AN INVESTIGATION INTO SIXTH GRADE STUDENTS' UNDERSTANDING OF RATIO AND PROPORTION

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ABSTRACT

Drawing on written assessments collected from 58 sixth grade students, this article discusses the results of a study that examined patterns in middle-grade boy's and girl's written problem solving strategies for a mathematical task involving proportional reasoning and their level of understanding of ratios and proportions. This work is a part of a larger, longitudinal project, Mathematics Identity Development and Learning (MIDDLE), that focused on the impact of mathematics reform on students' development as mathematics knowers and learners and identifying processes the explains changes in students' mathematical learning and self-conceptions. Findings the current work speaks to student strategy use, errors, and levels of understanding.

Keywords: Proportional Reasoning, Problem Solving Strategies, African American Students, Conceptual Understanding, Middle School.

RESUMO

Com base em avaliações escritas coletadas de 58 alunos da sexta série, este artigo discute os resultados de um estudo que examinou os padrões de meninos e meninas na escrita de estratégias de resolução de problemas para uma tarefa matemática envolvendo o raciocínio proporcional, e seu nível de compreensão de razões e proporções. Este trabalho é parte de um projeto maior, projeto longitudinal, Aprendizagem e Desenvolvimento de Identidade Matemática (MIDDLE), que incidiu sobre o impacto da reforma da matemática no desenvolvimento de estudantes como conhecedores e aprendizes de matemática e identificando processos de mudanças na aprendizagem matemática e auto-concepções dos estudantes. Os resultados do trabalho apontam para o uso de estratégias por parte do estudante, os erros e os níveis de compreensão.

Palavras-chave: Raciocínio Proporcional, Estratégias de Resolução de Problemas, Estudantes Afro-americanos, Entendimento Conceitual, Ensino Médio.

1. Introduction

This study examines the patterns in African-American sixth grade students' problem solving strategies and errors for a task involving proportionality concepts. The students in this study were a part of the Mathematical Identity Development and Learning (MIDDLE) project, a larger study examining the impact of mathematics reform on students' development as mathematics knowers and learners and identifying processes that explain changes in students' mathematical learning and self-conceptions. All the students in this study participated in MIDDLE for three years. For this paper, I am looking specifically at the written solutions of African-American students to categorize their strategy use, errors, and level of understanding. The focus on strategies and errors frames this paper as a detailed investigation and analysis of the patterns of strategies and errors that emerged and how these patterns connect to students' conceptual understanding of proportionality concepts.

2. Related Scholarship

2.1. Problem Solving and Problem Solving Strategies

Researchers and educators agree that problem solving is the substance of the mathematics discipline and is an important tool in students' development as mathematics learners (Polya, 1981; NCTM, 2000). Problem solving is necessary for the development of mathematical understanding because when students are allowed to grapple with situations that involve important mathematical concepts, they construct a clearer understanding of the mathematics (Kroll & Miller, 1993).

Problem solving strategies serve as a guide in the problem solving process; although they do not guarantee a solution, they may provide a pathway to solutions (Gick, 1986). In essence, problem solving strategies are "cognitive or behavioral activities" (Siegler, 1998, p. 191) employed by students to reach a problem solution. Student reasoning in mathematics has been analyzed based on the strategies they use when problem solving (Steinthorsdottir, 2003). While some strategies are labeled as more sophisticated than others, the strategies students use and the successful application of these strategies provide insight into students' mathematical thinking and level of understanding (Cai, 2000).

2.2. Gender Differences in Mathematics and Strategy Use

An analysis of TIMSS1 data documented a statistically significance difference favoring males in the area of problem solving strategies (Che, Wiegert, & Threlkeld, 2011). Prior research supports findings from the TIMSS data. Researchers have documented small but consistent gender differences in strategy use when solving mathematical tasks involving computations, rational numbers, measurement, and spatial visualization. Reported differences in these areas usually indicate that males are more adept and females have a less sophisticated and more rote approach to solving mathematical problems. (McGraw & Lubienski, 2007). However, these studies were based on findings from predominantly White samples. In the gender literature, when performance differences were broken down by ethnicity, gender differences among African Americans were smaller and sometimes nonexistent (Leder, 1992; McGraw & Lubienski, 2007).

