MATHEMATICS AND LANGUAGE: PERSPECTIVES OF WITTGENSTEIN'S PHILOSOPHY FOR MATH EDUCATION

Marisa Rosâni Abreu da Silveira[1]
marisabreu@ufpa.br

Federal University of Pará

ABSTRACT

This text aims to analyze the importance of the emphasis on the use of language in the teaching activities and the learning of mathematics. The Mathematics Education with emphasis on the communication between teacher and student can elucidate the meanings of a mathematical statement. On educational actions, such communication allows the clarification of the mathematical vocabulary, as well the necessity to search in natural language the support for the translation of the mathematical language. The interpretation of mathematical rules that are linked to the contexts where they are inserted, can find many senses, because in the application of mathematics in empirical situations, we do not find the logical necessity that is part of the self-movement of mathematics. In order to choose strategies that enhance the teaching this subject it is important that teachers know the characteristics of mathematics, such as its intra theoretical movement and the consequences of its applications in empyrean. These issues discussed here, are based on the philosophy of Wittgenstein and some of his commentators, as well as in research from mathematics educators that corroborate with this line of research.

Keywords: Language. Mathematics, Teaching and Learning. Math's Self-Movement. Wittgenstein's Philosophy.

RESUMO

Este texto tem o objetivo de analisar a importância da ênfase no uso da linguagem nas atividades de ensino e a aprendizagem da matemática. A Educação Matemática, com ênfase na comunicação entre professor e aluno, pode elucidar os sentidos de um enunciado matemático. Na ação educativa, tal comunicação permite o esclarecimento do vocabulário matemático, bem como a necessidade de buscarmos, na linguagem natural, o amparo para a tradução da linguagem matemática. A interpretação de regras matemáticas que estão atreladas aos contextos em que estão inseridas pode encontrar sentidos diversos, pois na aplicação da matemática em situações empíricas não

[1] Graduated in Mathematics (1985), has Specialization in Mathematics (1988), Specialization in Philosophy of Knowledge and Language (1995) from the University of Vale Rio dos Sinos. Is Master in Education (2000) and PhD in Education (2005) from the Federal University Rio Grande do Sul. Doctoral Internship at the University of Paris and Post-Doctoral Internship at the Institut d'Histoire et de Philosophie des Sciences et des Techniques of Université Paris (Sorbonne). He is currently an adjunct Professor at the Federal University of Pará in the Institute of Mathematics and Science Education, acting on the following themes: teaching and learning mathematics, pedagogical discourse, the construction of mathematical concepts, mathematics and languages, language translation of mathematical texts.

encontramos a necessidade lógica que faz parte do automovimento da matemática. Para eleger estratégias que favoreçam o ensino desta disciplina é importante que o professor conheça as características da matemática, tal como seu movimento intrateórico e as consequências de suas aplicações na empiria. Essas questões aqui discutidas são pautadas na filosofia da linguagem de Wittgenstein e de alguns de seus comentadores, assim como em pesquisas de educadores matemáticos que corroboram com essa linha de investigação.

Palavras-chave: Linguagem. Matemática. Ensino e Aprendizagem. Automovimento da Matemática. Filosofia de Wittgenstein.

1. Introduction

This article aims to discuss some relationships between language, mathematics and knowledge, taking as its starting point the fact that we consider that language exerts a strong influence on the process of teaching and learning math. For this, we first discuss what we think the teacher needs to understand when he teaches mathematics, such as the characteristics of mathematics and its language. The success or failure of teachers in their teaching activity depends somewhat on such knowledge. Baruk (1996) nicely illustrates the failure of the teacher, when he is betrayed by the educational theories that bolstered his teaching practice. In this sense, we can see the struggle of the teacher to contextualize mathematics in the day-to-day of the student, currently claimed by the official documents containing guidelines for school curriculum. Such demand can lead the existing conflict between mathematics and its applications. Mathematic is normative and follows its self-movement; however, its application in everyday situations does not depends on the rules established by the logic of mathematics.

Secondly, we discuss the teaching and learning of mathematics in the perspective of the philosophy of language, because we understand that communication through language, between teacher and student, can elucidate the words of the mathematical vocabulary, the interpretation of mathematical rules and the translation from mathematical language to natural language.

