PROJECT-BASED LEARNING IN STATISTICS: A CRITICAL ANALYSIS

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Abstract: Number of studies in didactics of statistics present proposals characterised as "project-based learning". We will present a critical review of these works in the light of the Herbartian schema to identify those elements that are most often called upon in the proposed projects, as well as those that seem to be least considered.

Keywords: Project-based learning, statistics education, anthropological theory of the didactic, Herbartian schema.

EL APRENDIZAJE BASADO EN PROYECTOS EN ESTADÍSTICA: UN ANÁLISIS CRÍTICO

Resumen: Un importante número de investigaciones sobre didáctica de la estadística presentan propuestas caracterizadas como "project-based learning". Presentaremos una revisión crítica de estos trabajos a la luz del esquema herbartiano para identificar aquellos elementos que se ven más convocados en los proyectos propuestos, así como aquellos que parecen menos considerados.

Keywords: Aprendizaje basado en proyectos, didáctica de la estadística, teoría antropológica de lo didáctico, esquema herbartiano.

INTRODUCTION

The study of the historical transition from the classical paradigm of visiting works to the emerging didactic paradigm of questioning the world constitutes an important field of research within the anthropological theory of the didactic (ATD) (CHEVALLARD, 2015). Bosch (2018) depicts the research methods developed in this direction, in which the notion of *inquiry* (of a given question) and of *study and research paths* (SRP) play a crucial role. When presenting this field of research, one cannot avoid being connected or even assimilated to instructional methods traditionally labelled as inquiry-, problem- or project-based teaching.

This paper considers some of the investigations or proposals carried out within what is known as *project-based learning* (PBL) that many authors define as a student-centred teaching method or instructional approach (SHPEIZER, 2019; see also DORIER & MAASS, 2020 for the analogous case of *inquiry-based learning* in mathematics education). For instance, the web PBLWorks from the Buck Institute for Education (www.pblworks.org) defines it as:

> Project-based learning is a teaching method in which students gain knowledge and skills by working for an extended period of time to investigate and respond to an authentic, engaging, and complex question, problem, or challenge.

The definition is broad and unspecific, thus embracing a diverse typology of teaching and learning activities. The blossom of PBL during these last years, mostly in the primary and secondary, but also more and more in tertiary education (KORKMAZ & KALAYCI, 2019), can be seen as a symptom of the decay of the paradigm of visiting works and its evolution towards the paradigm of questioning the world. However, this interpretation might be a little optimistic. When considering certain PBL proposals from a closer perspective, one can find relics from the old paradigm that tend to maintain it and restrict its evolution. At the same time, it seems that some key features of the new paradigm struggle to emerge or are not even considered.

From the perspective of the ATD, putting PBL and SRP face-to-face can seem as comparing apples and oranges. In fact, SRPs have their origins as research devices designed and implemented to overcome a specific didactic phenomenon and to study its conditions of existence. In addition, the SRP design, implementation and analysis is framed by an explicit methodology: the so-called Didactic Engineering (BARQUERO & BOSCH, 2015) which contrasts with the more general approach of PBL proposals. However, when ATD researchers

present their studies about SRPs and the conditions needed to make the paradigm of questioning the world come into existence, the connection with PBL is unavoidable.

This paper aims at proposing some elements to clarify why SRPs can be conceived as a PBL instructional proposal but also, and overall, what new perspectives it provides that we did not find in current research on PBL. We will consider the university level, which is where PBL emerged in the training of architects and engineers. Moreover, we will restrict our study to the case of statistics in tertiary education, which seems especially appropriate for this purpose given the number of PBL proposals in the domain, but also its role as core content for some university degrees and its proximity to professional practices.

After the theoretical framing of this paper within the PBL approaches and the ATD, we will provide general insight into some PBL implementations chosen from the literature and highlight the main mentioned projects' characteristic. This empirical material will help us point to some critical aspects of SRPs and the paradigm of questioning the world that do not appear in the PBL proposals, because they are discarded, assumed as not questionable or simply irrelevant for the authors who describe them.

