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Mathematical Intuition: impact on non-math major undergraduates

Intuición matemática: impacto en estudiantes universitarios no especializados en matemáticas

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Abstract

The research question being investigated in this paper is how to improve the learning of calculus for non-math major students whose interests are primarily non mathematical. The study explores the impact of an intuitive approach applied to the concepts of limits and integration. The survey was carried on 1st year engineering students taking the Calculus-1 course. The students had already had a brief encounter with the topics in high school. They were surveyed about their outlook towards the concepts of limits and integration using the same pre and post inquiry form. The post analysis was conducted after introducing both the topics through an intuitive approach. The survey was analyzed for statistical significance using the Mc Nemar's statistical test, which studies the impact of an intervention. Survey analysis indicated an improved understanding for the students and a positive change in their approach towards the topics.

Keywords: Calculus. Integration. Intuition. Limits. Non-math majors.

Resumen

La pregunta que guía la investigación presentada en este artículo es cómo mejorar el aprendizaje del cálculo, para los estudiantes que no se especializan en matemáticas y cuyos intereses, principalmente, no son matemáticos. El estudio explora el impacto de un enfoque intuitivo aplicado a los conceptos de límites e integración. La encuesta se llevó a cabo en estudiantes de primer año de ingeniería que tomaban el curso Cálculo-1. Los estudiantes ya habían tenido contacto con los temas en la escuela secundaria y, luego, fueron encuestados sobre su perspectiva hacia los conceptos de límites e integración, utilizando el mismo formulario de pre y post indagación. El análisis posterior se realizó después de presentar ambos temas a través de un enfoque intuitivo. La encuesta se analizó para determinar su significación estadística mediante la prueba estadística de Mc Nemar, que estudia el impacto de una intervención. El análisis de la encuesta indicó una mejor comprensión de los estudiantes y un cambio positivo en su enfoque hacia los temas.

Palabras clave: Cálculo. Integración. Intuición. Límites. No especializaciones en matemáticas.

1 Introduction

The paper begins by addressing the issue of student motivation while studying Mathematical service courses as a part of their study plan. This issue has been addressed

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through an intuitive approach and its impact on students' attitude towards calculus has been analyzed to some extent. This paper focuses on the attitudes, challenges, and conceptual understanding of engineering students enrolled in the Calculus-1 course. The analysis was carried out within the theoretical framework of motivation and intuition. The first part of the paper introduces a "Mathematical impact chain", which implies that intuition triggers a behavioral change in student approach at all levels and the product of this chain is, improved learning. The second part of the paper applies this approach to existing Calculus-1 for engineering students and analyzes the results.

The declining trend in the student enrolment for Mathematics major degrees is no secret. Low student enrollment and high attrition rates in Science, Technology, Engineering, and Mathematics (STEM) are major challenges in higher education (SITHOLE; CHIYAKA; MCCARTHY; MUPINGA; BUCKLEIN; KIBIRIGE, 2017). Science and mathematics education fall behind in fostering talent and enthusing young people seeking a career in these domains as mentioned in the Organization for Economic Co-operation and Development (2006) report. This problem involves low popularity, as well as perceived difficulty, National Research Council (2001). Countries like the U.S., Germany, and Australia have already raised their concerns and are researching this big question.

Hence, this puts additional responsibility on the mathematics educators to improve the teaching methodologies, especially to cater to the needs and demands of a vast number of non-math majors they encounter during the service courses. To bring about a change in the attitudes and interests of students, whose interests are not primarily mathematical, is no easy venture. The branch of mathematics most encountered by the undergraduate students of various majors is Calculus. A lot of research has been done on how to tackle issues related to students' understanding and motivation in learning Calculus, but little focus has been given on how to differentiate the issues of non-math majors who end up in these courses as part of the package. Most research on students' mathematical beliefs, attitudes, and emotions show that most students have an attitude where they believe that mathematics merely requires facts and procedures to be remembered (SVEGE, 1997; YUSOF; TALL, 1996). This superfluous treatment leaves gaps in the minds of those for whom calculus comes easily, while for others, it may breed discontent. Secondary school education has often been blamed for not giving enough emphasis on this aspect of mathematics education and the consequent trend of declining student enrolment in the mathematical programs themselves.

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2 Mathematical Impact Chain

In this research, an attempt has been made to draw a connection between different aspects of mathematical learning for math students from different disciplines, as optimizing "learning" is always the key objective of any mathematics educator. For this purpose, a "Mathematical Learning Impact Chain" for non-math major students is being introduced.