The mathematical language has certain features that distinguish it from natural language. The first, intends to have only one meaning, while the second is polysemic, i.e., it may have different meanings, which comes from changes in the contexts in which the words are being applied. The translation from one language to the other, seeks the meanings that we can find in what is implicit in the language codified by mathematical symbols. Thus, the interpretation of mathematical texts depends on a reading that meets the characteristics of each one of these languages, as well as the understanding of the mathematical rules governing the texts.

To discuss this problematic we sought support in Wittgenstein, an Austrian philosopher who treated both the problems of the philosophy of language, as well of the philosophy of mathematics. He abandoned certain philosophical schools of mathematics, such as intuitionism, formalism and logicism, stating that mathematics is based on *language games*. The philosopher points out that the application of mathematics makes it a language, understanding language as an activity guided by rules. Besides the contributions of his philosophy to our theoretical knowledge, we add that, according to Janik and Toulmin (1998), Wittgenstein, as a teacher, despite not having had a good relationship with the parents of his students, and with school authorities, he becomes famous for his teaching, especially in mathematics.

Wittgenstein brings to light the much-debated philosophical question about mathematics and its applications, in an unique way, as well as the importance he gives to language in knowledge processes. Hence our interest in addressing the relationship between mathematics and language in the view of this philosopher.

For our discussions, we will use the philosophy of Wittgenstein (1983, 1989, 1996, 2000, 2003, 2004) and of some commentators of his works, such as: Bouveresse (1987), Chauviré (2008), Dumoncel (2010), Granger (1974, 1990), Hebeche (2000) and Schmitz (1988), and we also sought support in some mathematics educators who recognize the importance of the use of language in performing their teaching activities.

2. Math's Self-Movement

According to Wittgenstein (1996), mathematics as a human creation, is based on language, more specifically, in language games, that is, even if a mathematical concept has "born" from an empirical problem, when it is formalized as a mathematical rule, it becomes independent of the empyrean, and its foundation is the agreement of linguistic judgments from our way of life.

> The expression "language-game" is meant to highlight the fact that the speaking of language is part of an activity, or of a way of life. Review the multiplicity of language-games in the following examples, and in others:
> Giving orders, and obeying them-
> -Describing an object by its appearance or its measurements-
> -Constructing an object from a description (a drawing) -
> -Reporting an event-
> -Speculating about an event
> -Forming and testing a hypothesis-
> -Presenting the results of an experiment in tables and diagrams-
> -Making up a story; and reading one-
> -Acting on a Play-
> -Singing rounds-
> -Guessing riddles-
> -Cracking a joke; telling one-
> -Solving a problem in applied arithmetic-
> -Translating from one language into another-
> -Requesting, thanking, cursing, greeting, praying (WITTGENSTEIN, 1996, p. 27).

The language game is an analogy between game and language due to the fact that they follow grammatical rules because, both the game and the language, are activities guided by rules. The language games are communication systems pointing to the ways of using language, such as when a child learns his mother tongue that is taught. According to Glock (1990, p. 226), teaching practices matter "because they show distinctive features of the use we make of the words".

Granger (1990) affirms that a language game should not be regarded as an arbitrary fiction but as a "form of life". He emphasizes that speaking a language is part of an activity, or a "form of life". However, the same words, the same isolated grammatical rules, become totally arbitrary in different languages games. To exemplify, Granger says that in the change from the algebra of real numbers to the calculation of complex numbers the considered signs subsist identically from one theory to the other, - considered by Wittgenstein as language games -, which, however, evoke different operating systems rules. .

Wittgenstein considers mathematical concepts and mathematical propositions as language instruments. A statement of the type 2 + 2 = 4 is a preparation to a certain use of the language. And it is only its application, its use "in mufti" which makes Mathematics, a language itself. Bouveresse (1987) says that for the Austrian philosopher, mathematical propositions have no cognitive content, and that they constitute expressions of forms, standards or rules for the description of reality. In this sense, intuition is not a source of mathematical knowledge. We do not discover by intuition that 13 follows 12. It is our counting technique that is learned, because counting is an empirical operation.

Wittgensteinian philosophy is not interested in mental processes, because it is in the use of words that we learn their meaning. Chauviré (2008, p. 114) affirms that for Wittgenstein knowledge is not a psychological state, but a capacity, and that the link between learning and capacity does not need to involve the psychology of the student, but to meet a purely conceptual viewpoint of words.

Such capacity is developed when the teacher, in his practical activity of teaching, uses examples and exercises which explicit techniques for the understanding of concepts. This allows the student to gradually learn until the moment of having autonomy and be able to go on by himself.