LOOKING AT PBL THROUGH THE HERBARTIAN SCHEMA

The PBL's "teaching philosophy" and its outcomes

In his article about Project Method in the *Encyclopedia of Educational Theory and Philosophy* (2014), the historian of education M. Knoll (2014) locates the origin of this instructional approach in the 16th century's Italian preparation of architects. The Project Method appeared at the time as a way "to advance [builders'] social standing by developing their profession into a science and improve the education of their apprentices by offering lessons in the theory and history of architecture, in mathematics, geometry and perspective" (KNOLL, 2014, p. 665). In this new form of education, architects "transferred their daily work of designing buildings from the studio to the academy" (Ibid.). This is why Knoll characterises this origin as "the academisation of a profession" and not the result of "abstract philosophical deliberations". In fact, many architecture (and engineering) university degrees maintain today specific subjects called "Projects".

The spread of the word and therefore also the evolution of the method lasted for centuries, taking different shapes and roles at the university and in general education, up until the present, when it is still in the process of expansion and development. The most common expression used today is "project-based learning" (PBL) (BATANERO, DÍAZ, CONTRERAS & ROA, 2013; GODINO, ARTEAGA & ESTEPA, 2013; HARMER, 2014). It is presented in all levels of education, and not exclusively in higher education, as a pedagogical approach whose purpose is to step away from the so-called "traditional teaching" – i.e. the teaching based on the direct transmission of a body of contents. It wishes to embrace the contemporary teaching demands such as soft skill development and critical thinking but always considering the purpose and extent of the educational institutions' conditions. In our search of the literature about the PBL, we have identified two main approaches that appear more elaborated: theoretical studies on the philosophy that stands behind PBL, and empirical studies about the outcomes it brings. As we will see later, the main absent seems to be the analysis of the PBL processes themselves.

According to many authors (LEE, BLACKWELL, DRAKE & MORAN, 2014; MARKHAM, LARMER & RAVITZ, 2003; SHPEIZER, 2019), the PBL teaching philosophy is based on engaging students in an active inquiry-based process, with carefully designed guidance, powered by a real-world driving problem. For instance, in his synthesis of the literature, Shpeizer (2019) distinguishes a set of basic characteristics whose combination form the PBL's unique character. He names it as *in-depth inquiry*, *authenticity*, *active learning*, *freedom and autonomy*, *challenging questions or problems*, *collaborative learning*, *the product and product presentation*, and provides a short description of each term. For instance, *in-depth inquiry* includes "phrasing questions, finding sources, collecting information, analysing and synthesising the findings, and applying the information" (SHPEIZER, 2019, p. 1766), activities that are almost absent in traditional transmissive teachings. The *authenticity* recalls that projects go beyond the simulations and hypothetical exercises usually used at school. *Active learning*, together with *collaborative learning* are usually presented as core features of PBL, while *product and product presentation* are less quoted and correspond to the fact that "projects should yield a final product" and "be presented and explained to an audience […] that goes beyond the confines of the classroom" (Ibid.).

The other type of studies assumes these key characteristics of PBL proposals and analyses the effects of their implementation. Many of them (CHEN & YANG, 2019;

HARMER, 2014) base their methodological approach on the comparison between PBL and traditional teaching, measuring aspects such as the *authenticity*, *active learning* and *collaborative learning* part of the projects. Others measure the differences in the students' perceptions or attitudes towards the subject related to the PBL (see, for example, KOPARAN & GÜVEN, 2012). All in all, as we will see later in the case of PBL in statistics, there is a tendency to focus on the students before and after the inquiry process, providing little information about what happens *during* it: how teachers manage these new instructional processes, how students act, what difficulties they all find, what strategies they deploy to deal with them, etc.

PBL components and implementation

It is interesting to approach PBL from the perspective of the anthropological theory of the didactic (ATD) as suggested by (SÁNCHEZ-JIMÉNEZ, GALLEGO, CHEVALLARD & BOSCH, 2020) using the Herbartian schema (CHEVALLARD, 2008). A PBL proposal can be seen as an instructional activity where a teacher or group of teachers Y propose a question Q_0 to a group of students X, thus forming the didactic system $S(X, Y, Q_0)$. Depending on the designations of "problem-", "inquiry-" or "project-based" learning, question Q_0 can be an open problem to solve or a project Π to carry out. We will include these different cases under the same umbrella, by considering the completion of a project Π as an answer to the question: "How to carry out Π?". The projects of PBL proposals always correspond to open tasks, that is, to activities that are not directly achievable by X. This corresponds to the "in-depth inquiry" and "challenging questions or problems" characteristics of PBL. In some cases, the whole group of students X is proposed to approach the same question; in other cases, different questions are presented to the students, to work individually or in small groups. About Q_0, the other characteristic that is mentioned is the "authenticity", which we can understand in contrast to the school fictional problems or situations and, therefore, meaning that Q_0 is a question that exists or could exist outside the school.