2.3. Proportionality Concepts

Proportionality concepts are important within the middle school mathematics curriculum in that they underpin upper-level mathematics concepts. Students in the middle grades must have a strong understanding of proportionality for success in the middle grades and subsequent mathematics courses (Lobato, J., Ellis, A., Charles, R., & Zbiek, R., 2010). The literature suggests that though an understanding of proportionality concepts is critical to students' mathematical development, middle school students often have difficulty learning, understanding, and applying these concepts (Behr, Harel, Post, & Lesh, 1992; Lamon, 1993; NRC, 2001; Lobato, J. et al., 2010). Students who lack an understanding of these concepts will have limited success in algebra, the gatekeeper to more advanced mathematics courses (Behr, Harel, Post, & Lesh, 1992; NRC, 2001). In a study examining the problem solving strategies of 119 sixth grade students, of which 61 were female and 58 male. Fifty-four percent of the female students and 27.5 % of male students did not show any evidence of understanding proportional relationships beyond rote procedures but an equal percentage of males and females demonstrated an understanding of proportions beyond the application of a procedure (Che, et. al, 2011).

When solving problems involving proportions, common strategies include unit-rate, factor-of-change, cross-multiplication (Cramer & Post, 1993; Cramer, Post, & Currier, 1993; Lamon, 2005), and build-up (Tourniaire & Pulos, 1985). The unit-rate strategy involves students recognizing the multiplicative relationship *between* ratios. Factor-of-change follows the same thinking pattern except students recognize the multiplicative relationship *within* ratios. Cross-multiplication is an effective and mechanical model but the use of the cross-multiplication method without understanding could hinder the development of proportional reasoning (Cramer & Post, 1993; Cramer, Post, & Currier, 1993; Lamon, 2005). The additive build-up strategy is dominant during childhood and adolescence, is common when students are unable to recognize the multiplicative relationship between rational expressions or measure spaces (Lamon, 1993, 2005; Tourniare & Pulos, 1985). Students sometimes use tally marks and other visual representations to help support their reasoning (Lamon, 1993). Common errors with proportional reasoning involve ignoring part of the data in the problem, using additive strategies when multiplicative strategies are more important, or faulty application of a correct strategy (Lamon, 2005; Tourniaire & Pulos, 1985).

2.4. Procedural and Conceptual Understanding

Kroll and Miller (1993) suggest that to solve problems efficiently students must possess appropriate knowledge and be able to coordinate their use of appropriate skills. Research has shown that students struggle with deciding how to approach a problem solving task when they lack conceptual understanding of the content in the task (Malloy and Jones, 1998). Conceptual understanding is flexible, generative, and is an important factor in students' strategy development. Conceptual understanding is also necessary for the development of mathematical thinking and successful problem solving (Carr & Hettinger, 2002; Carpenter & Lehrer, 1999). The flexibility in students' knowledge results in flexibility in strategy use and the successful implementation of these strategies (Malloy & Jones, 1998; Carr & Hettinger, 2002).

3. Framework for looking at conceptual understanding

In this study, students demonstrate their level of conceptual understanding during problem solving moments by their ability to apply concepts to new situations, to connect new concepts with existing information, and to use mathematical principles to explain and justify problem solutions. Students' level of conceptual understanding serves as a lens to examine the development of problem solving strategies and the students' decision to employ certain strategies to reach a problem solution. These dimensions are directly related to the five forms of mental activity of Carpenter and Lehrer (1999) that are important in attaining conceptual understanding of mathematics. They characterize understanding not as a static attribute but as emerging in learners as they engage in the following activities: (a) constructing relationships, (b) extending and applying mathematical knowledge, (c) reflecting about experiences, (d) articulating what one knows, and (e) making mathematical knowledge one's own and thus available for use in future situations (p. 20). For the purposes of this study, students' conceptual understanding was measured in relation to proportionality and problem solving. These domains for conceptual understanding items were selected based on two major mathematical domains stressed in middle school content strands of the Principles and Standards of School Mathematics (NCTM, 2000), the State Standard Course of Study for Mathematic, and the district's pacing guides for middle grades mathematics. For this paper I will investigate:

1. What strategies African American students employ when solving a mathematical task involving proportionality concepts?
2. What do these strategies reveal about African American students understanding of proportionality concepts?
3. Do differences exist between African American male and female's strategy use during mathematical problem solving and students' understanding of proportionality concepts?

