COLLABORATIVE TRAINING FOR TEACHING PROBABILITY AND STATISTICS: EMPIRICAL APPROACHES AND SIMULATION WITH ELEMENTARY SCHOOL STUDENTS

Leandro de Oliveira Souza
olileo@ig.com.br

Ana Paula Faccio
apfaccio@yahoo.com.br

Universidade Federal do Amazonas

Celi Espasandin Lopes
celilopes@uol.com.br

Universidade Cruzeiro do Sul

ABSTRACT

This paper presents a discussion of a case study conducted with the voluntary participation of sixteen mathematics teachers in the final years of elementary school in the city of São José dos Campos in São Paulo, Brazil. Research has shown that mathematics teachers feel insecure and sometimes use a linear and deterministic approach to teaching statistics. This form of teaching arises often in the course of initial training when teachers study statistics through formal, procedural and decontextualized training. Based on this problem, a collaborative action research project has been developed. In this paper, we analyze one of the activities planned in pairs and implemented by one of the teachers. Participants developed empirical activities on probabilistic and statistical themes and planned lessons with a focus on problem solving by simulation. It was evident that the teacher was a producer of knowledge that mobilizes that knowledge for the sake of generating teaching situations that promote student learning. During the teaching process, students misconceived luck, chance and causality, requiring the teacher to manage the process that allowed them to re-elaborate these ideas and the appropriation of new elements for understanding statistics.

Keywords: Teaching statistics; Teaching probability; Teacher training; Collaborative work; Empirical activities.

RESUMO

Neste artigo apresenta-se uma discussão sobre um estudo de caso realizado com a participação de 16 professores dos anos finais do Ensino Fundamental na rede municipal de São José dos Campos-SP, voluntários, especializados em Matemática. Pesquisas têm evidenciado que professores de Matemática se sentem inseguros e, por vezes, utilizam uma abordagem linear e determinista no ensino de Estatística. Essa forma de ação docente tem origem, muitas vezes, no curso de formação inicial, quando os professores estudam Estatística por meio de treinamento formal, procedimental e descontextualizado. Com base nessa problemática, desenvolveu-se um projeto de pesquisa-ação colaborativa. Os

participantes desenvolveram atividades empíricas sobre temas estatísticos e probabilísticos e construíram aulas com foco na resolução de problemas por simulação. Neste texto analisa-se uma das atividades planejadas em dupla, implementada por uma professora. Evidenciou-se que o professor se constituiu um produtor de conhecimento que mobiliza seus saberes em prol de gerar situações de ensino que favoreçam a aprendizagem discente. Durante o processo de ensino, os alunos apresentaram concepções equivocadas sobre os conceitos de sorte, azar e causalidade, exigindo que o professor gerenciasse um processo que permitiu a eles a reelaboração de tais ideias e a apropriação de novos elementos para a compreensão do fazer estatístico.

Palavras-chave: Ensino de Estatística; Ensino de Probabilidade; Formação de professores; Trabalho colaborativo; Atividades empíricas.

1. Introduction

The integration of statistical analysis with probabilistic distributions and the assumptions underlying these models are still real puzzles in teaching (Pfannkuck, 2005). We identified, through questionnaires, that teachers were reluctant to introduce concepts of probability in elementary education, and that it is often due to lack of knowledge about the connections between probability and statistics. Two other reasons identified by Serradó, Azcárate and Cardeñoso (2006) are that teachers believe that probability is not a basic subject; and that textbooks continue to present the contents of probability in a traditional, formal way, guided by mathematical problem solving, and without any connection with statistics.

Despite efforts to dispel traditional approaches to teaching statistics through new curricula, guidelines and research (Lopes, 2003; Carvalho, 2008; Batanero and Diaz, 2010; Batanero, Godino and Canizares, 2005; Gattuso, 2008; Shi, He and Tao, 2009), several paradigms still need to be overcome, particularly regarding the understanding of the way in which statistical reasoning develops and the processes of teacher training. There is great demand, and there is a need for further research to analyze informal reasoning (not systematized scientifically) and formal statistical inference, as well as the best ways to train teachers and students to make connections between probability and statistical inference (Pfannkuch, 2005).

New pedagogical approaches with an investigative focus would allow students to appropriate concepts and procedures to deal with the complex network of knowledge necessary to serve the wide range of social, economic, political and scientific interests of contemporary society. The construction of a more politicized, just and egalitarian society demands citizens that are capable of understanding and investigating what statistical data have to say, in addition to knowing how to produce and interpret measurements, using them to make decisions. Moreover, it is imperative that people are prepared to understand and investigate behavior of phenomena by means of quantitative and qualitative data in different kinds of situations. For these reasons, we believe it is essential, in the school environment, to promote reflection on the knowledge and actions necessary to implement statistics education, based on the experience and practice of teachers in the classroom.

With the recent reforms in the curriculum documents (e.g. Ministry of Education, 1998, Franklin *et al.*, 2005 National Council of Teachers of Mathematics, 2006, Ministry of Education, 2007), teachers have been challenged to modify their teaching strategies, mainly

because these documents and contemporary society have positioned themselves in favor of a more critical and democratic participation of citizens in relation to the learning process.

In this paper, when we speak of statistics education, we are referring to the interface between statistics and probability (Heitele, 1975; Greer and Mukhopadhyay, 2005), with the understanding that such subjects, when studied in an integrated manner in primary education, enable the development of particular ways of thinking and reasoning, involving random phenomena, interpretation of samples and drawing of inferences (Lopes and Moran, 1999).