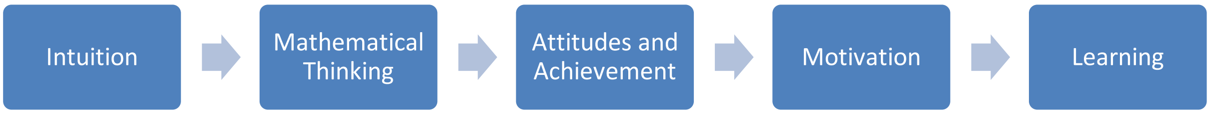

Figure 1 – Mathematical Impact Chain
Source: Prepared by author

Affective factors in students' learning include self-efficacy, beliefs, motivation, engagement, and attitudes towards mathematics; these factors play an important role in success or failure of mathematics learning (ABDULWAHED; JAWORSKI; CRAWFORD, 2012). The importance of each of this chain's components in the field of mathematics education has already been well-established by several educationalists. Going backwards, it is discussed here how one impacts the other and triggers the chain reaction which has learning as its end-product.

2.1 Motivation impacts Learning

Learning is defined as the acquisition of knowledge and skills through study, experience, and being taught, whereas student motivation naturally has to do with the students' desire to participate in the learning process. The Necessity Principle states that students are most likely to learn when they see a need for what we intend to teach them, where need refers to their intellectual requirements (HAREL, 1998).

In summary, learning can only take place when the heart and mind come together. A student may be achieving, yet not learning, as doing something because you must do it is very much different from doing it because you want to do so. In the context of mathematical education, we can think of motivation as a desire to get involved in the process of acquiring mathematical skills. Optimal learning cannot take place unless we create an environment where students are motivated to learn Calculus. Mathematics educators must be creative in their approaches to achieve the desired learning as motivation has a direct impact on learning.

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2.2 Attitudes and Achievement impact Motivation

Motivation is the inclination to do certain things and avoid doing others (HANNULA, 2006). In the context of mathematical education, attitude can be considered to be an approach or a set of beliefs. Mathematical beliefs that have an adverse effect on how students understand mathematical concepts disaffirm the assumptions about the nature of mathematics (AYEBO; MRUTU, 2019). The impact of attitudes and achievement in mathematics on motivation in mathematical learning has been well established in many research studies. Students who believe they are able and can do well are much more likely to be motivated in terms of effort, persistence, and behavior than students who believe they are less able and do not expect to succeed (ECCLES; WIGFIELD; SCHIEFELE, 1998; PINTRICH; SCHUNK, 2002). There is a strong relationship between motivation and mathematical achievement along with the interrelationship with Mathematical anxiety (ZAKARIA; NORDIN, 2008).

Some research advocates that achievement is a by-product of motivation, but it would not be appropriated to generalize this assumption. This may be true for math major students, who are intrinsically motivated, but for non-math majors, since the motivation is primarily extrinsic, academic achievement serves as a positive instigator for motivation while the opposite may not be true. Thus, for non-math major students, attitudes and achievement play a vital role in motivating them to learn.

2.3 Mathematical Thinking impacts Attitudes and Achievements

It is well established through various research studies that there is a relationship between attitudes and achievement. On the other hand, studies also show the predominance of one over the other. A review of research examining this relationship concluded that significant causal predominance between attitude and achievement is uncommon, but when it does appear, attitude assumes causal predominance over achievement (MINATO; KAMADA, 1996). Furthermore, a study by Ma and Xu (2004) concluded that achievement clearly demonstrated causal predominance or priority over attitude. We can think of them as fluctuating between one to the other, subjected to factors ranging from personality variables to social and cultural factors.

However, mathematical thinking abilities bring about a great change in attitudes and achievements. Aiken's (1996) definition of attitude can help us understand this better. Attitude consists of cognitive (knowledge of intellect), affect (emotion and motivation), and performance (behavior or action) components (AIKEN, 1996). Knowledge of intellect, the first

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component of attitude in this definition, arises from mathematical thinking. A mind that is ready to think mathematically can bring about positivity in the attitude of the thinker and help pass one of the biggest hindrances standing in the path of mathematical beliefs and achievement. The students' mathematical beliefs serve as a filter that influences mathematical thoughts and actions (PEHKONEN, 2001). So how do we bring about this conviction in our students' mathematical beliefs? Intuition may be a possible solution.