In the short version of the Herbartian schema – $S(X, Y, Q_0) \hookrightarrow A^{\bullet}$ –, the inquiry process finishes when the didactic system $S(X, Y, Q_0)$ produces an answer $A^{\bullet}$ to Q_0. The admissible conditions to assume $A^{\bullet}$ as the final answer are usually implicit but at the same time strongly determine the prevailing didactic paradigm assumed by the institution. The type of final answer

that is expected appears an important characteristic of the proposal and determines the meaning of Q_0 and the conditions for the completion of the process: what students will do under the teacher's guidance, and also what the teacher will do to guide the students depend on the final product expected and on who is receiving it. In Shpeizer (2019), the characterisation of PBL includes the "product and product presentation" as something interesting for an audience that goes beyond the limits of the classroom and, we can add, even beyond the limits of the school. The "authenticity" of the process depends on this final product and corresponds to the same type of exigency.

The last aspect that is valued to characterise PBL is the type of work students do, usually compared with traditional teaching. Collaboration is often included, except when the projects are chosen individually, but the emphasis is put on "active learning" and on "freedom and autonomy". These aspects can give us some clues about the type of contract that is established between X and Y. The choice of Q_0 is sometimes the responsibility of the teachers, but not always. In some cases, under certain limitations, students are required to choose their own project. For instance, in architecture, the limitations can correspond to the characteristics of the building's lot and some demands about the type of construction. Students can also be given a set of questions Q_{0i} to choose one among them. Anyway, the responsibility for preparing the proposal corresponds always to the teachers. From here on, the teachers' responsibility is differently specified, without much details about it.

In an often-quoted definition, PBL is presented as a teaching method that "engages students in learning knowledge and skills through an extended inquiry process structured around complex, authentic questions and carefully designed products and tasks" (MARKHAM ET AL., 2003, p. 4). The expression "carefully designed products and tasks" indicates the guidance that is expected from the teachers, especially in preparing the "complex, authentic questions" that structure the process. There is a certain ambiguity in the kind of autonomous work students carry out and the responsibilities they assume in it. Expressions such as "active learning" and "freedom and autonomy" indicate that students have to take important responsibilities in the process, but they do not specify which ones.

We can now consider the Herbartian schema in its developed form, including the *milieu* $M = \{Q_i, A^{\Diamond}_j, W_k, D_s\}$. According to Chevallard (2019), the milieu M comprises *derived*

questions Q_i, *labelled answers* $A^{\Diamond}_j$ ("ready-made" answers to Q that the students X, supervised by the teacher(s) Y, have discovered in the institutions around them), *data* D_s (sets of data of all natures—both quantitative and/or qualitative in nature, for example—gathered in the course of the inquiry on Q_i) and other *objects or works* W_k (works specifically drawn upon to make sense of the $A^{\Diamond}_j$, analyse and "deconstruct" them, bring appropriate answers to the questions Q_i and build up $A^{\blacktriangledown}$):

$$[S(X, Y, Q_0) \rightharpoonup M] \hookrightarrow A^{\blacktriangledown} \quad \text{or} \quad [S(X, Y, Q_0) \rightharpoonup \{Q_i, A^{\Diamond}_j, W_k , D_s\}] \hookrightarrow A^{\blacktriangledown}$$

With this schema, other features of PBL processes can be put to the fore. As mentioned before, authors consider the derived "complex, authentic questions" as an essential component of the process, and the same happens with the search for empirical data. What is missing in many descriptions of PBL proposals are the two other components of the schema: $A^{\Diamond}_j$ and W_k. If PBL aims to reproduce typical real inquiries, then it should consider labelled answers and objects or works not contained in the initial equipment of the students. Indeed, during an inquiry, real researchers do not only search or gather information but typically they also encounter and need to activate new pieces of knowledge that did not belong to their initial milieu. These moments of *study*, which certainly precede the "active learning", are almost absent from the PBL usual descriptions. It is not easy to find details about the way students access the new knowledge and the guidance provided by teachers in this endeavour.