Probability can be difficult to learn and teach, due to the special characteristics involved in looking more deeply at broader issues through data analysis, making value judgments about the appropriate models for looking at data, and mainly going through the process of reflection on controversial ideas such as chance and causality (Souza and Lopes, 2011). In Stohl's opinion (2005), the improvement of the teaching of probability must include both the understanding of statistical concepts, and reflection on the deterministic and non-deterministic nature of the world.

In this paper, we analyze a probabilistic simulation activity that was designed and planned by a pair of teachers participating in continuing education, which was then applied to a class of students, and then analyzed and communicated to the group in training by one of the teachers. Our goal was to build data for a collaborative training program so that, through observation of teaching activities, we could understand how the development, planning and implementation of a probabilistic simulation activity, after a cycle of workshops, would modify the pedagogical approaches and attitudes of teachers during the teaching process. The analysis was done using questionnaires completed during training and video-recordings of the meetings.

The training was based on an action research proposal based on the collaborative work of sixteen mathematics teachers who teach students in the final years of elementary school (aged 10-14). In pairs, they developed educational activities targeted to their students beginning with a problem that required the collection of data for analysis and understanding of the problem. To do so, we exposed the teachers to an initial phase of workshops where they studied and participated in empirical activities and were exposed to Pfannkuch and Wild's (1999) model that describes the statistical reasoning processes in research environments. During the planning phase, teachers were instructed to use the strategies outlined in the model, so that they would lead students to infer - starting with a problem, then planning, collecting data, building charts, analyzing and finally drawing conclusions about the proposed problem.

Last, we discuss evidence that shows how the training program helped to modify the conceptual view of the teachers on how to approach probability content.

2. Teacher Training for Statistics Education

The current situation of statistics teaching reinforces the need to investigate appropriate and efficient ways to improve the mathematical and professional knowledge of teachers, leading them to have pedagogical mastery of investigative approaches, empirical activities and production of information guided by data analysis. In the view of Batanero and Diaz (2012), just as students build their knowledge in an active way, through problem solving and interaction with classmates, teachers should be prepared for teaching, especially if we expect

them to use a constructivist and social approach in the classroom. We also believe (Smith & Goulart, 2008) that the sharing of lessons and the analysis of the results obtained by teachers can generate original ways of acting and thinking that will be absorbed and given new meaning in the future.

The multidisciplinary and systemic nature of statistical knowledge requires multiple strategies for the development and understanding of situations where statistics is necessary (Godino, Ortiz, Roa & Wilhelmi, 2011). This suggests that in continuing education teachers are involved in research activities in which they can share similar problem situations encountered in their classrooms, and through these planned activities, discuss what they learned by investigating their educational practices. Training teachers to utilize statistical research strategies, probabilistic simulations and games in their classrooms can be extremely useful, so that the students can learn how to create arguments, based on evidence, and make informed decisions on data that drive them to a smaller chance of error.

It is important to allow teachers to have more responsibility and more autonomy in their own training, so that they have support to develop creativity and critical thinking. Teachers who teach statistics do not necessarily need to build only statistical concepts, but also need to understand how students develop their reasoning (Pfannkuck, 2008). In addition, they need to learn different ways to prepare and use activities that stimulate the process of empirical learning.

It is important that teachers understand that teaching statistics should not occur in a linear process, since it requires knowledge of the context from which the data emerge to be able to choose the most appropriate of the variables. It is necessary to understand the phases of collection, grouping, analysis and comparison of the data, as well as perception of the characteristics of the distribution, identification of clusters and outliers, measurement analysis, and reconfiguration of the procedures applied to the context (Gattuso & Ottaviani, 2011). We can also add knowledge of combinatorics and the non-deterministic view of probability.

Mathematics teachers teaching statistics through investigative processes need to be prepared to support, guide and, if necessary, correct the mistakes of students in mathematical solutions. At the same time, they should not try to influence students to express opinions about their interpretations and arguments. It is important to provoke questions that allow the student some reflection (Bridge, 2011). The communication must be handled impartially, but promoting argumentation. The teacher's role is to refine the statements of the students through questions that make them reflect on the process employed in the analysis of each step of the statistical investigation cycle. This is essential for the development of the argumentative and discursive ability in statistics of those involved. The teacher has to understand and direct the pedagogical actions that need to be employed in the teaching process for the improvement of student learning.

In this study, when we analyzed a lesson developed and implemented by a pair of teachers, we found that the training program was essential to develop the perception of these teachers about the various forms of reasoning of their students, allowing them to redirect pedagogical activities when necessary. We describe the training and research methodology that we have adopted to help teachers participating in the research to modify their approaches to teaching.

3. Methodology

This case study refers to action research conducted during the PhD program. Here, we assume the perspective of Barbier (2004), which considers action research as a process conceived and decided upon by the researcher to facilitate intentional changes in teachers' practice. Actions are priority in this type of research; however, during the training process, the researchers explore the consequences of actions for the purpose of academic research. We chose to analyze and collect data during the process of collaborative training that, for Tripp (2005), is the process by which people work together as co-researchers on a project where everyone has equal participation.

According to Barbier (2004), this type of research begins in the context of a group in crisis, since researchers cannot cause it to happen. The problem of teacher training in the teaching of statistics was caused by the statistical content of the information processing block that has recently been introduced into the Brazilian curriculum. Therefore, our aim with the training was to help all involved to be aware of the most important concepts to be taught so that they could develop a collective action to improve their own practice.

In this project, the action research was designed to encourage intentional changes in teaching technique decided by the researchers. There were sixteen mathematics teachers who participated voluntarily in the study. They taught students in the later years of elementary school in the municipality of São José dos Campos in São Paulo. They attended eleven bi-weekly meetings that lasted two and a half hours.

