991). The examples speak for an approach where cognition is not considered as a natural feature of living beings, but rather as a cultural sensuous construction. Such a shift makes room for considering pedagogical actions through new lenses.

Summarizing her research with Anna, del Zozzo tells us that

> Overall, this experience was very encouraging: Anna has discovered a way of "doing geometry" in which she was deeply involved, keeping her attention sharp . . . she certainly has had access to mathematical concepts, even sophisticated ones (for instance the sections of the cube, the classification of solid figures and the non-uniqueness of their plane development) that would remain inaccessible for her in traditional teaching. Another aspect that we cannot overlook is that . . . Anna has experienced a different emotional relationship with mathematics, which she undertook with interest, curiosity and fun. (del Zozzo, 2010, p. 91)

Of course, much still has to be learned. The contribution of the examples that I discussed in this article, and the theoretical approach through which they were analyzed, may rest in the attempt to shed some light on the manner in which children with visual impairment can encounter, and gain fluency in, a culturally and historically constituted form of thinking (i.e., school geometric thinking) where visuality is supposed to be the main feature. Children with visual impairment, Anna shows us, can think, see, and visualize with their hands. Obviously we still need much more refined research to find our best options for special education.

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