> One learns the word 'think', i.e. its use, under certain circumstances, which, however, one does not learn to describe.
> But *I can teach* a person the use of the word! For a description of those circumstances is not needed for that. I just teach him the word *under particular circumstances* (WITTGENSTEIN, 1989, p. 38).

The thought experiment *may* simply be the experience of saying, since thinking is a kind of language. The word taught in certain circumstances can open the way for its use in other circumstances. It is in the practice of using the word, in different contexts of application, that the student grasps its meaning. The word triangle, for example, can be learned as a polygon with three sides and three angles; subsequently the same word can be used to describe the traffic instrument, a musical instrument, and can also be understood as the expression love triangle, noticing that all these different expressions containing the word triangle present family resemblances. However, the mathematical proposition that designates the concept of triangle constitute a norm, a convention. The polygon with three sides and three corners implies that the polygonal line that forms the triangle should be closed.

According to Alarcón (2003), Wittgenstein, understanding the meaning as action in the use of the words, retrieves the role that plays the learning and the construction of the meaningful universe of the student. The social and individual student's development is associated with the way he assimilates (technique) the different uses of the words and learn how to follow rules. Learning may arise from the reliance on the adult, which allows starting the process of linguistic coding assimilating the patterns that are taught.

The trust that is manifested initially is what allows one to settle basic certainties, something recognized as right, from which we can teach the student to doubt.

Corroborating with the author and in the attempt to clarify the meaning given when we say that we assimilate techniques, we will use the own words of Wittgenstein (1983, p. 209): "in that we are educated to a technique, we are also educated to a point of view, which is also firmly settled as this technique". As to the issue of the necessary confidence that students place in teachers, we illustrate that Wittgenstein (2000, p. 45) highlights that: "When someone tries to teach us math, he begins not by ensuring us that he *knows* that a + b = b + a", using the word "know" in italics, denotes that he who teaches have to ensure the student "how" he knows what he is teaching.

As we teach the uses of words, we are also teaching the use of rules, because it focuses attention on how to emit words in a meaningful way, which helps to distinguish correct and incorrect uses. This enables us to say that to educate means to introduce the student into an image of the world. May it be so! For example, the convention $(-1)\times(-1)=1$ cannot be weird at all, because we need to provide a direction to teach the student about how to judge about the true and the false.

In the gardeners method to design and construct an ellipse with two poles and a rope (Wittgenstein, 1983), Wittgenstein provides clues that point out to the possibility of understanding his mathematical empiricism, which, according to Dumoncel (2010), is a heuristic empiricism. The construction of the ellipse allows the gardener to create a new concept, that is, such a construction shows how a mathematical concept can be applied to the empyrean. From there, says Dumoncel (2010), the problem of Wittgenstein is to determine the difference between the empirical creation of a concept and the mathematical conceptual creation. The conceptual connection can also be illustrated by the experience of "seeing a circle in a distorting mirror".

Another example discussed by Wittgenstein is:

> "The equation 4 apples + 4 apples = 8 apples is a substitution rule which I use if, instead of substituting the sign "8" for the sign "4 + 4", I substitute the sign "8 apples" for the sign "4 + 4 apples".
> But we must careful in presuming that "4 apples + 4 apples = 8 apples" is the concrete equation and 4 + 4 = 8 the abstract proposition of which the former is only a special case, so that the arithmetic of apples, though much less general than the truly general arithmetic, is valid in its own restricted domain (for apples). There is not any "arithmetic of apples", because the equation 4 apples + 4 apples = 8 apples is not a proposition about apples. We may say that in this equation the word "apples" has no reference. (And we can always say this about a sign in a rule which helps to determine its meaning.) (WITTGENSTEIN, 2003, p. 243).

The statement 4 + 4 is a preparation for the use of the statement 4 apples + 4 apples, i.e. 4 + 4 applied to the apples inserts the rules of the calculating system in the grammar of natural language. The statement 4 + 4 is a logical relationship that can relate to many objects, including apples. The application of the enunciation 4 + 4 is the grammar of the arithmetic operation 4 apples+4 apples, just like applied geometry is the grammar of spatial objects.

> But what does the application add to the calculation? Does it introduce a new calculus? In that case it is not any longer the *same* calculation. Or does it give it substance in some sense which is essential to mathematics (logic)? If so, how can we abstract from the application at all, even only temporarily?
> No, calculation with apples is essentially the same as calculation with lines or numbers. A machine is an extension of an engine, an application is not in the same sense an extension of a calculation (WITTGENSTEIN, 2003, p. 244).