The meetings were divided into three phases - workshops, planning and communication. The first phase consisted of workshops over the course of four meetings in which teachers had their first contact with investigative approaches and technological resources (Fathom software and free internet applications) for teaching statistics. The second phase was also organized into four meetings. In this phase, we randomly selected statistics and probability content so that teachers could develop and plan a lesson in pairs with pedagogical assistance from the researcher. After that, we asked the teachers to develop the planned lesson with a group of students making notes of their observations to pass the experience back to the whole group. The last phase consisted of the communication. In this phase, the teachers did the sharing of the exercise, clarifying how the development of the lesson that was planned and implemented occurred by communicating the observations made during the class and explaining how they acted in the teaching process during the answers from their students. In the process of discussion and reflection, the teachers themselves listed actions that were beneficial to the teaching and learning process and those that were not.

Early on at the first meeting, data from the teachers was collected through an initial questionnaire, with the goal of helping us understand the professional experiences that they had in teaching statistics, and also their personal educational experiences with regard to this matter in their undergraduate courses. In addition, we sought to know what kind of teaching approaches they used in their classrooms. Five more questionnaires, all with open questions, were distributed throughout the sessions, for a total of 40 questions. These questionnaires were designed to understand how teachers saw the teaching of statistics compared to the teaching of mathematics. In addition, we assessed, through a self-evaluation, what kind of teaching approaches teachers used in their classes, how they were feeling about teaching statistics using a process focused on research and data collection, and finally what changes

had occurred in their teaching practice after participating in the project team. All meetings were recorded on video recordings that were also used in the analysis.

In response to our goal of understanding how the development, planning, implementation and communication of a lesson of probabilistic simulation would modify the teaching approaches and attitudes during the teaching process, we analyzed a case study in this paper. The focus was on two periods of the training: first, the questionnaires applied in the workshop phase that have brought to light the practical needs of teachers with regard to teaching statistics; and second, the planning, implementation and communication phases of a lesson by one of the teachers.

The purpose of this study was not to generalize a theory, but to learn, by analyzing the practice of the teacher, how to deal with the beliefs of the students regarding the concepts of luck, chance and causality during the statistics lessons, and also how teachers should deal with their own beliefs on these concepts. We chose to analyze the activities of this teacher because it is a case where she worked with probabilistic simulation, bringing forth new knowledge for her, which changed her feelings of insecurity and her approaches to teaching statistics.

4. Initial Data Analysis and Methodological Path

Through the initial questionnaire, we found that none of the sixteen teachers surveyed had had contact with any theoretical framework for teaching statistics, probability and combinatorics, though the new curricular guidelines have been proposed. We observed that most respondents did not see statistics as a science, nor did they see its potential for data analysis. They considered it a mathematical tool and made references only to its descriptive nature of data. Evidence that motivated this misconception in the reports pointed to the influence of procedural university education. During the analysis of responses and reports of teachers in the first two questionnaires, we found aspects of approaches and deficiencies in training that made teaching practices through investigative activities more difficult.

Due to the problems found in those early questionnaires, over the course of the project, we had to adapt the format of the training to the pedagogical needs of the participants. For example, initially, we expected that, after the workshop phase, the teachers would apply the activities developed in their classes, and then, through discussion and reflection on this, we would collect data on the cognitive development of students. This did not happen. When we asked teachers to choose and implement one of the activities, we received negative responses from them. Their difficulties were evident in the fourth and fifth questionnaires that asked what obstacles they would have in implementing the activities. Despite the commitment and motivation with the findings presented by the teachers, we observed some complicating factors to the process of teaching and learning which concerned a variety of infrastructural problems at each school and a certain divergence between the interests of students and teachers. There was no uniformity to allow us to identify the real needs of each school. Some evidence can be seen in some of the teachers' responses to the questionnaires:

> [ ] ... I have some questions regarding the concepts of statistics. Actually when we discussed here in our meetings, I could understand these concepts a little better, however I still have difficulties. I think my problem is insecurity. This year I have not used the computer lab yet because it is under renovation. I like to use it, however the students are really having a lot of trouble. Also, their bad behavior impedes their learning, but yet I keep on trying. (Teacher S. August 4, 2011)

> [ ] ... At the school where I work, the students have extreme difficulty that hinders the application of any special activity. (Teacher R. August 4, 2011)
> [ ] ... My challenge is to hold the students' attention for the purpose of the lesson, as the students offer great resistance to accepting the proposed topics. (Teacher M. August 4, 2011)
> [ ] ... I have never given a statistics lesson. I'll have to study the topics more. Our school is open all day long, so the computer lab is almost always in use. (Teacher I. August 4, 2011)
> [ ] ... I'm enjoying the workshops because it changed the way that I teach statistics and especially understanding its importance and use. I thought if there were more meetings, we could do it better and learn more and apply these techniques. (Teacher W. September 22, 2011)

As the meetings went on, we began to notice that we would not be able to help them change their teaching practice only by making them participate in educational activities previously prepared in the workshop phase. We discussed that with the teachers and then collectively decided that, starting with the fifth meeting, some content, such as probability, mean, mode and median and different types of charts would be distributed randomly among the teachers. Based on this, they would work in pairs, which would also be drawn at random. Then each pair would have to build an investigative or exploratory lesson to teach statistics to their students using the content drawn at random. This approach was not initially planned in the project. It happened that as the training went along, pairs of teachers would design and plan activities with the support of the researcher, who followed the progress of each pair. This organization allowed the researcher to intervene, gather data and understand some pedagogical and practical needs that teachers reported to the researcher during participation in the professional training program.