PBL's reflecting position

The kind of details authors choose to provide about the PBL proposals they are describing is undoubtedly related to the position they adopt towards them. We are distinguishing two main choices, even if intermediate cases can also be found: the *insider's position*, the one supporting the PBL movement, and the *outsider's position* of the one who remains neutral – or tries to – to the movement. The position taken in presenting or analysing a specific PBL implementation or its general philosophy affects the features that are announced or disregarded.

For instance, in his literature review, Shpeizer (2019) summarises the results obtained in PBL research as the identification of *advantages* and *challenges*. The *advantages* are "enhanced student motivation", "learning of various skills", "good preparation for a

professional career", "suitability for a wide range of students" and "learning and suitability for the information age". The *challenges* are "changes in roles of lecturers and students", "collaborative work" and "evaluation". This classification coincides in great measure with the *teacher's position* of PBL. Many *advantages* correspond to educational goals that PBL helps to reach, like skills, preparation, etc. Others refer to teachers' concerns about students' motivation or diversity, etc. About the *challenges*, they mainly regard the in-classroom management. The *outsider* position is less frequent in the investigations we have accessed. It would adopt a more sceptical approach, present more details about this new kind of teaching process that is proposed and worry about the conditions that are required to implement it. For instance, in this perspective, the difficulties a teacher can encounter when implementing PBL would be related to her professional background and, therefore, included as part of the challenges. Also, the traditional pedagogical culture that prevails in many national educational systems would be mentioned as a challenge, while the evolution of the curricula descriptors in terms of competences and skills can appear as advantages.

TEACHING STATISTICS WITH PROJECTS

To give more concretion to the above considerations, we will focus on the domain of statistics and its teaching. Given the recent evolution of this domain and its growing importance for general as well as professional education, many authors, among them Batanero et al. (2013) consider that basing statistical classes on the work with projects is "the best way to help the student develop her statistical sense" (p. 12 our translation). According to these authors, projects can be directly proposed by the teacher or freely chosen by the students. They describe this instructional option as follows:

> Instead of introducing the concepts and techniques out of context, or applied only to standard problems that are difficult to find in real life, the aim would be to present the different phases of a statistical investigation: posing a problem, deciding on the data to be collected, collecting and analysing data, and drawing conclusions about the problem posed (BATANERO et al., 2013, p. 12, our translation)

This proposal follows the recommendations of the American Statistical Association presented in the *Guidelines for Assessment and Instruction in Statistics Education* (GAISE) College Reports (CARVER, COLLEGE, EVERSON & OHIO 2016). The report contains six recommendations about the teaching of statistics, supported by an analysis of the evolution of

the nature and role of statistics and the proposal of nine learning goals. In the first recommendation about "teaching statistical thinking", the report proposes to "teach statistics as an investigative process of problem-solving and decision-making" (*op. cit.*, p. 13). Projects arrive linked to this goal: "a way of incorporating the investigative process into a first statistics course is to ask students to complete projects that involve study design, data collection, data analysis, and interpretation. (*op. cit.*, p. 14)

In the direction of studying the implementations and experiences of the projects as part of the education process, we present a summary of a series of articles discussing the topic of PBL in statistics. The chosen references consider the PBL at the primary, secondary and tertiary educational level in various countries and are, in our opinion, representatives of the papers concerning the PBL published in English, mainly during the last decade in education and statistics-oriented journals. Considered articles are presented in Table 1. The aim was to analyse PBL examples from the outsiders' perspective considering what is repeatedly emphasised in different papers and to what extent they provide information about the specificities of the projects and the environment in which they are incorporated.

The characteristics of the chosen PBL examples that form Table 1 are: geographical area (Country), level of education the PBL is implemented in (Level), name of the project if it is mentioned (PBL name), type of frame the PBL forms part of (Description), initial question, problem or general area of interest (Q_0), the origin of the data used in the PBL (Dataset), organisation of the students' work (X), the position of the person responsible for the PBL organisation (Y), type of implementation of the PBL in the education process (Subject integration) and the required way of presenting final students' answers for the project ($A^{\blacktriangledown}$).