4. Methods

4.1. Participants

Participants were 58 (43 females and 15 males) sixth grade students from an economically and ethnically diverse school district in Southeastern United States with a population of approximately 30,500 students. The district and participating schools served a student population that was approximately 54% African American, 24.3% White, 15.7% Latino, 3.4% multiracial, 2.4% Asian, and 0.2% Native American (Malloy, 2004). Students in this study were participants in the Mathematics Identity and Development (MIDDLE) Project, an NSF[1] funded three-year longitudinal, cross-sectional design study whose purpose was to (1) better understand how mathematics reform[2] affects students' development as mathematics knowers and learners; (2) provide a longitudinal analysis of students' mathematical development during the middle school years; and (3) identify the processes that explain changes in students' mathematical learning and

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[1] Mathematical Identity and Development (MIDDLE) Project, Dr. Carol Malloy, Dr. Jill Hamm, & Dr. Judith Meece, NSF Grant REC 0125868

[2] School reform is defined as a teacher's use of instructional practices and curricular materials that are aligned with the NCTM Curriculum and Teaching Standards (1989, 1991) and the Principles and Standards for School Mathematics (2000).

self-conceptions. Fifth-grade end of grade (EOG) scores showed that 93% of males and 100% of females in the current study entered middle school at or above grade level.

4.2. Data Collection

The current study analyzed student written responses on a mathematics task given as a part of the MIDDLE Project. MIDDLE's mathematics tasks were selected from released Trends in International Mathematics and Science Study (TIMSS) (1994) and National Assessment of Educational Progress (NAEP) (1990, 1992) tasks and then modified into tasks that have multiple solution paths leading to a single correct solution (Malloy, 2004). The task assessed students' understanding of a fraction always representing part-to-whole relationships and a ratio[3] representing part-to-part or part-to-whole relationships. It also assessed students' understanding and application of proportional reasoning in scaling. Additionally, the task could not be solved using the traditional algorithm with the numbers in the problem.

Sixth Grade Conceptual Understanding
A class has 28 students. The ratio of girls to boys is 4 to 3.
How many girls are in the class?
Explain why you think your answer is correct

Figure 1. Sixth Grade Mathematics Task

The sixth grade task was assessed using an item-specific conceptual understanding rubric (see figure 2). The scores in the conceptual understanding rubric ranged from 0 to 4, with 4 being the highest. The rubric scores were the following: *0* demonstrates *no attempt*; *1* demonstrates-*no conceptual understanding*; *2* demonstrates *no to limited conceptual understanding*, *3* demonstrates *procedural understanding, but conceptual understanding is not demonstrated or incomplete,* and *4* demonstrates *conceptual understanding.* The qualitative data collected from the conceptual understanding items were quantified into a numerical score of 0-4, based upon the item-specific rubric. This numerical score represented the student's level of conceptual understanding of the concepts assessed by the task (Author, 2008).

Level	Identifiers	Examples of student responses	Understanding
0	No work or states they do not understand with no answer given.	"I don't understand."	No attempt
1	No evidence of understanding concepts related to fractions or proportionality.	"21 girls. I think it is right because I used my × ÷ skills." "There are 12 girls because 12 × 3 = 28 and there are 28 students."	No understanding
2	Written or symbolic explanation shows an understanding the meaning of a ratio, but does not apply the ratio to solve the problem.	Ans. 16. "1. set up proportion, 2. cross multiply, 3. reduce fractions. $\frac{4}{3} = \frac{\_\_\_}{28}$ " Ans. 16 girls.	Understands different representations of ratio.

[3] I am defining ratio as the relationship between two different quantities.

	Correct written or drawing work but provides no explanation of how the answer was found.	"16 girls 12 boys"	
3	Explanation is accurate does not thoroughly explain the rationale used in solving the problem. The explanation is procedural rather than conceptual.	"12 boys and 16 girls equal 28 students." "There are 16 girls. If you multiply 4×4 you get 16. Then you multiply 4 ×3 to get 12. Then you add 16 & 12 to get 28."	Understands the meaning of the ratio and proportionality
4	Evidence of full understanding of proportionality either verbally or visually (scaling 4:3 or using and explaining the proportion 4/**7** = 16/**28**).	"For every 4 girls there is 3 boys, 4 3 4 3 4 3 4 3 There are 16 girls." "If you add the numbers together and multiply by 4 you get twenty-eight. So you just multiply the individual numbers and get 16:12."	Understands and the meaning of the ratio, proportionality, and how to apply and explain their application.