Our intention, describing how the development process of one of the teachers of the group occurred, is to provide data on how the training was developed, reporting changes that occurred in the teacher's practice and how her perceptions were reported to the training group. We took into account her entire creation process, from the initial questionnaires to the final product. The teacher will be called Alice in this story, a name chosen to preserve her identity from the group of teachers.

We describe below how the teacher led this process of lesson planning and implementation, in which she became a researcher of her own practice. In her approach to teaching, she prioritized the understanding of probabilistic concepts by collecting data, constructing tables, graphs and simulation, rather than procedures. From the data collected during the various stages of the training, we will analyze the knowledge that was modified during the research and we will point out implications that this process of formative research brought to her teaching practice. Finally, we will discuss pedagogical actions that we believe are necessary for the educational process of the students and that emerged from our interpretation of the data.

5. Development Process

At the beginning of the project, Alice, who has about three years of teaching experience and a degree in mathematics, claimed to be clear on all topics that must be presented in elementary education, pointing out the main themes that are described in the Brazilian Curricular Parameters - PCN (Brazil, 1997). Additionally, she revealed that she had never read anything about the information processing theme (combinatorics, probability, statistics), because she had not attended a continuing education course that dealt with this. She also commented that

in their initial training, the approach had been practically based on the problem solving paradigm.

In the first questionnaire, we asked teachers about how content related to statistics was given in their initial (undergraduate) training. Alice reported: "The methodology used was to fill tables, calculate average, standard deviation and construct graphs using paper for this. There has never been any attention to inference or the teaching methodology that would be passed on to students." On technological resources, Alice confirmed: "I never had contact with any software to teach statistics, although I had already used software to teach the four basic operations and first degree equations."

We asked in which grade level she normally introduced statistics. The teacher replied: "I teach 9th grade, but this is the first year that I am teaching this grade level and have not come to this content yet. In fact, I have never actually taught statistics". This statement is not totally at odds with the PCN of mathematics, because they do not indicate the grade level in which the content should be introduced, only the cycles (high-level groupings of grade levels). However, the assertion of the teacher shows that this content may be addressed by some teachers only in the final grade level of elementary school, and this indicates that statistics and probability may be being treated as separate content, with no relation to the other blocks, which is not recommended. When asked about the obstacles she encountered to teach this subject, the teacher replied: "In my training, I learned the content, however I did not learn how to approach it and so do not feel prepared." Regarding her expectations about the training, Alice said: "I hope to refresh my mind about statistics. It's been long since I studied this subject. I hope to learn methodologies that contribute to the dynamics of my classes and learn how to use useful software to address statistics."

In the initial questionnaire, in general, we confirmed that besides having learned statistics in a procedural way, teachers reported feeling insecure about teaching this subject. Alice's responses were very similar to those of the other teachers in the group.

About how Alice saw the differences between teaching statistics and mathematics, she said: "[ ] ... I believe that statistics is within mathematics. Statistics deals more with data collection, research and interpretation, while mathematics has more to do with algebra." (Alice, May 19, 2011)

Regarding teaching methodology, she believed: "[...] The best way to teach statistics is through fieldwork so that students are participating agents, building surveys, collecting data, designing, transforming and drawing conclusions about information" (Alice, May 19, 2011). Furthermore, she believed that probability and technological resources could be of fundamental importance.

There were many difficulties in modifying teaching approaches, which was pointed out by the group in training. To prepare a lesson that they considered to be good, they lacked the mastery of content, availability of simple and appropriate teaching materials, time for planning (which, in most cases, was done alone at home) and available computer equipment.

Based upon the story of Alice and other teachers in the initial phase of research, we believe that in many Brazilian universities, statistics is still taught in a procedural, hierarchical, linear and traditional way. It is very likely that this educational perspective described by teachers implies, for them, pedagogical difficulties to contextualize the content of statistics and apply

them to problems in the students' reality, in the same way as real problems that permeate other sciences.

In the next section, we will describe how Alice's activity was planned and developed in collaboration with another teacher, twenty years more experienced, who was randomly chosen. In addition to this lesson, the process of collaborative action research that we applied generated six other lessons planned by teachers, and each was subjected to individual analysis. We chose to discuss the activity performed by this pair, since it was planned and implemented by a teacher who had recently graduated, which allowed us to analyze data from a teacher who had just recently gotten out of college. We believe that by being a recent graduate, this teacher would be less likely to have her approach and vision influenced by problems that permeate the teaching practice within the school system.

5.1. Planning and Implementation Phase

The planning and construction of the activity of the pair of teachers began with the topic *Urn and Roulette Probability*. The pair at first searched the internet for some software that could assist them in planning the lesson. As both found only one application with a tutorial classified for high school, they struggled to adapt it to elementary school and gave up on using it. The major complication for the pair was that the graphics system of the software was not visually clear and, moreover, it treated probability in such a way that did not allow for the exploration of data collected. Basically, the application simulated the picking of colored balls from an urn, building a chart with columns containing the chosen balls.

In general, we realized that teachers needed more time to adapt to the tools. Furthermore, they complained that they could not find anything like it in textbooks. Activities found there, most of the time, dealt only with the interpretation of charts and tables, calculating and reading, leaving aside important concepts such as the study of the variation, the context of the problem at hand and relations of probability with statistics.

Because the software was not accepted by teachers, we suggested that they plan an activity that would use real materials, for example, colored balls and opaque bags. Thus it would be possible to understand the law of large numbers through actual simulation. We proposed that they build graphs from data collected by the students in groups and to add more data gradually accordingly to the color of each ball picked from the bag, so they could make observations and inferences during the construction of the graphs.