Dierker, Evia et al. (2018) treat a one semester-long university course for students of various degrees. The project-based curriculum is delivered in the context of research, and the project topic is chosen freely by students' choice. The source of data is the US public data sets (e.g. alcohol consumption, adolescent health, life survey, world health, Mars Craters, etc.).

Table 1. Summary of the PBL organisation according to the literature considered

Reference	Country	Level	PBL name	Description	Q_0	Dataset	*X*	*Y*	Subject integration	*A*♥
Dierker, Evia et al. (2018)	USA	3°	Passion-Driven Statistics	course	Free choice - US public data sets	given	Individual	Teacher	S = P + T	Poster
Dierker, Flaming et al. (2018)	USA	3°	Applied Data Analysis	course	Different disciplines	given	Individual	Teacher	S = P + T	Poster
Fawcet (2017)	UK	3°	The *CASE project	quantitative methods course	Business, Accounting and Finance context	given	Individual	Teachers + postgraduate support	S= L + T + P	Written work
Feo & Gómez-Blancarte (2019)	Colombia	3°	/	course for nursing	Nursing	/	Groups of 5	Teacher-mentor	S = L + P	Presentation
Marshall (2019)	UK	3°	/	course for mathematics	Medical trial	given	Groups -	/	S = L + P	Group presentation
Smith (1998)	USA	3°	/	course	/	web search, polling, data collection	Groups of 3	/	S = 6×P	Written report
Vignola et al. (2020)	USA	3°	"A World Without Statistics"	course for engineering	Free choice	simulations without datasets	Groups of 3-4	/	S = L + 4×P	Presentation
Koparan & Güven (2012)	Turkey	2°	**ATS	mathematics	Real life and familiar context	students collecting	Groups of 3	/	S = L + P	Presentation
TLS (2019)	Indonesia	2°	Statistics in school	mathematics	School pupils related	given	Groups of 6	Teacher	/	Presentation
Howley & Roberts (2020)	Australia	1°& 2°	"X + STATS"	National poster competition	Free choice	to be created	Groups of 2-5	Mentors	∉S	Poster

/ - not mentioned
1° - primary education, 2° - secondary education, 3°- tertiary education
*Case-based Approaches to Statistics Education, **Attitude towards statistics
Teacher = regular teacher
S - subject, P - project, L - lectures, T - tutorial teaching, ∉ - "not element of"

X- student(s) work organisation

Y - project manager(s)

Source: The authors.

Students activity consisted in generating testable hypotheses, preparing data for analysis, selecting and using descriptive and inferential statistical tools; and finally evaluating, interpreting and presenting research findings orally, graphically, and/or as text. However, the research methodology of the article concentrates on the students' perceptions of statistics and gives little information about the PBL process.

Dierker, Flaming et al. (2018) describe one-semester courses open to all university students with no prerequisites for enrolment. Titled Applied Data Analysis, the course was described in the university's online catalogue as a project-based course. As described by the authors, in the first week, students develop their own research question after an introduction to several large data sets representing different disciplines such as ecology, psychology, economics, and planetary science according to GAISE recommendations. All statistical analyses are said to be done within the context of the students' research question culminating within a poster presentation. As in Dierker's previously mentioned article, the research is about the data drawn from administrative records and student surveys completed before and after each course. Such data consider, e.g. students' increases in confidence, experiences, hard-working, interest, etc.

Fawcett (2017) gives an insight into a two-semester long quantitative methods module for first-year business undergraduates. The module implements the basics of descriptive statistics, probability, inference and modelling. The content of the course lists types of data, data collection methods, graphical/numerical summaries of data, introductory probability and discrete probability models in the first semester, and then continuous probability models, basic statistical modelling, and inference for normally distributed populations for the second one. The paper presents an analysis based on a questionnaire distributed to elicit students' thoughts and opinions regarding the case studies implemented in the last quarter of the overall module. The first part consists of lecture/tutorial teaching. In the second part, some students are randomly assigned to tutorial classes in which case study materials and activities are implemented; the other students follow standard tutorials in which more routine data-response questions, like those in lectures, are tackled.