Figure 2. Scoring Rubric for Sixth Grade Mathematics Task

5. Analysis

The analysis of student strategy use was based on students' written responses because "[w]riting has been viewed as 'thinking-aloud' on paper" (Pugalee, 2004, p.29). Verbal protocols are powerful to gain information about students' approach to solving problems, but research has shown the feasibility and validity of using written responses from open-ended tasks to assess students' problem solving approaches (Cai, 1997). Students' written responses also provide robust accounts to their mathematical reasoning, therefore reading students' explanations provided insight into students' understandings and misconceptions (Moskal, B. & Magone, M., 2000).

5.1. Qualitative

Participants' responses on the mathematics task were analyzed in three ways. First, responses were separated by gender and sorted according to their final response as correct, incorrect, or no solution/attempt (including blank tasks and those stating some variation of "don't know" or "don't understand"). Secondly, students' written solutions were analyzed to determine the solution plan (problem-solving strategies) they devised and carried out to solve the task. Using an open coding strategy, each written response was read and reread in order to code and classify the different strategies students generated to solve the tasks. This strategy involved comparing problem-solving strategies and searching for similarities and differences between them (Liamputtong & Ezzy, 2005). Thirdly, student's level of understanding, as indicated by solution strategies and written explanations, were coded according to the item-specific conceptual understanding rubric using the following categories: "no attempt", "no understanding", "understands difference representation of ratio", "understands the meaning of the ratio and proportionality", "understand the meaning of the ratio, proportionality, and how to apply and explain their application". See Figure 2 for a detailed description of each category.

5.2. Quantitative

Descriptive statistics (mean, maximum, minimum, and standard deviation) and frequency counts were conducted to describe the data. Since the students in this sample were not randomly assigned and prior knowledge can affect students' demonstration of conceptual understanding (Carpenter & Lehrer, 1999), univariate correlations were ran to determine if students' conceptual understanding scores were correlated with the students' fifth grade End-Of-Grade (EOG) scores and an Analysis of Variance (ANOVA) was used to determine if gender difference exist in students' EOG scores. In this study, the students' fifth grade End-Of-Grade (EOG) raw scores were used as an indicator of students' knowledge, because they provide a baseline of student's prior knowledge before entering middle school mathematics. A one-way analysis of covariance (ANCOVA) was conducted to compare the mean conceptual understanding scores of male and female students to determine if a statistically significant difference existed among the scores.

6. Results

6.1. Qualitative

Students in this study experienced difficulty when asked if "**A class has 28 students. The ratio of girls to boys is 4 to 3. How many girls are in the class?"** Nine students (4-males and 5-females) reached a numerically correct solution, 28 (9-males and 19-females) generated an incorrect numerical solution and 21 (1-male and 20-females) left the task blank or stated "I don't know". The number of males and females written responses in each strategy and conceptual understanding category along with the percentage the category represents of the total responses are presented in Table 1 and 2.