After the suggestions, the pair decided that they would not use any software and started planning an activity. At first, they thought about doing a simulation where they would put three green balls and seven red balls in an opaque bag, so that students would know neither the number nor the color of the balls.

In pairs, students (about 12 years old) would pick a ball, note its color in a table, return it to the bag, mix and take another ball, also noting its color. This procedure would be repeated at least 20 times for each pair. After completing each table, the teachers would add the data collected to construct a graph on the blackboard, asking the students, during construction, about which ball had been taken out more often.

In theory, this process would help students in the cognitive construction of probability skills. Probably, after some balls had been picked from the bag, students would be able to say for

sure which color occurred more often. The initial idea was to increase the number of picks gradually, noting them in a table, transforming the results into a percentage to then construct a bar chart or a pie chart with the results. While this process of construction was taking place, the teachers, with some pre-prepared questions in hand, would lead the cognitive development of students, so that they could predict the percentage of balls of each color, after a series of picks.

With the initial plans outlined, we then suggested that the teachers take advantage of this activity to introduce issues of gambling. When they accepted, they suggested allowing students to make bets, using fake bank notes. The idea was that, with the room divided into pairs and after revealing how many balls of each color were in the bags, all the students would reach a consensus and bet on the green balls or the red balls. The teacher would bet on the color not chosen by the students.

The rule of the game would be: Every time that a green ball was selected, the bettor who had chosen it would receive R$16.00 from the opponent, and every time a red ball was selected, the bettor would pay R$8.00. The question in this case would be: Who will win the most money?

There were three green balls and seven red balls in the bag. It was predicted that after 100 picks, for example, those who had chosen the red balls would get around R$560.00 (70 x 8), while those who had opted for the green balls would get around R$480.00 (30 x 16). Encouraging students to reason about the bets, teachers would propose the simulation a few more times. After that, they would lead students through questions to predict the results of bets, demystifying the idea of luck when the game does not provide equal chances for both players.
The goal of the activity outlined by the teachers would be to lead students to understand that even if they could sometimes win the bet, knowing the rules of the game and betting on the right color would give them control of the situation and allow them to earn much more money than they would lose, according to the number of balls picked. Following this plan, students would be prepared to empirically understand the law of large numbers - "the first fundamental theorem of probability."

5.2. Communication of Results Phase

One month after the discussion of this activity, Alice began her seminar by explaining how the planning and development of the class with her partner had occurred. Though she seemed really excited with the development of the project, which they had named "Statistics in the Urns," the teacher reported that they had much trouble finding material that would be helpful, especially in textbooks that, in her view, did not address statistics in an exploratory way, limiting the tasks to reading and interpreting graphs. This caused them to lose time.

To construct the activity, the teachers adapted the idea of an application that can be found at the site http://www.mais.mat.br/wiki/Probabilidade_com_urnas (last accessed on August 14, 2014) and applied it to students in the 8th year of primary education. The procedure adopted by teachers was the same as previously described.

The activity was applied by the two teachers in two different groups, one of 30 and the other of 16 students. In this paper, we will describe only Alice's report, who applied the activity in the smaller group.

According to her, the students were instructed to choose one of the colors, in such a way that the whole class would reach a consensus. She told the students that she would make one bet also. They would remove the balls from the bag, one at a time, and replace the selected ball after each pick. Once set, each pair made 20 picks, noting the results in a table. The teacher explained to the students that honesty was essential for the activity to work, and fully trust the results noted, understanding that nobody would cheat. Alice reported:

> [ ] ... Before starting the activity, I distributed R$200.00 of fake money to each pair and asked them to choose. Either they would choose green and would receive R$16.00 for each green ball picked or they would choose red and receive R$8.00 for each red ball picked. When the ball selected was the other color, they had to pay according to the agreed amounts. Each pair received a bag with 10 balls. The students would bet against the teacher, i.e., the room had to come to a consensus and choose a color. When I asked the students to choose a color, they were undecided because they did not know which color could win them the most money. The students discussed for about five minutes, and after that there was one student who was good at math who took leadership of the discussion. She chose the color that had seven balls, but some students were opposed. They said that green would pay R$16.00. I knew I would lose, but I did not interfere. The leadership of this student prevailed. (Alice, video recording, September 29, 2011)

Observing Alice's initial report, we see that the activity started with an investigative issue that struck a chord with the students. Who would win more money in the game: he who chose the green balls or he who chose the red? This way of presenting the problem led the students to formulate hypotheses. The first hypothesis was that the students had more red balls. The second hypothesis was that the green balls would pay a higher value. When teachers planned the activity, they had predicted that this would occur. The fact that the students did not know what the answer would be meant that they had to negotiate. Then the anxiety to answer the questions and a desire not to lose money allowed them to observe the results of the game carefully.

Alice continued:

> [ ] ... As they picked the balls, students noted the number of balls of each color in a table. When they finished, the students on their own multiplied the number of green balls by sixteen and the number of red balls by eight. When I went around to the groups, some had already done the subtraction to find out how much money they made. During the activity I asked some questions: "Who do you think will win the game? Do you think it's a matter of luck?" After the game, most students said it was a matter of luck, and one student replied that it had to do with statistics. Some students, influenced by the response of their classmate, said it was a matter of statistics, but they probably did not even know the meaning. Many were in doubt and said they still believed that the issue was luck. This occurred in the first round, and obviously I had lost because the students had chosen the balls that were worth R$8.00. So I said, "Let's play again. Who do you think will win? Do I have any chance of winning?" The students responded that I also had a chance to win. I asked, "Do you believe this?" They students answered yes, saying that it was a matter of luck. I said then let's play. (Alice, video recording, September 29, 2011)

At this point in the game, Alice realized that she would have to deal with the beliefs of the students. And so she went along thinking and rethinking the questions, so that the students would change their beliefs. We consider this approach to be an important step so that students begin to realize that an investigative process does not allow for preconceived answers. Another aspect that we see in this approach was the emphasis that Alice put on observation.