Feo and Gómez-Blancarte (2019) present a qualitative study about a one-semester university statistics course for nursing students. The program of the course suggests studying

three principle statistical topics: data exploration, introduction to probability and inferences about variables. The article focuses on students understanding and applying elements of the statistical Problem-Plan-Data-Analysis research cycle (PPDAC).

In her article, Marshall (2019) introduces a first-year probability and statistics module for mathematics undergraduates. The project is a cooperative group presentation involving analysis of data from a medical trial. According to the author, it enabled students to fully express statistical reasoning, thinking and communication. The discussion in the article focuses on the students' subjective description of their attitudes, effectiveness of the course and further interest in statistics.

Smith (1998) represents a pioneer of PBL in statistics among the literature we chose for this overview. He describes a university course where he includes six biweekly projects. During the semester, each student writes two project reports and gives brief oral presentations (five minutes maximum) of their project results. The team members can discuss their project with the teacher (Smith himself) or the teaching assistant; they do not help collect data but would answer questions about project design and data analysis. The article provides results of a post-course short survey about the course in general and the comparison of the midterms and finals grades with the previous years.

The work of Vignola, London, Ayala and Huang (2020, p.2) raises the research question: "How can project-based learning be used to cultivate the entrepreneurial mindset among engineering student taking an undergraduate statistics course?" The authors called their proposal "A world without statistics". They oppose the mindset that the PBL does not allow the content coverage as the traditional lectures do by stating that **"**assumed that the coverage of materials is not the goal of the course, but instead that facilitating students' attainment of the course learning outcomes is the goal" (*Ibid*.) Their PBL included four mini-projects added to an undergraduate engineering statistics course. The main goal is to make students "able to articulate and defend the role of statistics in their engineering discipline, anticipated career, and the world." (*Op. cit.,* p. 3)

Koparan and Güven (2012) present an 8th-grade mathematics course (compulsory education). Of the 70 students who participated in the study, 35 were in the control group. The remaining 35 students were in the intervention group. Project-based learning approach was

applied to the experimental group. According to the authors, an "authentic learning approach" was applied to the experimental group. In the projects, it is only said that students select their own variables, craft their own research questions, and collect and analyse their own data sets. This study aimed to evaluate the effects on students' attitude towards statistics (ATS) by conducting quasi-experimental research.

TLS (2019) presents a senior high school project "Statistics in School". The students were asked to write an individual report describing group project progress along with an individual's contribution to finish the project. The article emphasises on the observations of students' collaboration skills development.

Howley and Roberts (2020) develop the story about The National Schools' Poster Competition (NSPC) where secondary school student teams undertake a small-scale real-world investigation and ultimately present their investigation in a poster format. The project involves academics across three principal fields of focus (statistics, environmental sustainability, indigenous peoples). The research methodology consists of anonymous online surveys for students and teachers prior and after the visit, to assess the effect of the initiative. The surveys consisted of a series of statements surrounding attitudes, interests and perceptions of statistics. Students or teachers indicated their level of agreement on a seven-point Likert scale (from 1 very strongly disagree to 7 very strongly agree). Among others, authors present a rather interesting result about surveyed teachers having expressed particularly low mean scores in the item about pre-activity surrounding feelings of being supported by tertiary (higher education or post-secondary) institutions.

A significant proportion of the articles mentioned in Table 1, and many others not mentioned here due to the incompatibility with the scope that the research aims to inform, base their PBL elaboration on a quasi-experimental study. The research questions are mainly the students' subjective views on statistics, where student's beliefs are analysed with a survey administrated prior and after their PBL experience. Such methodological approach based on study and control groups almost always confirms that traditional lecturing is not the most appropriate way to teach statistics anymore, considering the current options and needs in statistics as an area of research and as a profession. However, it also hinders the PBL specificities when paying little attention to the description of the instructional activities that

define what is done in the study group. It seems clear that PBL is more effective, but it is less clear what PBL is.

RESULTS AND DISCUSSION

What is shown and valued?

Project-based learning (PBL) research area seems fruitful and nowadays progressively accepted and applied. As we have seen in the first section, many of the research questions driving the articles in the area are the generic *advantages* of such projects' results. Such emphasis on the pedagogical results, often compared with the standard lecturing or the students' starting point before the initiating the project, overshadows the component that precedes the results: the process of the project work itself.