2.4 Intuition impacts Mathematical thinking

Intuition as a word has been defined by oxford dictionary as "direct perception of truth or fact, independent of any reasoning process; immediate apprehension." In the field of mathematics education, intuition has already been embraced by many researches (FISCHBEIN, 2010; FISCHBEIN; GROSSMAN, 1997; TALL, 1991; TSAMIR, 2007; TSAMIR; TIROSH, 2002).

In their study, Roh and Lee (2017) refer to intuition as cognition directly accepted without or prior to any rigorous justification. Intuition is a direct, global, self-evident form of knowledge, which main properties are syncretic structure and obviousness or being spontaneously accepted (PRZENIOSLO, 2004). Deep intuition is a blend of the basic meaning of a concept and its properties derived from relationships with other objects, both mathematical and non-mathematical (SEMANDI, 2008). In Tall's theory of the three worlds of mathematics, the first step towards mathematical thinking is a conceptual embodiment, which builds on human perceptions and actions. This embodiment develops mental images that are verbalized in increasingly sophisticated ways and become perfect mental entities in our imagination (TALL, 2004).

Another view of intuition is, that there are many ways in which one understands something. The most gratifying is a sudden wave of insight in which suddenly something becomes clear (BURTON, 1999). Burton's "wave of insight" is a mere description of intuition which can trigger a complete mathematical idea in the mind of a student. Intuition may be regarded as one of the approaches which can assist in conceptual embodiment, thereby providing the necessary stimulus for the students to embark on the mathematical journey.

In this research study, mathematical intuition is seen as an informal approach, which improves the understanding of mathematical ideas and pushes the boundaries of mathematical thinking beyond the subject itself, and into the pool of common sense. The exact impact of intuition on mathematical thinking has never been measured till date, but it is a well-known fact

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that it was Newton and Leibnitz's initial intuition on the rate of change and area under the curve that gave birth to Calculus. It was only later that they formalized these ideas.

3 Calculus understanding and intuition

The students' discussion boards across various social media platforms as well as the student feedback for the introductory calculus courses suggested that many of the non-math major students believe calculus to be a necessary evil, which they just want to "get over with". As mathematics educators, it comes down to us to come up with pedagogies and interventions which can alter such student perception and make them more enthusiastic about the subject. If the students are excited and curious about the subject, there is something that drives them to make sense of what they work with (JUTER, 2005). Experience has shown that to convince non-math majors to study proof is a losing battle. Also, there has been much concern regarding the failure to develop a conceptual understanding of calculus topics because of the rote, manipulative learning that takes place in an introductory course (CIPRA, 1988; STEEN, 1988; WHITE; MITCHELMORE, 1996). One of the explanations for this is that the modus operandi for professional mathematicians tends to be: intuition; trial and error; speculation; conjecture; proof. But the teaching often focuses only on the end point of the flow. This focus may marginalize and hide from students the inherent sense-making activity in mathematics (ALMEIDA, 2000). Hence, this leads to our research question: *Are there any alternative strategies which can make calculus relatively more meaningful for the non-math major students?*

Mathematical intuition can be presented as a preferable alternative here. The intuitive approach helps make calculus more natural and helps students connect with it and feel it. Once this connection is established, learning mathematics can become a thought-provoking and adventurous journey and it is here that more formal mathematical ideas can be introduced in a frame of mind which is now ready to absorb more details about a concept that has become less abstract. Although some members of the mathematics community such as Carey (2000) disagree with this approach and believe that when students enter the science classroom with rich, pre-instructional intuitive theories, they typically interfere with the learning, a more accurate, scientific theory of the same domain, most feel that this may not be the case. As Fischbein (2010) points out, when students face a notion of advanced mathematics that is intuitively unacceptable, they produce, deliberately or unconsciously, more acceptable intuitive substitutes - the intuitive models that can be understood, represented, and manipulated like other

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concrete realities. Tall (1991) also feels that intuition and rigor need not be at odds with each other. By providing a suitably powerful context, intuition naturally leads into the rigor of mathematical proof. It is important for educators to understand that the complex way the brain functions is often at variance with the logic of mathematics. It is not always pure logic that gives us insight (TALL; VINNER, 1981).