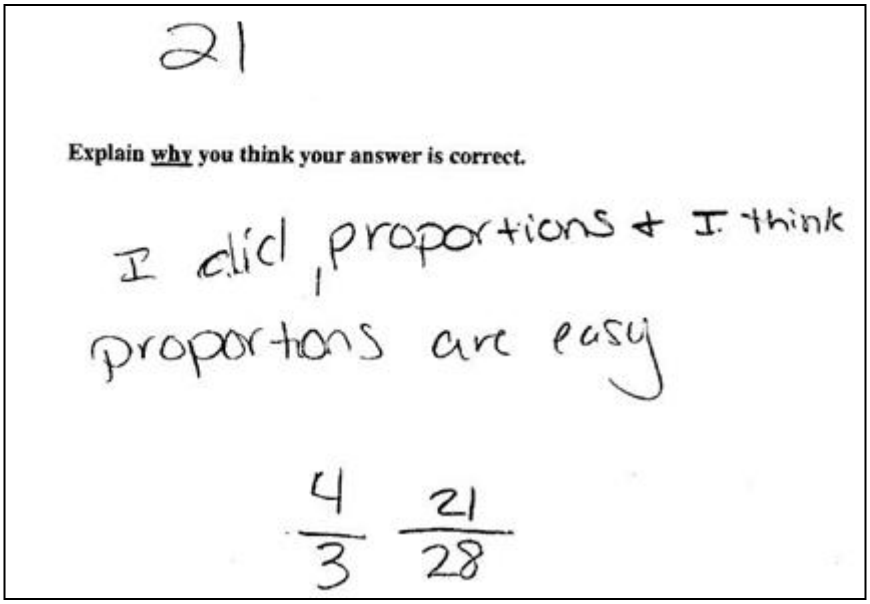
Table 1. Strategy classification of sixth-grade males and females (Parenthesis indicated percentages of respective gender and totals rounded to the nearest tenth)

Response classification	Females (n=43)	Males (n=15)	Total (n=58)
Non-response of state "I don't know"	20 (46.5)	1 (6.7)	21 (36.2)
Guess	8 (18.6)	3 (20)	11 (19)
Operations with numbers in the task	10 (23.3)	8 (53.3)	18 (31)
Build-up additive	2 (4.7)	0 (0)	2 (3.5)
Multiplicative Strategy	3 (6.9)	3 (20)	6 (10.3)

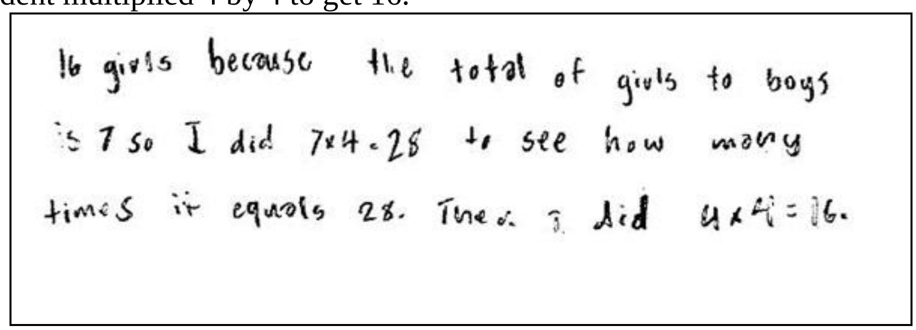
Table 2. Conceptual Understanding Score Distribution (Parenthesis indicated percentages of respective gender and totals rounded to the nearest tenth)

Response classification	Females (n=43)	Males (n=15)	Total (n=58)
No attempt	20 (46.5)	1 (6.7)	21 (36.2)
No conceptual understanding	17 (39.5)	10 (66.6)	27 (46.6)
Limited understanding or significant errors	1 (2.3)	1 (6.7)	2 (3.4)
Some understanding, but incomplete	4 (9.4)	2 (13.3)	6 (10.4)
Complete understanding	1 (2.3)	1 (6.7)	2 (3.4)

The following response represents the most common error of generating and solving a missing value proportion problem with the three numbers in the problem; that is, students set up a proportion comparing the number of girls to boys with an "unknown" to total number of students. A correct proportion would compare four girls to seven students with "unknown" to total number of students.



For students who attempted the task (provided a response) and based on the item rubric demonstrated some (score of 3) and complete (score of 4) understanding, finding the multiplicative relationship was the most common strategy. The students found the total number of "parts" (girls to boys) to be 7 and determined that there were 4 "parts" in 28. Since 7 * 4 "parts" results in 28 (the total number of students in the class), to find the number of girls in the class, the student multiplied 4 by 4 to get 16.