For Wittgenstein, it is vague to say that mathematics form concepts, since the concept of a rule is not specifically a mathematical concept, but a concept that comes from the connection of the subject's activity with the application of the rule. Mathematical concepts have use outside of mathematics, for example, I have three pairs of shoes, three shirts, etc. If we want to compare mathematical concepts with non-mathematical concepts, we should not compare mathematical propositions with non-mathematical propositions. We must take into account empirical propositions that contain mathematical concepts (WITTGENSTEIN, 2004, p. 115).

The application of mathematics makes its concepts make sense, however one cannot compare the mathematical proposition $\frac{1}{2}+\frac{1}{2}$ with the non-mathematical proposition $\frac{1}{2}$ apple + $\frac{1}{2}$ apple, because whereas for the 1^{st}, the result is 1, for the 2^{nd} the result is a whole apple. According to Hacking (2011), the mathematical proposition must be accepted by logical necessity; however, its application on empyrean is contingent, it may or it may not validate it because it may or may not be accepted.

The constructions of arithmetic are autonomous and, thus, they themselves guarantee their applications. The arithmetic seems grounded in itself, and according to Wittgenstein (2003), by teaching it we will be laying its foundations. "You could say: why bother to limit the application of arithmetic that takes care of itself. (I can make a knife without bothering about what kinds of materials I will have cut with it; that will show soon enough.)" (p. 241).

This means that arithmetic may have no immediate applications, but we may find it in the future, because when we construct mathematics we are not concerned with its applications.

Math is normative, it follows norms set by the needs of their own intra theoretical movement. This internal relationship, from the own field of mathematics, Caveing (2004) calls self-movement. For Wittgenstein (2003), mathematics is a field of its own, autonomous and independent.

Mathematics is a human construct, but as we saw, it follows its self-movement for its own needs of its theoretical development. The history of mathematics points to the creation of mathematical rules by different people and these creations, over time, become norms.

The fact that some people have created certain mathematical concepts and that they lived in locations far away from each other, seems to point to this inevitable construction predicted by the self-movement of Mathematics. The concepts of differential and integral calculus of functions with one variable were created simultaneously in Germany by Leibniz and Newton in England. The needs of the mathematical knowledge of their time led to such creations, Leibniz on mathematics and Newton in physics. They were distant and probably did not maintain communication but the result of their research presupposes that they had some similar mathematical knowledge.

“The necessity which governs rule deduction in math is echo of the necessity which expresses the logical laws. Necessity, despite being a human invention, has the intention to prevent and avoid human error” (SCHMITZ, 1998, p. 193). Schmitz highlights that what seems mysterious in mathematics is that we can pass from what may seem just a game of writing and empirical manipulation to the recognition of a game need for accuracy which seems radically distinct.

We created the set of integers’ numbers by conceptual necessity, not because we have debts, claims Bouveresse (1987). The set of natural numbers could not manage to explain the negative numbers, thence, we create the set of integers’ numbers, and thus, we invented the other number sets, - rational, imaginary, real- according to our conceptual needs and thus, our creations shall be preceded by others. Still, according to the philosopher, the invention is the mother of necessity that relies on an element of discovery. We may find, for example, in the case of 2 + 3 = 5, that if I receive 2 objects and then 3 more objects and count the total number of objects, the observed result is regularly and usually 5 objects, this because an empirical statement is not a rule. In this case, according to Chauviré (2008): “2 + 3 = 5” is a necessity and men believe that 2 and 3 are 5 is an anthropological statement, as the first sentence points to a norm and the second, expresses an agreement among men.

In the empirical world necessity does not exist, necessity is only logical. Math is logical and normative, therefore, *we must accept* their propositions. In practical activities, *we can* simply *accept*. In this sense, Wittgenstein (2003) illustrates: “Two men who live at peace with each other and three other men who live at peace with each other do not make five men who live at peace with each other. But that does not mean that 2 + 3 are no longer 5; it is just that addition cannot be applied in that way” (p. 264).

The logic of mathematics in the operation 2 + 3 requires that the result be 5 and it is necessary to be 5, so, *we must accept* the proposition 2 + 3 = 5. The empirical proposition “two men who live at peace with each other and three other men who live at peace with each other do not make five men who live at peace with each other”, *can be accepted*, because there may be a special case in which this fact happens.