After realizing that some students had not modified their initial argument, she decided to do the simulation again, to try to convince the students that it was not just a matter of luck.

> [ ] ... We played once more. Each pair picked twenty balls and of course I lost again. At the end I asked, "Do you still think that the issue is luck?" Some students changed their answer and began to argue that the issue was logic because I had lost twice. I asked, "If we keep playing, do you believe that I will continue losing?" They said that I would probably keep losing. So I said, "Let's see what are the chances? I took the red balls and asked, "We have seven balls of this color of ten balls total, so what are the chances that we pick a ball of that color?" That good student replied, "Seventy percent, teacher." And then everyone in the class said seventy percent. And then I asked about the other color, and she replied, "Thirty percent, teacher." (Alice, video recording, September 29, 2011)

The investigative process requires reflection on the possibilities of an event occurring. Through questions, Alice led her students to refine their thinking. This does not mean that everybody had achieved the expected response. However, in the future each one would be more prepared to deal with these issues, improving their responses about investigative processes. Changing beliefs is not a simple process - it takes time and understanding of the reasons that lead someone to believe in something. The deterministic approach in teaching mathematics does not help people deal with these issues in statistics.

Alice ended by saying:

> [ ] ... At the end, even doing the exercise multiple times and asking questions, some students went away believing that it was a matter of luck. I found it very interesting to apply this activity because I realized that some students already have a notion of the issue of probability, but there aren't many. Most believe it is a matter of luck. I really enjoyed doing this activity. I had never worked with statistics that way, only with graphs and tables. I think it was worth it. (Alice, video recording, September 29, 2011)

Although Alice had observed that some students still believed that the issue was luck, she realized that her change of approach helped to evaluate the cognitive behaviors of students she had not noticed yet. This stimulated her and made her happy with the results of her class. Probably in future classes, with different experiments, these students would also modify their beliefs about statistics.

The activity with balls developed by Alice showed the importance of developing knowledge about possibility analysis. When the students tried to understand why one of the colors brought greater financial returns, they thought about the possibilities and brought arguments to justify their inferences. This phase of the process is important for people to develop reflective knowledge, so that they do not accept the first result as being correct. Developing teaching through questioning that brings an uncertain response creates a need to explore a quantitative measurement of the chance of an event occurring. This is an important step to demystify the issue of luck on events that occur randomly. Teachers need to experience different investigative situations involving probability, so they have the ability to adapt their activities.

Alice concluded that, despite never having worked with statistics in this way, it was very interesting to observe the processes of learning construction in students, since she herself previously had only learned by interpreting graphs and tables. During the lesson, she realized that not many of the students had any idea of the meaning of probability and, though some

have changed their minds, others continued stating that the game was just a matter of luck. In the report, the teacher made it clear that she was thrilled with the results of the experiment.

5.3. Self-evaluation and final thoughts from Alice

Throughout the construction process with Alice and other teachers, we observed significant changes in their understanding from the moment they had to plan their class to present to colleagues. So the teachers realized that a procedural approach would not be appropriate and therefore revisited the workshops making efforts to adapt them. Some evidence revealed in self-evaluation:

> [ ] ... At the beginning of the research, I felt that we had no direction, learning some software, but without a common goal, much less contributing to the research. I believed that we would only learn different ways to teach statistics to our students in an easier way, so I felt a little lost in the first meetings. The activities that we performed in the computer lab were important for me, since I did not know and had never seen applications of probability as they were shown in the workshops. The training really took hold for me when we began to involve the students. I learned a lot from the lesson that I had to plan based on the programs we had seen in previous meetings. (Alice, questionnaire, August 4, 2011)

The teacher's report showed us that, even though she thought it was important to have participated in the workshop phase, she found it difficult to transform her practice. What we did starting from the workshop phase was to encourage teachers to build and plan materials together. According to Roseth, Garfield and Ben-Zvi (2008), this is extremely important for student learning. The involvement with the material helps teachers connect their teaching goals to the curriculum and assessments. The group discussion and decision about what to teach, how to teach and how to assess encourage an atmosphere of team work and improves engagement and satisfaction with the teaching profession (Rumsey, 1998). In addition, the teacher feels responsible as the "owner" of the activity, and it sharpens his/her curiosity and his/her efforts to see the activity work.

In our view, planning one's own activity collaboratively is critical. A teacher who plans an activity developed by him/herself seeks to understand the difficulties that students have and will have. If this is done in collaboration with a colleague, the work is more productive because a teacher learns by gaining knowledge when he/she sees the way the other one works. Understanding students' difficulties is of paramount importance, when it is expected that the teacher develops knowledge for statistical research. Through Alice's reports, we found that teachers who plan their own lessons guided by references to research in education are more likely to use a non-deterministic and less procedural approach for statistics education.

Many mathematics teachers have not learned to work with statistics, and therefore need to be immersed in training that allows the process of experimentation, using new tools that should be constructed, and not offered. For Alice, the training became meaningful for her when she began to involve her students. Moreover, the challenge of assembling her lessons based on new knowledge acquired during training motivated her to learn and participate.

