

THE TEACHING OF ELEMENTARY MATHEMATICAL KNOWLEDGE IN TIMES OF THE INTUITIVE METHOD AND THE ANALYTICAL INTUITIVE METHOD

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ABSTRACT

This paper aims to understand the methodological recommendations for the teaching of elementary mathematical knowledge in Brazilian primary school, in the 19th-20th century transition. The investigative approach adopted here consists of the analysis of a few writings by Oscar Thompson and Sampaio Doria, which were published in pedagogical São Paulo journals. They are: “*The Public School*” (“*A Eschola Publica*”) and “*Journal of the Education Society*” (“*Revista da Sociedade de Educação*”). In these journals, Oscar Thompson and Sampaio Doria circulated mainly methodological concepts for the teaching of arithmetic knowledge in primary school. From the standpoint of Arithmetic, we saw that, by appropriating Pestalozzian pedagogical concepts, Oscar Thompson recommended the teaching of numbers and calculation based on the intuitive method. Oscar Thompson’s article revealed that the Pestalozzian intuitive method already presented a teaching proposal based on groups of objects. In the methodology presented by Doria, the first calculation learning would not lie on memorizing Arabic symbols or fundamental signals, but rather on recognizing a set of concrete things resulting from an addition, a subtraction, a multiplication, or a division of objects.

Keywords: Arithmetic; Intuitive method; Analytical intuitive method; Oscar Thompson; Sampaio Doria.

RESUMO

O presente texto objetiva compreender as recomendações metodológicas para o ensino dos saberes elementares matemáticos do curso primário brasileiro, na transição do século XIX para o XX. O caminho investigativo aqui adotado consiste em analisar alguns escritos de Oscar Thompson e de Sampaio Doria, os quais foram publicados em revistas pedagógicas paulistas. São elas: *A Eschola Publica* e a *Revista da Sociedade de Educação*. Nesses periódicos, Oscar Thompson e Sampaio Doria fizeram circular, sobretudo, concepções metodológicas para o ensino dos saberes aritméticos do curso primário. Do ponto de vista da Aritmética, viu-se que ao se apropriar das concepções pedagógicas pestalozzianas, Oscar Thompson recomendou o ensino de número e

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cálculo a partir do método intuitivo. O artigo de Oscar Thompson revelou que o método intuitivo pestalozziano já apresentava uma proposta de ensino a partir de grupos de objetos. Na forma metodológica apresentada por Doria, a primeira aprendizagem de cálculo não estaria na memorização dos símbolos arábicos e nem nos sinais fundamentais, mas no reconhecimento de um conjunto de coisas concretas como resultado de um acréscimo, de uma subtração, de uma multiplicação e de uma divisão de objetos.

Palavras-chave: Aritmética; Método intuitivo; Método intuitivo analítico; Oscar Thompson; Sampaio Doria.

1. Introduction

As a partial result of a doctoral research², this paper aims to understand the methodological recommendations for the teaching of elementary mathematical knowledge in Brazilian primary school, in the 19th-20th century transition. The questions that guide this text are as follows:

- How are recommendations established for the teaching of arithmetic knowledge in São Paulo primary school under the aegis of the intuitive method and analytical intuitive method indications?
- How have these methods provided possible articulations in the Read-Write-Count Triad?

In an attempt to find answers to these questions, the investigative approach adopted here consists of the analysis of a few writings by Oscar Thompson and Sampaio Doria, which were published in pedagogical São Paulo journals, between the late nineteenth century and early twentieth century. They are: “*A Eschola Publica*” (“The Public School”) and “*Revista da Sociedade de Educação*” (“Journal of the Education Society”). We chose these magazines because, although both had a relatively short lifespan, they safeguard fragments of a history of mathematical education in Brazilian primary school. In these journals, Oscar Thompson and Sampaio Doria circulated mainly methodological concepts for the teaching of arithmetic knowledge in primary school. I believe that the analysis of such conceptions reveals configurating elements from different proposals for primary school Arithmetic teaching.