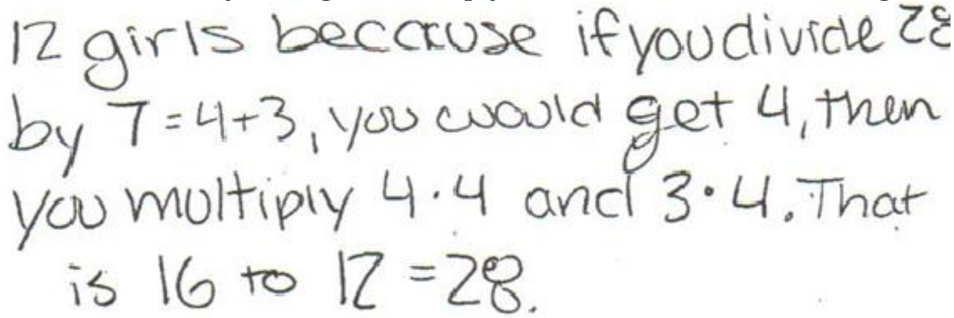


6.2. Quantitative

Correlational analyses revealed that students' fifth grade EOG scores were significantly correlated with students' conceptual understanding scores. The ANOVA showed no statistically significant gender differences in students' fifth grade EOG scores. The ANCOVA results revealed no significant gender differences in the mean conceptual understanding scores of African American male and female students and the chi-square test found no statically significant differences between the strategies employed by African American male and female students when solving the proportionality task.

7. Discussion

In this study the most common strategies used by students to find the number of girls in the classroom were additive building up and finding the multiplicative relationship. To solve the task, participants in this study utilized strategies documented in the literature. Students often reason additively or multiplicatively when solving problems that involve proportions (Behr, Post, & Lesh, 1992). Students' level of conceptual understanding was measured by the conceptual understanding item rubrics that were (but were not solely) based upon students reaching a correct numerical solution. The response below represents the work of a student who understood the concepts assessed in this task but gave the answer of 12 instead of 16. The student may have read the ratio as 4 boys to 3 girls or simply made a minor error in writing 12 instead of 16.


12 girls because if you divide 28 by 7 = 4+3, you would get 4, then you multiply 4·4 and 3·4. That is 16 to 12 = 28.

The data in this study indicated similarities in African male and female students' problem solving strategies and level of understanding. Although the literature reports that differences exist between the strategies employed by male and female students, these studies were often based on predominately white samples (Gallagher and DeLisi, 1994; Gallagher et al, 2000; Carr and Jessup, 1997; Fennema, Carpenter, Jacobs, Franke, & Levi, 1998). Studies in the gender literature tend to suggest that females use less sophisticated strategies than males and have more procedural approaches to mathematics than males (Fennema and Peterson, 1985). Using both qualitative and quantitative analyses, this study revealed no gender difference in students' strategy use and conceptual understanding scores.

In addition to male and female students utilizing similar strategies and demonstrating similar levels of understanding it is important to note that approximately 36.2% of the students in this study wrote no response or I don't know resulting in a conceptual understanding level of 0. About 50% of students guessed or performed various numerical operations with the numbers presented in the task resulting in 46.6% of students demonstrating an understanding level of 1 (no conceptual understanding) and 3.4% demonstrating an understanding level of 2 (no to limited conceptual understanding). Limited conceptual understanding means that students did not understand different representations of ratio. The remaining students guessed the correct response but provided no explanation (3.5%) or used the additive build up or multiplicative strategy (10.3%) to solve the task. This resulted in 10.4% of students demonstrating procedural understanding but conceptual understanding is not demonstrated or complete (level 3: understood the meaning of ratio and proportionality), and 3.4% demonstrated conceptual understanding (level 4: understood the meaning of the ratio, proportionality, and how to explain their application). The vast majority of students in this study utilized strategies that did not demonstrate procedural or conceptual understanding of a fraction always representing part-to-whole relationships, a ratio representing part-to-part or part-to-whole relationships and proportional reasoning in scaling and were unable to effectively solve the task. Research suggests that students must possess appropriate knowledge and coordinate the use of appropriate skills to decide how to approach and solve problems effectively and efficiently (Kroll & Miller, 1993; Malloy & Jones, 1998).

8. Conclusions and Implications

This study contributes to our understanding of how African American male and female students solve a task assessing students' understanding of a fraction always representing part-to-whole relationships; a ratio representing part-to-part or part-to-whole relationships, and application of proportional reasoning in scaling. From the results, male and female students employed similar strategies when solving this task and demonstrated similar levels of understanding. It is apparent from the results that students' need additional support to develop their understanding of fractions, ratios, and proportional reasoning; important concepts that underpin upper-level mathematics such as Algebra (Lobato, J., Ellis, A., Charles, R., & Zbiek, R., 2010). In this study, one of the greatest hindrances to students successfully solving problems was their lack of knowledge of the concepts assessed.