In general, it would be a good idea to develop among the students an attitude to challenge their intuition whenever it conflicts with a formal definition and modify or eliminate it to maintain the sanctity of the formal mathematical logic. But to strip students of intuition out of this fear is to deny them the outlook that was the starting point of calculus itself. Without intuition, the students would find themselves with a collection of tools which they can use at will but unaware about the full functioning of the machine.

Juter (2005) points out that students who use rote learning are more likely to see the parts of mathematics they have learned as disjoint units of methods or rules, with the result that they cannot see connections between different concepts or underlying crucial properties. Their mathematical understanding becomes weak and incomplete, and it takes a lot of effort to sustain their mathematics studies as the pressure to remember rules, methods, theorems, and definitions mounts. Such a dysfunctional pattern is difficult to reverse since the students' foundational mathematical knowledge has gaps, which must be filled, at least partially.

4 Intuitive approach

4.1 Intuitive introduction to Calculus

Mathematics is traditionally taught deductively beginning with definitions and formulas, followed by some practical applications. As such, the engineering students come to the Calculus-1 course with a pre-conceived notion that the aim of taking the course is to study theorems, solve problems, and get the right answer. As Prince and Felder (2006) put it rightly, the only motivation that students get, if any, is that the material will be important later in the curriculum or in their careers. Most of the students, thereby, have little or no concept or image of what calculus is at its core, its significance and relevance.

Hence, in this case, the Calculus course started with the Big Picture of Calculus, 'Why study calculus?' To say to the students that it has many applications in physics and other disciplines and make them solve the application problems later certainly does not give them any insight into the soul of Calculus. Rather, this question was approached with intuition. Some

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experiences create patterns and framework in our mind so that the next time we come across a similar experience, our mind can process it. Calculus does something similar by creating a framework in our mind, which is programmed to process real-life information accessed in fields such as engineering, physics, and chemistry.

Usually, Calculus courses are taught as parts of a puzzle, while the big picture created by the puzzle is quite often ignored. Empirical data shows that most Calculus curriculums usually follow the path of limits, differentiation and its applications, and integration and its applications. They treat each one of them as a separate entity. The limits usually start with graphical and numerical approach followed by algebraic manipulations. While this may be appropriate if the sole aim of the process is to teach computational techniques, but it fails badly in providing students an insight into the relevance of limits. Usually, it is only during derivative through first principle and later Riemannian sums that the connection between the limits and derivatives/integrals is explained. However, neither the books nor the curriculum gives enough emphasis to the fact that the idea of "limiting value" was introduced in the first place to solve the tangent line problem and the area problem. It is important to emphasize and revisit the intuition behind the development of calculus right from the beginning of the course so that the students know where each part fits in the big picture as they traverse the path of calculus. Current teaching of calculus mostly focuses on how and where calculus works, but hardly mentions why calculus works. The intuition behind the bigger picture gives the students something to ponder about as to why calculus works.

4.2 Intuitive introduction to Limits

In their research related to limits, McLeod (1992) and Pehkonen (2001) observed that if their first experiences with limits are positive, then the attitudinal development triggered by emotional reactions has a chance to grow strong and be sustained during the demanding nature of the learning process. Moreover, Juter (2005) concluded that the relationship between attitudes and achievement in solving limit problems implies that time should be spent on enhancing positive attitudes. In the early stages of development of the theory of functions, limits, and continuity, visualization is a key source of ideas. To deny these ideas to students is to cut them off from the historical roots of the subject (TALL, 1991). Szydlik (2000), in his research, showed that students who preferred authorities such as lecturers and textbooks to establish mathematical truth owned ill-defined concepts and a faulty understanding of limits. This contrasted with students who relied on intuition, logic, and empiricism.

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Currently, in most curriculums, the limits' computation and its formal definitions are taught at various levels. However, nowhere is the intuition behind the limits emphasized. Limits are more than often described as y approaches "what" as x approaches a particular value. To make things simpler, the epsilon delta definition is only introduced after a graphical and numerical approach. It is assumed that the gaps that might have been created during the graphical and the numerical approach will probably get filled during the epsilon delta definition. While the importance of the definition cannot be overlooked, for non-math majors, this definition does nothing more than add to the already existing vagueness that they associate with limits. To address this issue, the limits were introduced intuitively as follows.