The *A Eschola Publica* journal, published during 1893-1894 and 1896-1897, highlights the diffusion of pedagogical models that should be incorporated into the teaching practices of primary school teachers. Such teaching practices would be configured around the intuitive method. According to Valdemarin and Aparecida Pinto (2010, p. 170), this pedagogical journal “came up with an explicit purpose of meeting the needs of an educational system in the implementation phase [...]”. At the time, the intuitive method adopted was the Pestalozzian, which is based on the perceptions of the senses. According to Souza (2000, p. 13), the first São Paulo Republicans saw this method as one that “[...] could triumph over pedantic, repetitive teaching, rooted in memory and in useless abstractions practiced in the primary schools of the Empire.” The use of

² The topic of the doctoral research are the processes of circulation and appropriation of the intuitive method for the teaching of Arithmetic.

different materials in the teaching of different school knowledge was one of the recommendations touted by the Pestalozzian intuitive method.

In the *Revista da Sociedade de Educação*, published between 1923 and 1924, we notice speeches that sought to characterize a possible infeasibility of the incorporation of a particular pedagogical model in primary schools. With that in mind, this journal served as a social vehicle of demarcation of the educational field. Three topics condensed the demarcations legitimized by Renato Jardim, Arnaldo Barreto, Sampaio Doria, BM Tolosa, and Albino Camargo: 1) the analytical method; 2) the teaching of reading; 3) the so-called “analytical method” in the teaching of reading. In addition to other private objections, the biggest question that mobilized these intellectuals to present the topic was “what is the best name for the teaching method: analytical method or analytical intuitive method?”

In the end, however, how did these nuances on the intuitive method and the analytical intuitive method circulate in Brazil in the late 19th and early 20th centuries, and provide other forms of teaching Arithmetic in primary school? One possible answer can be found in the writings of São Paulo intellectuals of the time, who recommended in the pedagogical journals *what and how to teach* in primary school. I believe that these writings also reveal the appropriations that these ‘men of flesh and blood’ made of pedagogical ideas of intuitive teaching.

2. The teaching of arithmetic knowledge by the intuitive method of Oscar Thompson

Oscar Thompson³ was the director of the São Paulo General Board of Public Instruction in the periods 1909-1911 and 1917-1920. In this position, he wrote the following on the Arithmetic teaching in primary school:

To teach arithmetic logically is to follow abstract ideas of unity and number to develop then all relations of numbers and the processes of calculation.
To teach arithmetic psychologically is to follow concrete objects and show how a number of things can be increased or decreased. (THOMPSON, 1910, p. 166)

The above quote shows that, for him, Arithmetic could be taught in two different ways: logically (starting from the abstract and following the order of knowledge) and psychologically (starting from concrete objects and working with more than one set of knowledge simultaneously). Therefore, it can be said that to teach Arithmetic following the order of contents would mean to work logically – that is, the numbers are taught first, followed by addition, subtraction, multiplication and division, and so forth. Unlike the other proposal, the psychological teaching of Arithmetic was that which made use of concrete objects and did not follow the order of contents – that is, a proposal that simultaneously taught various contents.

³ In 1893, Oscar Thompson took over as Acting Director of the Model School attached to the Normal School. In 1901, he was head of the Board of the Normal School of Praça, remaining in that position for nearly two decades.

Which of these modes did Oscar Thompson adopt? How did he structure a teaching proposal for number and calculation knowledge based on his choice for one of these modes? What pedagogical principles could assist Oscar Thompson in the development of this distinction in the mode of teaching Arithmetic? To do that, we invite the reader to reflect historically with us in order to identify, in other writings of his authorship, elements that could serve as answers to both these questions and the problem herein.

We thus return to the year 1895, following the publication of the book *A Eschola Publica: ensaio de Pedagogia Pratica* (“Public School: Practical Pedagogy Teaching”) by Typographia Paulista, totaling 360 pages. Oscar Thompson, Joaquim de Sant’Anna, Benedito Mario Tolosa and Antonio Rodrigues Alves Pereira organized this work. They elected primary school teachers as their readership aiming to introduce them to the “São Paulo Pedagogy”. According to Monarcha (1999, p. 16), Thompson, Sant’Anna and Tolosa belonged to the “first generation of normalist Republicans.”

The 31 articles that compose that book had already been published in the São Paulo pedagogical magazine *A Eschola Publica* (1893-1894). In general, the articles discuss methodological issues for the teaching of elementary school knowledge, such as Reading, Arithmetic, Language, Lessons of Things, Civics, and others. This reveals that the issues addressed were adjusted based on the educational program. Of this total, four address school knowledge of Mathematics in primary school, or rather, Arithmetic, with the titles: “Elementary Arithmetic” (p. 29-54.) “Multiplication Table” (p. 139-143), “First Drawing Lessons” (p. 145-182), “Arithmetic School” (p. 271-272). For this text, only the first article, authored by Oscar Thompson, will be considered.