The National Council of Teachers of Mathematics (2000) proclaims that, "students must learn mathematics with understanding, actively building new knowledge from experiences and prior knowledge" (p. 20). In the middle grades, Sowder and Philipp (1999) suggest that to promote understanding, teachers must understand the interrelated nature of mathematical concepts and how students develop understanding of these concepts. Along with understanding these concepts and how students develop these concepts, teachers must provide challenging, appropriate tasks and allow students the opportunity to wrestle with those tasks. Learning mathematical concepts for understanding will greatly enhance students' development as mathematics problem solvers and increase their overall mathematics understanding and performance.

9. References

Author. (2008). Making the Invisible Visible: An Examination of African American Students' Strategy Use During Mathematical Problem Solving. Unpublished doctoral dissertation, University of North Carolina at Chapel Hill.

Behr, M., Harel, G., Post, T., & Lesh, R. (1992). Rational Number, Ration, and Proportion. In D. Grouws (e.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 296-333). Macmillan, New York.

Cai, J. (2000). Mathematical Thinking Involved in U.S. and Chinese Students' Solving of Process-Constrained and Process-Open Problems. *Mathematical Thinking and Learning*, 2(4), 309-340.

Cai, J. (1997). Beyond Computation and Correctness: Contributions of Open-Ended Tasks in Examining U.S. and Chinese Students' Mathematical Performance. *Educational Measurement: Issues and Practices*, *Spring*, 5-11.

Carpenter, T., & Lehrer, R. (1999). Teaching and learning mathematics with understanding. In E. Fennema & T. A. Romberg (Eds.), *Classrooms that Promote Mathematical Understanding (pp. 19-32).* Mahwah, NJ: Erlbaum.

Carr, M., & Hettinger, H. (2002). Perspectives on Mathematics Strategy Development. In J. Royer (Ed.), *Mathematical Cognition* (pp. 33-68). Greenwich, CT: Information Age Publishing.

Carr, M., & Jessup, D. L. (1997). Gender differences in first-grade mathematics strategy use: Social and metacognitive influences. *Journal of Educational Psychology, 89*(2), 318-328.

Che, M., Wiegert, E., & Threlkeld, K. (2011). Problem solving strategies of girls and boys in single-sex mathematics classrooms. *Educational Studies in Mathematics*, doi:10.1007/s10649-011-9346-x.

Cramer, K., & Post, T. (1993). Connecting research to teaching proportional reasoning. *Mathematics Teacher*, *86*(5), 404–407.

Cramer, K., Post, T., & Currier, S. (1993). Learning and teaching ratio and proportion: Research implications. In D. Owens (Ed.), *Research ideas for the classroom* (pp. 159–178). New York: Macmillian.

English, L. (1993). Children's Strategies for Solving Two- and Three-Dimensional Combinatorial Problems. *Journal for Research in Mathematics Education*, *24*(3), 255- 273.

Fennema, E., Carpenter, T., Jacobs, V., Franke, M., & Levi, L. (1998). A Longitudinal Study of Gender Differences in Young Children's Mathematical Thinking. *Educational Researcher*, 27, 6-11.

Fennema, E., & Peterson, P. (1985). Autonomous Learning Behavior: A Possible Explanation of Gender-Related Differences in Mathematics. In L.Wilkinson & C. Marrett (Eds.), *Gender Influences in Classroom Interaction* (pp.17-35). Orlando, FL: Academic Press.

Gallagher, A., De Lisi, R., Holst, P., McGillicuddy-De-Lisi, A., Morely, M. & Chalan, C. (2000). Gender Differences in Advanced Mathematical Problem Solving. *Journal of Experimental Child Psychology*, 75, 165-190.

Gallagher, A. M., & De Lisi, R. (1994). Gender differences in Scholastic Aptitude Test – Mathematics Problem Solving Among High-Ability Students. *Journal of Educational Psychology, 86*(2), 204-211.

Gick, M. (1986). Problem-Solving Strategies. *Educational Psychologists, 21*(1 &2), 99-120.