Figure 2 below shows a screenshot of a football match at different stages on www.youtube.com. Using figure 2 Azad (2006), students were asked to guess the position of the ball at the 4:00th minute when the internet connection was buffering. The students gave their predictions based on the position of the ball at 3:59th and 4:01st minute. They were sure that their guess was accurate because the position of the ball at the 3:59th and 4:01st minute was almost the same. Building on this introduction, limits were introduced as, "A process which confirms our prediction or guess about a doubtful scenario". Of course, the guess should always fall between its neighboring points, no matter how much we zoom in, and the before and the after has to agree (L.H.L = R.H.L). The techniques of evaluation of limits confirm the accuracy of our guess or prediction, and whether a guess or a prediction is possible. Valid exceptions would be the doubtful scenarios of mathematically undefined results when zooming in is "getting as close as possible", but this introduction made the limits come to life for the students This real-life situation gave the students an intuitive outlook towards the mathematical concept of limits. Equipped with this intuition, the students' response was overwhelming when asked why they thought that the limit would exist if and only if the L.H.L = R.H.L. Hereafter, the students comfortably embarked on the graphical and numerical journey of the limits as the vagueness had been addressed in the beginning and the intuition had given them an interesting conceptual image.

The standard textbooks usually follow up the graphical and numerical approach concept with a few practice questions and then introduce the infamous $\lim_{x \to 0} \frac{sinx}{x} = 1$ numerically. It is only during the later chapters of linear approximation that students learn about the linear approximation of $y = sinx$ is the line $= x$. It is gaps like these that make Calculus a nightmare for many non-major students. A math student may eventually be able to draw the link, but for the other students $\lim_{x \to 0} \frac{sinx}{x} = 1$ and the linear approximation of $y = sinx$ is the line $y = x$ exist

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as two separate results of their Calculus course with no connection whatsoever.