This process was also efficient such that, throughout the research project, Alice's vision of statistics was modified. After a few meetings, in her questionnaire, we noted that she ended up stating: "Mathematics and statistics are dependent on each other. I believe that

mathematics is a tool of statistical science because it is required during the collection and synthesis of data." This argument is contrary to her previous statement, where she said that statistics was within mathematics.

6. Final considerations

By observing the development of the activity described and the data collected over the course of the project, we noticed that, for any change to happen in the approach of teachers for teaching statistics, it is necessary to adopt some strategies to enable the exchange of experiences in the training process and experimentation in the classroom.

Another point which we believe is of fundamental importance is the shared analysis of the results. Capturing and understanding the movements of the teacher during a lesson is a complex task because the events that occur in this environment are difficult to repeat and are influenced by various factors such as social conditions, affective relationships between teachers and students, emotional state, and others. Sharing together, through lessons that are planned and developed in pairs, can enable teachers to provide new possibilities of action that may be passed along in the social structures in which they move. In our opinion, this teacher training strategy neither occurs nor works immediately. It will take a while for the process to stimulate comradery and trust among teachers and researchers, so that they feel safe and prepared to expose their ideas. Although we have not used this approach in our research, we understand that it is of fundamental importance for the evolution of approaches to teaching.

Regarding the workshop stage, it is recommended to perform different experiments. Teachers are not used to teaching statistics by collecting data. Typically, statistics lessons in primary education are developed through an approach that involves only reading and interpretation of data, which is clear in Alice's report. This approach to teaching deprives the student of the necessary knowledge to develop a critical and investigative sense in society. So it is also necessary to improve perceptions about the difference between math and statistics. It is important, in the processes of teacher training for statistics, to explore the difference between doing math and doing statistics. One approach that we believe would be efficient to explore this discussion is to start the training with questions that require data to be answered, without any *a priori* knowledge of the answers to these questions.

Investigative situations that involve probability usually begin by observation. In this case, the process of data processing must be done so that different experiments involving probabilistic situations are developed. Simulations using only dice, coins, and balls are not enough to explore these perceptions, because when we perform experiments with these objects, we already know the results. It is necessary to conduct experiments where the results are not known *a priori*. It is also extremely important to focus on the improvement of the knowledge of statistical content of teachers. Nobody teaches what he does not know. There is a consensus in the educational literature that knowledge about the content taught is an essential component of the teacher's competence.

The process of constructing this activity, and others that are not described in this paper, emerged from the ideas and experiences of each teacher. From the moment that teachers began planning, they began to get involved with their activities. During the planning phase, the teachers sat down and told stories about their own practice. After going through the process of listening, those who were listening responded with their own stories, with questions that allowed them to investigate details or critical questions that explored the

essence of the teaching and learning process that happened during these events. This constituted a reflective process that generated the construction of new ideas.

The reports of the teachers participating in the research project about their frustrations regarding continuing education shows that it often does not prepare them to deal with problems relevant to their area, much less to allow the exchange of experiences. Oftentimes, continuing education in the area of mathematics offers models of lessons ready for teachers to follow as examples. Other times, the discussion becomes so open that those involved lose sight of the goal. We hear reports of teachers saying that even when they believe that the activities given in the textbooks are not appropriate, they end up using them, due to lack of choice and lack of time to prepare. At the end of the project, teachers stated that the best way to learn new teaching methodologies was through interaction with other teachers and with an appropriate theoretical support to the topic being studied.

Finally, another important element to be reported is that, although we have evidence that teachers overcame feelings of insecurity, a change in approach requires continuity, training cycles and more research to ensure the improvement of statistics education for teachers and students.

7. References

Barbier, R. (2004). *A pesquisa ação*. Brasília, Brasil: Líber Livro Editora.

Batanero, C., Godino, J. D. & Cañizares, M. J. (2005). Simulation as a tool to train Pre-service School Teachers. In: J. Addler (Ed.), *Proceedings of First ICMI African Regional Conference* (pp. 1-8). Johannesburg: ICMI.

Batanero, C. & Diaz, C. (2010). Training teachers to teach statistics: What can we learn from research? *Statistique et Enseignement*, 1(1), 5-20. Retrieved September 18, 2014, from, http://math.univ-lyon1.fr/irem/IMG/pdf/Batanero_Diaz.pdf.

Batanero, C. & Diaz, C. (2012). Training school teachers to teach probability: reflections and challenges. *Chilean Journal of Statistics*, 3(1), 3-13. Retrieved September 18, 2014, from, http://chjs.soche.cl/index.php?option=com_content&view=article&id=58&Itemid=56.

Brasil (1997). *Parâmetros Curriculares Nacionais: introdução aos parâmetros curriculares nacionais.* Secretaria de Educação Fundamental. Brasília: MEC/SEF.

Carvalho, C. (2008). Collaborative work in statistics classes: Why do it? In: C. Batanero, G. Burrill, C. Reading, & A. Rossman (Eds.), *Teaching statistics in school mathematics. Challenges for teaching and teacher education. Proceedings of the Joint ICMI Study 18 and 2008 IASE Round Table Conference*, Mexico: ICMI. Retrieved September 18, 2014, from, http://www.stat.auckland.ac.nz/~iase/publications/rt08/T1P7_Carvalho.pdf.

Feldman, A. (1998). Implementing and assessing the power of conversation in the teaching of action research. *Teacher Education Quarterly*, California, 25(2), 27-42.

Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M. & Scheaffer, R. (2005). *Guidelines for assessment and instruction in statistics education (GAISE) report: A*

pre-k-12 curriculum framework. Alexandria, VA: American Statistical Association. Retrieved September 18, 2014, from, www.amstat.org/Education/gaise/.