Hembree, R. (1992). Experiments and Relational Studies in Problem Solving: A Meta-Analysis. *Journal for Research in Mathematics Education*, 23(3), 242-273.

Hibert, J., & Lefevre, P. (1986). Conceptual and Procedural Knowledge in Mathematics: An Introductory Analysis. In James Hibert (Ed.), *Conceptual and Procedural Knowledge: The Case of Mathematics* (pp. 1-27). Hillsdale, NJ: L. Erlbaum Associates, 1986.

Kroll, D.L., & Miller, T. (1993). Insights from Research on Mathematical Problem solving in middle grades. In D. Owens (Ed.*), Research ideas for the classroom: Middle grades Mathematics* (pp. 58-77).. New York: MacMillan Publishing Company.

Lobato, J. Ellis, A, Charles, R., & Zbiek, R. (2010). *Developing Essential Understanding of Ratios, Proportions, and Proportional Reasoning*. Reston, VA: NCTM.

Lamon, S. (2005). Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers. Mahwah, NJ: Lawrence Erlbaum Associates.

Lamon, S. (1993). Ratio and Proportions: Children's Cognitive and Metacognitive Processes. In T. Carpenter, E. Fennema, & T. Romberg (Eds.), *Rational Numbers: An Integration of Research* (131-156).

Leder, G. C. (1992). Mathematics and Gender: Changing Perspectives. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 597-622).. New York, NY: Macmillan.

Liamputtong, P., Ezzy, Douglas (2005). *Qualitative Research Methods* (2nd ed.). Oxford; New York, N.Y.: Oxford University Press.

Malloy, C. E. (April 2004). National Council of Teachers of Mathematics Research Pre-Session, Symposium Leader: Students' Perception of and Engagement with Mathematics Reform Practices, Philadelphia, Pa.

Malloy, C. & Jones, G.M. (1998). An Investigation of African American students' mathematical problem solving. *Journal of Research in Mathematics Education*, 29, 143-163.

Meltzoff, J. (1998). *Critical Thinking About Research: Psychology and Related Fields.* Washington, DC. American Psychological Association.

McGraw, R., & Lubienski, S. T. (2007). 2003 NAEP mathematics findings regarding gender. In P. Kloosterman & F. Lester (Eds.), *Results from the ninth mathematics assessment of NAEP* (pp. 261–287). Reston: NCTM.

Moskal, B. M. & Magone, M. E. (2000). Making Sense of What Students Know: Examining the Referents, Relationships, and Modes Students Displayed in Response to a Decimal Task. *Educational Studies in Mathematics, 43*, 313-335.

National Council of Teachers of Mathematics (1989). *Curriculum and Evaluation Standards for School Mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics (2000). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: Author.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Polya, G. (1981). *Mathematical Discovery: On Understanding , Learning, and Teaching Problem Solving* (Combined Edition). New York, NY: John Wiley & Sons.

Pugalee, D. (2004). A Comparison of Verbal and Written Descriptions of Students' Problem Solving Processes. *Educational Studies in Mathematics, 55*, 27-47.

Schoenfeld, A. (1985). *Mathematical Problem Solving*. Orlando, Florida: Academic Press.

Siegler, R. (1998). *Children's Thinking* (3rd. Ed.). Upper Saddle River, New Jersey: Prentice-Hall.

Silver, E., Shapiro, L., & Deutsch, A. (1993). Sense Making and the Solution of Division Problems Involving Remainders: An Examination of Middle Students' Solution Processes and Their Interpretations of Solutions. *Journal for Research in Mathematics Education, 24*(2), 117-135.

Sowder, J., & Philipp, R. (1999). Promoting Learning in Middle –Grades Mathematics. In E. Fennema & T. A. Romberg (Eds.), *Classrooms that Promote Mathematical Understanding (pp. 89-108).* Mahwah, NJ: Erlbaum.

Steinthorsdottir, O. (2003). Making Meaning of Proportion: A Study of Girls in Two Icelandic Classrooms. Unpublished doctoral dissertation, University of Wisconsin-Madison.

Tourniaire, F., & Pulos, S. (1985). Proportional Reasoning: A Review of the Literature. *Educational Studies in Mathematics, 16,* 181-204.