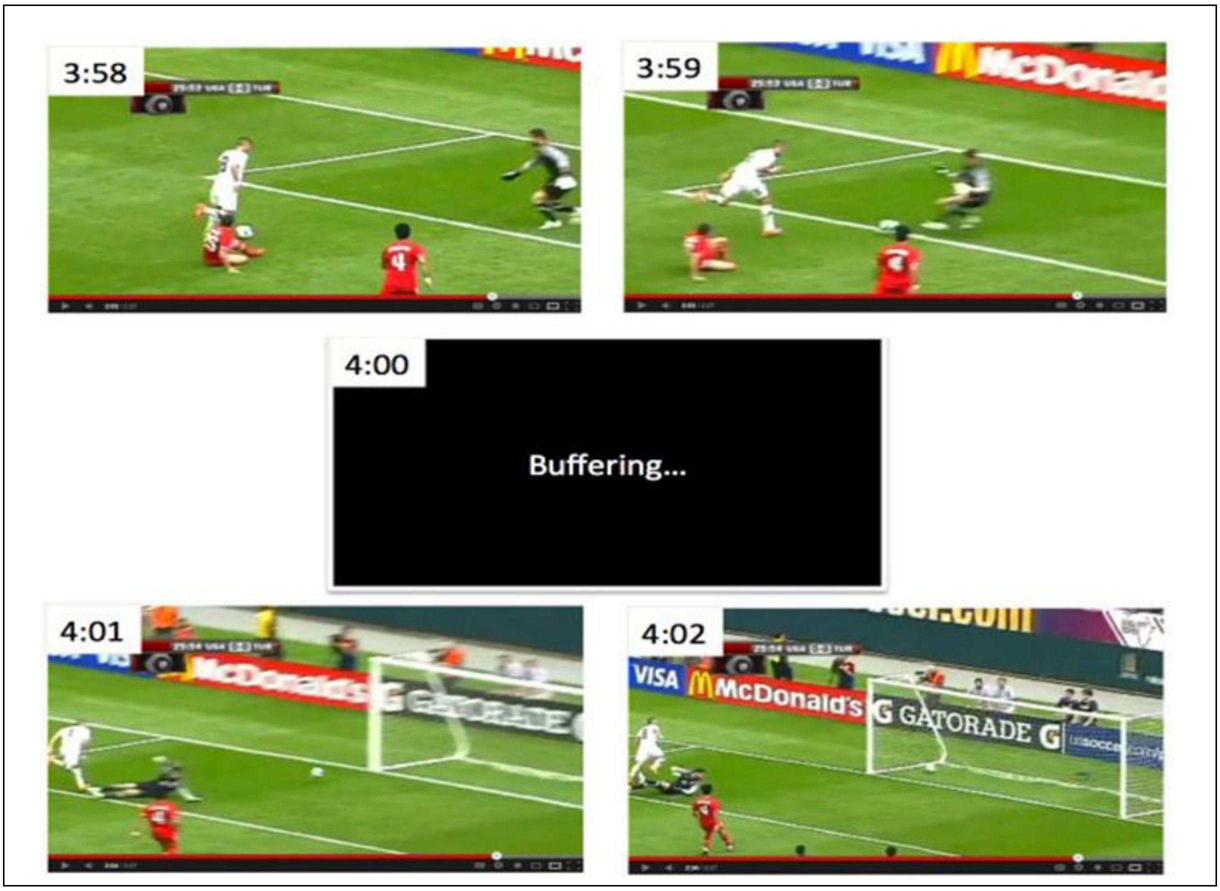

Figure 2 – Intuition behind limits
Source: https://betterexplained.com/articles/an-intuitive-introduction-to-limits/. Access in: 19th Sep. 2018

To address this issue, once again a simple intuitive graph of $\lim_{x\to 0}\frac{sinx}{x} = 1$ was introduced along with its zoomed-in result as shown in figure 3 (AZAD, 2006). The students already had the intuition on how limits are a prediction or a guess about a mathematically undefined situation. Hence, a zoomed-in graph gave them an immediate insight as to why $\lim_{x\to 0}\frac{sinx}{x} = 1$, without the need of any numerical calculation or the graph of $y = \frac{sinx}{x}$ itself. The bigger achievement was when a student exclaimed that this would mean the result $\lim_{x\to 0}\frac{tanx}{x} = 1$ should also be true and inquired if he was right. Most of them were also able to explain why $\lim_{x\to 0}\frac{cosx}{x} \neq 1$. This was very heartening to see as it was non-math major students using logic in Calculus as opposed to memorizing procedures.

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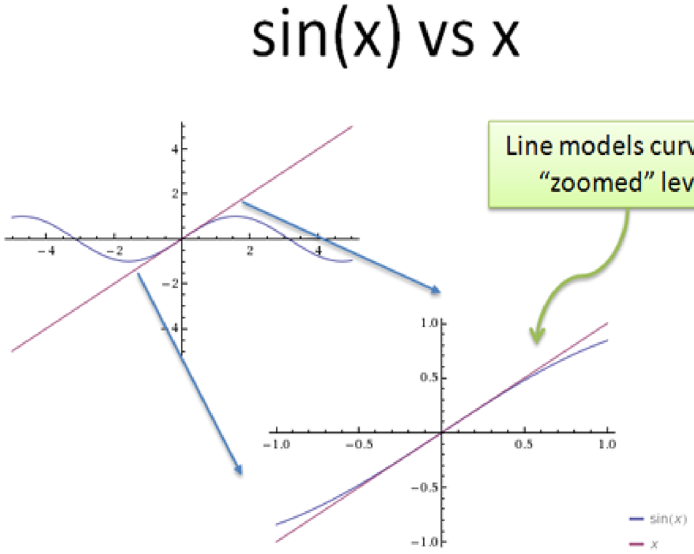

Figure 3 – Intuition behind $\lim_{x \to 0} \frac{sinx}{x} = 1$
Source: https://betterexplained.com/articles/why-do-we-need-limits-and-infinitesimals/. Access in: 19th Sep. 2018

However, the most effective change of this introduction with intuition was visible when the students were able to relate and predict the linear approximation for $y = sin\,x$ and $y = tan\,x$ near 0 without any calculations.

4.3 Intuitive approach to Integration

Coming to the Calculus class, most of the undergraduate students have only one definition of integration, which was clear during the survey where they believed it to be a technique for finding the area under a curve. Again, they knew how and when integration worked but were clueless about the "why". This is mainly because elementary Calculus courses introduce integration as simply how to find Anti-derivatives, followed by the Riemannian sum, definite integral, and area under the curve without giving enough emphasis on the fact that integration is the reverse of derivatives. The fundamental theorem of Calculus is only introduced in the later stages and as such the intuition behind integration is lost between all the theorems and formal definitions. However, neither calculus nor integration that we see today ever existed in this form in the earlier stages of Mathematics. It was only the intuitive idea of summing up infinitesimals, which was later formalized to integration.