Initially, our analysis was intended to gather characteristics of PBL that researchers consider essential to organise and implement such teaching approach in the classroom. Nevertheless, some features appeared in basically none of the quoted articles; hence, we will now distinguish the *shown* and the *omitted* parts of the PBL. Starting with what is *shown*, the most common ones are as follows:

PBL performs. Researchers present PBL as a part of the university courses, a component that varies in duration (from a couple of sessions to a whole semester), includes real data, usually implies group work and hence encourages the development of communication and collaboration skills. (DIERKER, EVIA et al., 2018; DIERKER, FLAMING et al., 2018; FEO & GÓMEZ-BLANCARTE, 2019; HOWLEY & ROBERTS, 2020; SMITH, 1998; TLS, 2019)

PBL satisfies. Researchers pay a lot of attention to the consequence of undergoing PBL processes. The results presented are formulated in terms of the students' feeling of getting the sense of statistics, a deeper understanding of course material, the coverage of the course content, a reflection on the nature of statistics, etc. (DIERKER, FLAMING et al., 2018; FAWCETT, 2017; KOPARAN & GÜVEN, 2012; MARSHALL, 2019; VIGNOLA et al., 2020)

PBL improves. Many authors state that implementing PBL in university courses results in improving the course structure and increasing students' performance compared to

exclusively lecture-based courses. (FAWCETT, 2017; KOPARAN & GÜVEN, 2012; MARSHALL, 2019; SMITH, 1998)

PBL attracts. In what refers to students' motivation and attitudes towards statistics, some investigations show that PBL drives the willingness to proceed with the statistics-oriented courses, raise awareness of the presence and importance of statistics in university studies, work and life in general. (DIERKER, EVIA et al., 2018; DIERKER, FLAMING et al., 2018; KOPARAN & GÜVEN, 2012; MARSHALL, 2019; VIGNOLA et al., 2020)

What is not highlighted and even sometimes omitted?

However, we find some aspects of teaching processes that are not developed enough or even omitted in the way researchers present PBL proposals:

PBL organisation details. Other than often specifying the in-course implementation timewise, it is surprising the lack of emphasis and details that are given about the project itself, the progress from the initial problem to the additional emerging questions, further helpful instructions, given by a teacher or a person of interest, on how to proceed with the project. It is difficult to find in the literature specific mention of the process dynamics; if it is boosted by the students participating in the teamwork or by the teacher.

PBL management. Teacher's or mentor's role seems to be not that often described, which leaves plenty of room to conceive this role from a pure observer to a strict instruction giver. It is unclear. Moreover, another not highlighted aspect of PBL is classroom management. Little is said about the pace of the project development or the problems that might arise during the process such as the ones caused by a higher instance's influence, delays with the data collection or technical difficulties that might occur.

Conditions for PBL. Implementing PBL is not a trivial enterprise. However, the papers are not very informative about the conditions under which the PBL is running. How do the teachers prepare for it? Do they get any assistance, before or during the process? How do they design the process? What kind of teaching materials do they prepare? How is the schedule within the degree programme positioned? How much flexibility do curricula allow for implementing certain projects, and to what extent? Lee et al. (2014) affirm that the faculty in university settings develop the curriculum tailored to the students' needs or the programs.

Unlike for university faculty, this is a condition that recurrently presents a constraint for primary and secondary school teachers. Unfortunately, the conditions of PBL practical implementations are poorly shared and practically invisible.

Constraints for PBL. In the same way, it is normal for many PBL proposals to face several hindrances, at least during their first editions. We all know the pressure that a curriculum based on lists of contents puts on the teaching organisation and assessment. Some universities can also be very rigid in the instructional formats they allow or consent, not to say support. What are the technological, professional and societal possibilities when it comes to implement teaching approaches in line with the paradigm of questioning the world? When it comes to the constraints for PBL experimentations, we found no such information provided.

What is questioned, what is assumed?