These characteristics can be identified in the mathematics taught in school, because the teacher needs to realize that mathematics is not based on the empirical, but on the logical relations immanent to his self-movement.

3. Teaching and learning of mathematics from the perspective of the philosophy of language

One of the main supports available for teachers to teach math is the use of mathematical propositions. To give meaning to mathematical propositions he often resorts to empirical propositions. However, these meanings can be problematic as mentioned previously.

Mathematics, as well as the Portuguese language, have rules that must be applied. Following a mathematical rule is a language game and may play the game just those who understand the description of the rule. The learning of the rules is always in a state of becoming, because it depends on the context, and this state of becoming is the passage of what is not and what will be. Following a rule may be a mechanical process but the application of the rule may encounter problems with the contingency.

"I can follow a rule alone, but I *cannot be the only* one to follow it, nor to follow it just once" (CHAUVIRÉ, 2008, p. 111). The rule is consensual, comes from a regularity of judgments. This regularity implies that different persons have verified the rule, not just one, and that it was not applied only once. The application of a rule in an empirical problem does not allow us to predict the outcome. The rule is not empirical because it follows norms; however, even if the algorithms are constructed with words, the understanding of mathematical rules by students seems to appear through magical processes. In this sense, such magic is discussed by Wittgenstein (1983) when he approaches the mystery of the calculation, that we believe is already fixed, just as a fortune-teller predicts future events.

Baruk (1985), when dealing with magic in mathematics classroom, draws attention to the fact that for the students -, there is a relationship between logic and magic, the magic of, for example, *a* to be equal to $\frac{1a^1}{1}$. In this sense, the student believes that he can also do this kind of magic, such as a Baruk's student shows in the calculation: $\frac{3}{2}+1=\frac{3}{2}+\frac{3}{2}=\frac{6}{2}$. The similarly to $\frac{1a^1}{1}=a$ which is an equality that comes from the logic of mathematics -, the equality $\frac{3}{2}=1$, comes from the logic of the student, however, we emphasize that these logics operate in different ways.

Within mathematics education, Sarrazy (1997), based on his reading of Wittgensteinian philosophy argues that the conditions of use of a rule and its application cannot be defined *a priori* by a mathematical model, and says that it is an illusion to think that the meaning of a task given to the student can be established independently of the situation.The teaching through mathematical models, intends that the student understand each model, and then, learn to identify them, and apply their methods of resolution, however, Sarrazy warns that the transposition of the application of rules in different models can not be guaranteed.

To illustrate this situation, we resort to an example narrated by Baruk (1985). One of her students claimed to know solving equations of the type $13x – 5 = 3x + 2$, however, he could not solve equations of the type $13x – 5 = 3x$. Maybe in 13x - 5 = 3x + 2 , he joins the similar terms 3x and 13x and 2 with - 5 however, in the equation 13x - 5 = 3x, he joins 13x with 3x, but there is nothing to join with - 5. Thus, in his perspective, when the rule changed, he did not know how to solve anymore. We can notice that the meaning of an equation is comprised in the use, in applying the rule to solve equations, not in models of equations. The result is not contained in the rule, as there is not only one application. To learn a rule in order to solve a particular type of equation is be able to apply it in different contexts of application. The contingency cannot be predicted by the teacher and therein lies a problem.

Students, often, do not understand the subtleties of mathematical transformations (logical), because sometimes the equation can be written as $2x - y - 4 = 0$ and sometimes it can be written as $y = 2x - 4$, as well as in a phrase *x* is an exponent, in another phrase *x* can be the base of a triangle, because the letters can perform various functions in mathematical sentences.

Operating with signs is part of the mathematical activity. The sign in writing seems dead when it has no meaning for students. In this sense, it becomes necessary to translate the coded language of mathematics into natural language, with the objective of clarifying the mathematical text. Previously, we saw that for Wittgenstein, translating from one language into another is a language game.

> Translating from one language into another is a mathematical task, and the translation of a lyrical poem, for example, into a foreign language is quite analogous to a mathematical *problem*. For one may well frame the problem "How is this joke to be translated (i.e. replaced) by a joke in the other language?" and this problem can be solved; but there was no systematic method of solving it (WITTGENSTEIN, 1989, § 698).