Therefore, the main intuition emphasized here was how integration is "the idea of combining quantities into a new result or better multiplication". Integration was further discussed as a tool to undo differentiation. This idea was intuitively explored in terms of its

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application. It was discussed how integration can undo not only the mathematical derivative that they are used to finding, but also every other physical implication of derivative. Students were then asked to list the physical implications of integration apart from area. It was only when students offered "height from slope", "distance from speed", and "velocity from acceleration" views of integration that they were led to the Fundamental Theorem of Calculus, instead of doing it the other way round. Thus, students were encouraged to explore different integration applications. So, one of the informal learning objectives was to erase from the students' minds the perception that finding the area under the curve was the purpose of integration rather than only one of its many applications.

5 Survey Methodology and Analysis

In order to analyze whether intuition had any impact on the attitudes and perceptions of students, two surveys were designed related to the topics of Limits and Integration, in particular. The choice of these two topics was based on the observation of a consistently low outcome achievement percentage for these two topics in contrast to the outcome achievement percentage of "Derivatives and their applications" in the Calculus-1 for engineering course. The observation came from the analysis of the course assessment reports of the Calculus-1 course for engineering for the past four semesters. The fact that students are comfortable and confident about the ideas and intuitions leading to derivatives but unsure about the intuition behind limits and integration, despite the intangibility of the three concepts, reflects the gross shortcomings of the mathematical education system across many places.

Since the aim of this study was to analyze whether an intuitive approach can modify the attitudes and perceptions of non-math major students studying Calculus, the analysis was approached using a pre-post survey instead of relying on their academic grades. So, the students were given the same survey at the beginning of the semester and towards the end of the semester. The questions in the survey were designed keeping in mind the aim of the analysis, which was to observe, understand, and produce attitude modification, if required.

The survey was carried out on two different sections of boys. The total sample size was 92 students who were present in both the stages of the survey. Out of these, 81 students had already studied limits, derivatives, and integrals in high school Calculus. The results were analyzed by running the data through Mc Nemar's test on SPSS version 20. The test is used to determine whether an intervention has an impact on two related categorical groups where each entity can exclusively belong to a single category. Hence, it is an appropriate tool for analyzing

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this study. More details about the test can be found on https://statistics.laerd.com/spss-tutorials/mcnemars-test-using-spss-statistics.php.

In this study, the test was used to determine whether an intuitive approach to the topics of limits and integration had any impact on the students' perceptions and attitudes towards them. Each Calculus idea mentioned in the survey is considered as a separate dependent variable and the intuitive approach is the intervention. The following assumptions were made:

- Due to the same participants being measured twice, we have paired samples.
- Each question in the survey was treated as an individual idea which is our single categorical dependent variable with two categories (i.e., a dichotomous variable)
- The two groups of our dependent variable are mutually exclusive.
- The participants are a random sample from the population of interest.

5.1 Limits survey

The limits survey comprised 3 questions. In a pre-post survey, the students were asked to choose one of the two options which they felt better suited the idea. The third question asked the students about their understanding of the concept of limits without any written assessments. The cross-tabulation table for each idea along with the test results for the same is given above in Table 1.

Table 1 – Mc-Nemars Test Statistics for the Limits survey

Idea	**Before and after Statistics**			**P-value**
	Before	*After*		
		Abstract Math concept	Underlies real life scenarios	
1. Limits	Abstract Math Concept	8	40	0.000
	Underlies real life scenarios	3	30	
	Before	*After*		
		L.H.L = R.H.L	Predicting the unknown	
2. Solving of Limits	L.H.L = R.H.L	21	20	0.004
	Predicting the unknown	5	35	
	Before	*After*		
		Average	Excellent	
3. My understanding of limits	Average	30	30	0.000
	Excellent	0	21	

Source: Prepared by author using SPSS V.20

5.2 Limits survey analysis and result interpretation

In each of the three cases above, the null hypothesis was: "Intuitive approach has no impact on students' perceptions of this particular idea". Since the "p-value" for each of the three cases was below 0.05, we can reject the null hypothesis for each case. Hence, it can be

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concluded that intuitive approach is a statistically significant intervention in bringing about attitude modification towards students' concepts of limits. A closer look reveals that the most significant shift in the students' outlook was observed in the response from "Limits being an abstract math concept" to the "Limits being hidden under real life situations." This may be attributed to the "YouTube video image" which provided the students with a strong intuition about what limits may imply. In contrast, quite a few students were still reluctant to embrace the idea of "solving limits is an attempt to predict the unknown" and continued to be more comfortable with the thought that the sole purpose of solving limits is to get to "L.H.L = R.H.L". This was expected, as the first idea had been rooted in the students' mind throughout their secondary education and hence required consistent challenging to bring about a total change. Similar pattern was observed in the response for the third idea where many students were still hesitant to rate their understanding excellent. This can be attributed to the culture of many students judging their understanding based on grades rather than insight.