Gattuso, L. (2008). Mathematics in a statistical context. In: C. Batanero, G. Burrill , C. Reading, & A. Rossman (Eds.), *Teaching statistics in school mathematics. Challenges for teaching and teacher education. Proceedings of the Joint ICMI Study 18 and 2008 IASE Round Table Conference*, Mexico: ICMI. Retrieved September 18, 2014, from, http://www.stat.auckland.ac.nz/~iase/publications/rt08/T6P1_Gattuso.pdf.

Gattuso, L. & Ottaviani, M. (2011).Complementing mathematical thinking and statistical thinking in school mathematics. In: C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics - Challenges for teaching and teacher education: A joint ICMI/IASE study* (pp. 121-132). New York: Springer.

Greer, B. & Mukhopadhyay, S. (2005). Teaching and learning the mathematization of uncertainty: Historical, cultural, social and political contexts. In G. A. Jones (Ed.), *Exploring probability in school: Challenges for teaching and learning* (pp. 297-324). New York: Springer.

Godino, J., Ortiz, J., Roa, R. & Wilhelmi, M. (2011). Models for statistical pedagogical knowledge. In: C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics - Challenges for teaching and teacher education: A joint ICMI/IASE study* (pp. 271-282). New York: Springer.

Heitele, D. (1975). An epistemological view on fundamental stochastic ideas. *Educational Studies in Mathematics*, 6(2), 187-205.

Lopes, C. E. & Moran, R. C. (1999). A estatística e a probabilidade através das atividades propostas em alguns livros didáticos brasileiros recomendados para o ensino fundamental. In: *Anais da Conferência Internacional "Experiências e expectativas do ensino de estatística – Desafios para o século XXI"* (pp. 167-174). Santa Catarina: Anais. Retrieved September 18, 2014, from, http://www.ime.unicamp.br/~lem/publica/ce_lopes/est_prop.pdf.

Lopes, C. E. (2003). *O conhecimento profissional dos professores e suas relações com Estatística e Probabilidade na Educação Infantil.* Tese de Doutorado, Faculdade de Educação, Universidade Estadual de Campinas, Campinas, Brasil.

Ministério da Educação. (1998). *Parâmetros curriculares nacionais: Matemática*. Brasília, Brasil: Author.

Ministry of Education. (2007). *The New Zealand curriculum*. Wellington, New Zealand: Learning Media Limited.

National Council of Teachers of Mathematics. (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics: a quest for coherence*. Reston, VA: Author.

Pfannkuch, M. (2005). Probability and statistical inference: How can teachers enable learners to make connection? In: G. A. Jones (Ed.), *Exploring probability in school: Challenges for teaching and Learning* (pp. 267-294)**.** New York: Springer.

Pfannkuch, M. (2008). Training teachers to develop statistical thinking. In: C. Batanero, G. Burrill, C. Reading, & A. Rossman (Eds.), *Teaching statistics in school mathematics. Challenges for teaching and teacher education. Proceedings of the Joint ICMI Study 18 and 2008 IASE Round Table Conference* , Mexico: ICMI.

Ponte, J. (2011). Preparing teachers to meet the challenges of Statistics education. In: C. Batanero, G. Burrill, & C. Reading (Eds.), *Teaching statistics in school mathematics - Challenges for teaching and teacher education: A joint ICMI/IASE study* (pp. 299-309). New York: Springer.

Roseth, J.; Garfield, J. & Ben-Zvi, D. (2008). Collaboration in Learning and Teaching Statistics. *Journal of Statistics Education***,** 16(1). Retrieved September 18, 2014, from, http://www.amstat.org/publications/jse/v16n1/roseth.html.

Rumsey, D. (1998). A Cooperative Teaching Approach to Introductory Statistics. *Journal of Statistics Education,* 6(1). Retrieved September 18, 2014, from, http://www.amstat.org/publications/jse/v6n1/rumsey.html.

Serradó, A., Azcárate, P. & Cardeñoso, J. (2006). Analyzing teacher resistance to teach probability in compulsory education. In *Proceedings of ICOTS-7 International Conference in Teaching Statistics Salvador*, Bahia, Brazil. Retrieved September 18, 2014, from, http://www.stat.auckland.ac.nz/~iase/publications/17/2E4_SERR.pdf.

Soares, E. S., Goulart, M. I. M. (2008). Aulas Compartilhadas na formação de licenciados em matemática. *Revista Brasileira de Educação*, 13(38), 306–412.

Stohl, H. (2005). Probability in teacher education and development. In G. A. Jones (Ed.), *Exploring probability in school: Challenges for teaching and learning* (pp. 297–324). New York: Springer.

Shi, N., He, X., & Tao, J. (2009). Understanding statistics and statistics education: A Chinese perspective. *Journal of Statistics Education, 17(3).* Retrieved September 18, 2014, from, http://www.amstat.org/publications/jse/v17n3/shi.html.

Tripp, D. (2005). Pesquisa ação uma introdução metodológica. *Educação e Pesquisa, 31(3),* 443-466.

Souza, L. & Lopes C. E. (2011). O uso de simuladores e a tecnologia no ensino da estocástica. *Bolema Boletim de Educação Matemática, 24(40),* 659-677.

Wild, C. & Pfannkuch, M. (1999). Statistical thinking in empirical enquiry. *International Statistical Review*, 67(3), 223-265. Retrieved September 18, 2014, from, http://www.stat.auckland.ac.nz/~iase/publications/isr/99.Wild.Pfannkuch.pdf.