The mathematical knowledge involves, among other things, texts to be interpreted, translated and communicated. The communication in formalized language, according to Granger (1974) is virtual. Mathematical symbols have no phonology. There are no specific sounds of mathematical language to express, for example, $\sqrt{3}$ (square root of three).

The formalized language has a residue, - that which can be interpreted out of the text, beyond the text- the missing senses that were suppressed by the formalization process, and the rescue of these senses is indispensable for its understanding.

Mathematical text tends to be objective, precise and concise. Despite objectivity and subjectivity walk together, the mathematical text seeks to exclude subjectivity. In this sense, Foucault (1995) says that the subject of the mathematical statement is a neutral subject, for example, "given the line *r*..." and "be the function...". The conventional style of mathematical text is impersonal and authoritarian (ALCALÁ, 2002), such as: "solve the operations", "calculate f (3)" etc.

A mathematical text can be written in mathematical language, (with symbols, algebraic expressions ...) or in natural language with expressions from the mathematical vocabulary, such as: apothem, factoring, geometric solid etc. It can also be written without specific words of that vocabulary, however present implicitly a mathematical rule, as in a problem involving the rule of three.

Mathematics works essentially through writing and its language intend to be universal and have only one sense. This language is formalized through algebraic expressions, graphs, tables and symbols such as: $\neq$, ∞, $\div$ etc., differently from mathematical language, the natural language is polysemic and, for this reason, the former expresses what the latter usually is not able to express, for example, the set of real numbers comprised between 2 and 5: $x \in \Re / 2 \le x \le 5$. It would be necessary to enunciate infinite numbers that mathematical signs can represent abbreviatedly.

The math symbols must be translated into the natural language. However, only this translation is not enough; we must find the meaning of the words that are beyond this translation, as well as the sense of mathematical rules that are immerse in the text. This is one of the tasks of the teacher; assist the student in the search of these senses, in the language game of the classroom.

The translation of the expression $\forall x \in A, \exists! y \in B / (x, y) \in (A \times B)$ correspond to: for all *x* belonging to the set A, there is one and only one *y* belonging to set B, such that the *x* and *y* coordinates, belong to the Cartesian product A × B. The residue consists of all the explanation given by the teacher, so that the student understands the translation that corresponds in examples and exercises with different mathematical relationships expressing the concept of function.

The pedagogical experience of Tolstoy (apud VIGOTSKI, p. 105) allowed him to understand that children learn with the use of the word. They use the language to communicate their ideas and it is in the use of the word that they understand its meaning.

> When she [the child] hear or read an unknown word in a sentence, for the rest understandable, and read again in another sentence, she begins to have a vague idea of the new concept: sooner or later she...feels the need to use a word- and once she has used it, the word and the concept belong to her...

When someone asks a student what he is studying in math at school, usually he does not know, for example, that he is studying trigonometry and often times, he might say that he is studying something with triangles. Actually, he does not know the meaning of the word trigonometry. What would he say if he were studying the factorization of algebraic expressions?

The importance of understanding the meaning of the uses of words in their contexts, that is what was sought by Baruk, Tolstoy and Wittgenstein. Baruk, from her extensive experience in mathematics education of students with learning problems in mathematics, created dictionaries for students; Tolstoy created primers for the use of students in his school; and Wittgenstein, believing that there is no essence in language, since the word does not carry its meaning, created a German orthography dictionary for his students. We believe that the use of words with meanings, was one of the reasons why they elaborated their works. The words used in different contexts, acquire meanings that satisfy the language games in which they are inserted. It is not the word alone, isolated from a context, which expands its meaning, it is its use in different situations that make it a word with meaning.

According to Tolstoy pedagogue, it is in the use of the word that students learn. Learn the meaning by the use of the word justifies the fact of assigning emphasis to language. Massot and Poulain (1999) and Pimm (2002) provide us suggestions on how to develop the practice of communication and the use of words in math classes.

The first authors make an inventory of the activities that illustrates the modes that involve the talking, reading and writing of a student. In their classroom practices, they initially propose individual activities; then group activities and; finally, discussion activities among the entire class of students. One of the activities that caught our attention asked that the whole class discuss four different writings of the same calculation that were exposed in Image 1.

"What do you think about the different writings of this calculation $(26 - 6 \times 2 - 2{,}6 \times 2 + 6{,}2 \times 6)$?"