The literature on PBL's philosophy background elaborates the motivation for PBL implementation, whether partially in traditionally established curriculum or in entirely PBL based university settings. Meanwhile, the literature regarding PBL experimentation and performance considers the observable changes in students' achievements and attitudes in a relatively short time interval. Many papers *question* the difference between PBL and traditional lecturing, together with its effects on the students' performance and attitudes towards statistics. How does PBL affect the courses' set up? How to adapt the project topic to the students' professional preferences? How does a teaching approach influence students' feelings and self-estimation? How does PBL influence students' formal examination? Those research questions, however, do not question the process of the PBL itself but the "before and after" state. We strongly agree with Ravitz (2009) when he suggests further investigation on how the PBL is defined and effectively practised across disciplines for establishing optimal PBL facilitation model.

In the papers about PBL implementations, there are also hidden, not discussed, assumptions regarding the conditions PBL demands. As *assumed*, we find the didactical skills of the PBL manager, who is usually a university teacher. This assumption agrees with the note of several authors that even though the PBL philosophies are well established, we are short on "how to" indications (GALLAGHER, SHER, STEPIEN & WORKMAN, 1995; LEE et al., 2014; WARD & LEE, 2002). Another assumption is about the curriculum content "covered"

by the project and the division of learning goals into disciplines and specialities, which often collides with the transdisciplinary or integrated character of the projects. Few of the articles from Table 1 mention the course content, and most of them assume it as a frame where the project fits. In any case, we have found the other way around that *the content emerges from the project*. PBL always appears as a means to reach a final goal which is formulated in terms of a given discipline and a given body of contents within this discipline. In a way, students *do not learn to do a project*, but they *learn statistics better by doing a project*. Shpeizer makes it clear:

> The goal of this method is to promote the learning of knowledge, skills, and dispositions by enhancing students' engagement in the learning processes and by emphasising the practical dimension of learning and its relevance to students' lives and to the society in which they live (SHPEIZER, 2019, p. 1766)

What remains unquestioned is the selection of the "knowledge, skills, and dispositions" that determine the teaching and learning process served by the PBL method.

CONCLUSIONS AND NEW QUESTIONS TO GO FURTHER

The academic literature about PBL in statistics shows a prevalence of the *insider position* in the way researchers present the instructional proposals and analyse them. This insider position locates researchers close to the teachers' concerns, focusing more on the students' learning outputs and motivations, than on the overall conditions needed for PBL to exist as a sustainable activity in the schools and universities. Researchers adopting more *outsider positions* would consider broader conditions than those under the teacher's reach. For example, talking about PBL in general, Lee et al. (2014, p. 29) state that:

> [F]or PBL to become more integrated into university curricula, institutional support must be put into place. First, induction programs for new faculty should include substantial training on how students learn and on assessing student learning […] Second, faculty members require ongoing professional development and mentoring in order to develop as reflective PBL practitioners […] Third, faculty resist pedagogical innovation because of the time it takes to retool courses; a perceived lack of collegial and supervisor support […] Faculty in this study indicated the need for systematic change that would support their use of PBL, including administrative support for implementation and a reconsideration of how teaching is evaluated. (LEE et al, 2014, p. 29).

In studies about PBL and the teaching of statistics, this institutional perspective seems to be still underdeveloped. In this case, and in coherence with the researchers' insider position, one expects to find information about the PBL process itself, the way it is implemented, the difficulties and obstacles found, together with the innovations introduced to overcome them. Paradoxically, and maybe because the attention is mainly put on the effects of PBL processes on the students, there is not much detail about the conditions established for PBL processes to exist. This lack of details can easily become an obstacle for its dissemination as a sustainable educational activity.

In the research approach proposed by the ATD, critical attention is put on the study of the *ecology* of teaching and learning processes, that is, of the conditions of any nature affecting their existence and evolution. The Herbartian schema and its dialectics (BENITO & BOSCH, this volume) appear to be productive tools for the design, implementation and management of *study and research paths*, an instructional format based on the inquiry of questions that arise within the development of a project. Its origins as research devices caused the development of a specific methodology and analysis tools that have been transposed to the practice in school settings. Because of that, SRPs have the virtue of supplying researchers and teachers with these descriptive elements to both provide details about the instructional proposals and help to design, manage, analyse and develop them. Adopting an insider's or an outsider's position concerning teaching and learning processes is not always a deliberate choice. It is strongly conditioned by the theoretical perspective we adopt as researchers, and by the methodological tools we use to develop it.

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