Since no understanding of mathematics is complete without drill and practice, this intuitive approach was followed by the regular drill and practice technique which prepares a toolbox for students to investigate when trying to attempt more complex mathematical ideas and problems. Equipped with intuition, students were inclined to be more motivated which resulted in an overall improved learning experience and student satisfaction. This was evident at the end of term, student satisfaction survey for the course, as well as the achievement outcome percentage for the topic of limits.

5.3 Integration Survey

The integration survey was comprised of six statements. In the pre-post survey, students were asked to select true or false for the given statements. The cross-tabulation table and the statistical results of the Mc-Nemars test are shown below in Table 2.

Table 2 – Mc-Nemars Test statistics for the Integration survey

Statement	Before and After Statistics			P-value
	Before		*After*	
		False	True	
1. Integration glues together pieces of a bigger picture.	False	8	19	0.054
	True	8	46	
	Before		*After*	
		False	True	
2. Integration finds area.	False	58	5	0.064
	True	14	4	
	Before		*After*	
		False	True	
3. Integration is the opposite of differentiation.	False	7	9	0.424
	True	5	60	

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Statement	Before	After: False	After: True	p
4. Integration can find the height of a curve at a point if its slope is given.	False	10	47	0.000
	True	7	17	
5. Integration can be used to find distance if the velocity at a point is given.	False	8	38	0.000
	True	4	31	
6. Given integrand and anti-derivative, integration can be used to find the area under the integrand.	False	9	23	0.012
	True	8	41	

Source: Prepared by Author using SPSS V. 20

5.4 Integration survey analysis result interpretation

For each of the six statements, the null hypothesis was “Intuitive approach has no impact on students’ perception of the given statement”. It was observed that the p value for the first three statements was over 0.05, which led to the acceptance of the null hypothesis for the first three statements. This is understandable because most of the high school syllabus does introduce integration as Riemannian sums, area, and the opposite of derivative. Hence, the intuitive approach here only reinforces the existing integration perceptions. However, the p values for the remaining three statements are less than 0.05, which lead to rejecting the null hypothesis for the last three statements.

This is an interesting observation, as despite being confident about the first three statements, it was only through the intuitive listing exercise discussed in section 4.3, that the students were able to understand the complete logic behind integration. This suggests that the intuitive approach adopted at the secondary school level, if any, was more mechanical than thought provoking. In fact, when used as a facilitating tool, the same intuition brought about a statistically significant perceptual modification among the students. This once again led to an overall improved learning which was evident through the increased outcome achievement percentages for the topic of integration in the semester’s final course assessment report.

6 Limitations, Conclusions and Implications for Teaching

This study produced some clear results. However, the sample in this research consisted of engineering students, which is, by far, the most math-heavy STEM field. More research needs to be done to see if similar attitude modifications could be achieved for calculus in other non-math major fields such as business, health sciences, pharmacy etc. It may also be argued that engineering being the most math-heavy STEM field, it is unlikely that students are not

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intrinsically motivated. This is not necessarily true as existing research shows that a career choice is influenced not only by student variables, but also by factors such as parenting style, background, ethnicity, gender etc. In the consensus study report by the National Academy of Engineering (2018), it was found that the factors influencing student's decision to opt for engineering careers differed based on both individual characteristics (e.g., gender, race, ethnicity, disability, personality traits) and external factors (e.g., family socioeconomic status, local community, school experiences, parental education, perceptions of the engineering culture).

The importance of a rigorous math curriculum cannot be ignored but research into mathematics education shows that students mostly lack intuition when it comes to limits and integration and treat it mechanically. This paper suggests that intuition can be used as an approach to bring about a positive behavior modification in the students' attitude towards these two topics, specifically. Despite the shortcomings of the intuitive approach as discussed above, the significant results shown are encouraging. Further research is required to develop its technique and efficacy and to test for validity and reliability on a larger scale.

As Einstein famously said, "Education is what remains after one has forgotten everything one learned in school". In order to ensure this, intuitive approach is an excellent start.

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