26 – 6 x 2 – 2,6 x 2 + 6,2 x 6	② 26 – 6 x 2 – 2,6 x 2 + 6,2 x 6	2 = 26 – 6 x 2 – 2,6 x 2 + 6,2 x 6	6 x 2 = 12
= 26 – 6 x 2		2 = 26 – 12 – 2,6 x 2 + 6,2 x 6	26 – 12 = 14
= 14	② 26 – 12 – 5,2 + 37,2	2 = 26 – 12 – 5,2 + 6,2 x 6	2,6 x 2 = 5,2
= 14 – 2,6 x 2		2 = 26 – 12 – 5,2 + 37,2	14 – 5,2 = 8,8
= 14 – 5,2	② 8,8 + 37,2	2 = 14 – 5,2 + 37,2	6,2 x 6 = 37,2
= 8,8		2 = 8,8 + 37,2	37,2 + 8,8 = 46
= 8,8 + 6,2 x 6	② 46	2 = 46	
= 8,8 + 37,2			
= 46			

Image 1 – Different writings of the same calculation.

After the discussions, students elaborate guidelines on how to write a good mathematical text. An activity like this provides a language game between students which can be mediated by the teacher. Pimm (2002), discussing the math talk of students, suggests an active reader, not those who read silently, but those who speaks loudly, discussing with their peers and with the teacher. Thus, in the language game, the student when listening to the other, is careful to their own perception, and prepares himself to be listen also by his interlocutor. Both Massotand and Poulain (1999) and Pimm (2002) seek strategies that lead to the communication, oral expression and reasoning of students, through language games.

Perhaps now we can understand why Wittgenstein analyzed math problems.

> There are mathematicians who say that mathematics is a game. Others do not. Both know what they do. Because some say: Everything about it consists in making signs. The others: 'There has to be more to it, otherwise it would deal with things without importance, lifeless, that is, only signs'. It seems that we are subject to two absurdities. It is enormously difficult to think of a completely different alternative (WITTGENSTEIN, 2004, p. 349).

Mathematical knowledge is important for students because we cannot neglect the development of their logical reasoning. However, with our discussions, we realize how complex it is to teach mathematics, considering its particularities and the enormous difficulty to think of an effective way of conceiving it.

4. Final Considerations

In this paper, we pointed out some relations between mathematics and language in the construction of mathematical knowledge. Mathematics is logical and the student has a logic that cannot be ignored. In the classroom language game, the teacher can understand the students' logic if he gives them voice, and the student, in turn, can understand the logic of mathematics with the help of the teacher. Through language, student and teacher can communicate, interact and play with words in seeking to understand and, thus, understand each other. According to Hebeche (2002), a commentator of Wittgenstein philosophy of psychology, we can only have access to what the subject thinks through what he says or does, thence we choose to value its expressiveness through language, -written, spoken and gestural -, in order to know what it is that is difficult for him to understand. We believe that the understanding of mathematics comes through language; thus it is necessary that the teacher listens to his student to know what he does not understand.

We understand that the classroom cannot only be guided by the teacher's voice; it is important that students express how they understand what the teacher explained. The situations that are confusing to the student may be clarified in language games with their peers and teacher. Thus, it is convenient that the teacher seeks strategies to make students speak. The dialogue established with the student help on the resumption and continuation of what he is trying to teach. We, the teachers, must constantly remind ourselves that our words, often, lack the clarity that we think they have when we teach. In the language game between master and apprentice, we can recognize the meanings of our words when the student question us. Thus, we resumed our speeches seeking to correct the misconceptions of our words.

Mathematics is constituted by the written of a coded language, which find a phonology by means of the natural language. In this sense, we try to show that it is in the educational activities, in the teaching and learning of mathematics, with emphasis in language that we can point to the senses of a mathematical text.

Under these circumstances, we understand that a mathematical rule applied in different contexts, assumes different meanings, just like a specific symbol used in different situations, may have different meanings. The mathematical rule must be followed, for the student to obtain the result of the problem in which it is inserted.

The rule is automatically updated, but we have difficulties to update it in different contexts of its application. In everyday life, the rules may be accepted, depending on how the subjects negotiate them. In the classroom, the mathematical rules must be accepted and, for that, the teacher has to teach the student using an appropriate language, so that there will not be a mistaken interpretation. In this sense, it is important to give proper attention to language, since a word can have many meanings, which is conducive to misunderstandings.

The teaching of mathematical rules through its applications in students' day-to-day life does not always guarantee their learning, precisely because the contexts of the classroom and everyday life are different. In the classroom, the student is involved with mathematical problems in a formalized language. In everyday life, it is no longer the student who needs to solve problems; it's a subject that is in situations of everyday practices, which can be solved with calculations involving approximations. Such practices are permeated by negotiations with other subjects, such as in business practices of facilitating the giving of change. Thus, for example, the teaching of arithmetic is not justified for the "giving of change", because the giving or confer of change, students learn outside of school.

It is not possible to relativize on logical procedures, and we know that logic is required for the foundations of the student's knowledge. In a dialogue, we argue in order to prove that which we have certainty and that was not well understood by our interlocutor. In mathematics, we argue from logical procedures that which we must demonstrate. In both cases, we use the language in our argumentative manifestations.

In this sense, language, mathematics and knowledge form a triad to the intellectual enrichment of the student. The Mathematics Education is improved when we teach students how to communicate using mathematical knowledge. Basic education provides a minimum, -the mathematical knowledge base, for students understanding of the world in which we live. We opted to fulfill our task as mathematics educators, valuing the language of mathematics, the language of the student and the language of the teacher, as well as providing language games that intertwine themselves in classroom activities.

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Bibliography

Alarcón, J. J. (2003). La educación en Wittgenstein. *Revista de Filosofía Paideía: Filosofía y Educación*, 30,117-122.

Alcalá, M. (2002). *La construcción del lenguaje matemático*. Barcelona: Editorial Graó.

Baruk, S. (1996). *Insucessos e Matemáticas*. Lisboa: Relógio D' Água Editores.

______. (1985). *L'âge du capitaine: De l'erreur en mathématiques*. Paris: Éditions du Seuil.

Bouveresse, J. (1987). *La force de la règle: Wittgenstein et l'invetion de la necessité*. Paris: Les Éditions de Minuit.

Caveing, M. (2004). *Le problème des objets dans la pensée mathématique*. Paris: Librairie Philosophique J. Vrin.

Chauviré, C. (2008). *Le moment anthropologique de Wittgenstein*. Paris: Kimé.

Dumoncel, J. C. (2010, mars). Les fondements des mathematiques selon Wittgenstein. *Conférence au Département de Philosophie de l'Université de Caen*, Mercredi, 3.

Foucault, M. (1995). *A arqueologia do saber*. Rio de Janeiro: Forense Universitária.

Glock, H. J. (1998). *Dicionário Wittgenstein*. Rio de Janeiro: Jorge Zahar Editor.

Granger, G. G. (1974). *Filosofia do estilo*. São Paulo: Perspectiva, Ed. da Universidade de São Paulo.

______. (1990). *Invitation à la lecture de Wittgenstein*. Aix-en-Provence: Editions Alinea.

Hacking, I. (2011). *Wittgenstein, necessity, and the application of mathematics*. South African Journal of Philosophy, 30 (2), 155-167.

Hebeche, L. (2002). *O mundo da consciência: ensaio a partir da filosofia da psicologia de L. Wittgenstein*. Porto Alegre: EDIPUCRS.

Janik, A., & Toulmin, S. (1998). *La Viena de Wittgenstein*. Madrid: Taurus.

Massot, A., & Polain, B. (1999). Dire, lire et écrire en mathématiques au collège. *Reperes - Irem*, 37.

Pimm, D. (2002). *El lenguaje matemático en el aula*. Madrid: Morata.

Sarrazy, B. (1997). Sens et situations: une mise en question de l'enseignement des stratégies méta-cognitives en mathématique. *Recherches en didactique des mathématiques*, 17, 135-166.

Schmitz, F. (1988). *Wittgenstein, la philosophie et les mathématiques*. Paris: Presse Universitaires de France.

Vygotsky, L. S. (2003). *Pensamento e linguagem*. São Paulo: Martins Fontes.

Wittgenstein, L. (2000). *Da certeza*. Lisboa: Edições 70.

______. (2003). *Gramática Filosófica*. São Paulo: Edições Loyola.

______. (1989). *Fichas (Zettel)*. Lisboa: Edições 70.

______. (1996). *Investigações Filosóficas*. Rio de Janeiro: Coleção Pensamento Humano.

______. (2004). *Lecciones de filosofia de la psicologia 1946-1947*. Madrid: Alianza Editorial.

______. (1983). *Remarques sur les fondements des mathématiques*. Mayenne: Éditions Gallimard.