Before proceeding to the analysis of the article, we should expose the desires of those who idealized the publication of the book, namely:

We work for the children. All our activity and all our efforts are aimed at them. We do not expect any reward other than the smiles offered by these delicate souls each time we provide them with an easy and nurturing teaching, able to be assimilated by your spirit. Our goal is the children’s interest [...]. (THOMPSON et al 1895, p. 5-6)

This quote makes it clear that the educational proposals presented in the work should take into account child development, based on their interests. In fact, they wish to tell primary school teachers in São Paulo that the goal of primary school was the child, as opposed to the knowledge to be taught, as expressions such as “we work for the children” and “our goal is the children’s interest” should be embedded in the activities and efforts of each teacher. In addition, the quote contains terms that allow us to raise the following questions: How to provide Arithmetic teaching that can be easily assimilated by the child’s spirit? How to awake children’s interest in learning arithmetic knowledge?

According to Oscar Thompson (1895), the study of Arithmetic arouses attention and increases the habit of research. These benefits, however, rely on the teaching method. In the past, he stated, “[...] young children were forced [...] to memorize numbers and multiplication tables” (THOMPSON, 1895, p. 29). Eager to overcome this negative legacy, efforts were made to make teaching more attractive, “[...] so that children could show interest in it since the first lessons” (THOMPSON, 1895, p. 29). How should this teaching be developed, however? The best way, he argued, is to *materialize* the study.

Such indication probably helped Oscar Thompson to argue, years later, that the teaching of Arithmetic adopted by him would use a psychological approach (starting from the concrete), as opposed to a logical one (starting from the abstract).

When proposing a psychological teaching of Arithmetic, Thompson believed that this subject should be taught not following the order of contents, but by teaching different knowledge that worked in the same activity. Thus, to teach Arithmetic psychologically in primary school would mean to articulate the method and processes with the laws of the learning mentality – in this case, the child’s mentality. With that in mind, we have at this point a redefinition of Pestalozzian aphorisms: “I shall proceed from the concrete to the abstract; not the order of the subject, but the order of nature [the child]” (THOMPSON et al., 1895, p. 198).

At the time, Oscar Thompson adopted the pedagogical conceptions of Pestalozzi, which revolved around the intuitive method⁴. This statement is attested by the very São Paulo educator: “São Paulo has always accompanied the educational progress of Europe and the United States, with a downright Pestalozzian organization, in principle, of intuitive teaching, [...]” (THOMPSON, 1935, p. 199). It is thus clear that, among the appropriations of educational advances, Pestalozzian intuitive teaching exerted strong influence on the school organization of São Paulo, in the period when Thompson wrote the article being analyzed. Regardless, these inferences do not yet say much about the appropriations of Pestalozzi’s intuitive method by Thompson in addressing the knowledge of numbers and calculation. Let us look at it more closely.

By indicating that the completion of the study would be the best way to teach arithmetic knowledge, Thompson makes another reflection in methodological terms: “It is crucial that the children do not consider numbers as mere symbols, but as groups of objects” (1895, p. 30). This indication causes a rupture of two past legacies: a) the teaching of numbers should occur by the very materialization of this knowledge, as opposed to symbols/numerals (1, 2, 3, etc.); b) the teaching of numbers should not be separated from the idea of additions and subtractions, or rather groups of objects would provide the art of counting and calculating in the same activity. These two findings are illustrated by an example set by Thompson himself, as follows:

Only then the addition, for example, of four and three, resulting in seven, will not be a verbal memory effort, but an act of insight; as children themselves paint it and understand that the result – seven – has a group of four objects and another group of three objects. (THOMPSON, 1895, p. 30)

The quote makes it clear that learning to count and calculate in the Thompsonian perspective would be an action of perception, as opposed to a “verbal memory effort”. To teach this way, however, the perceptual action would be controlled by processes⁵. In

⁴ According to Pestalozzi, intuition is the foundation of all knowledge produced by sensory impressions of things, or objects of reality, ([1827] 2003, p. 140). In the pedagogical dimension of intuition, sensorial impressions would act in the process of development and formation of an individual’s faculties. The senses would be excited through contact with concrete things. For that reason, Pestalozzi ([1818-1819] 2012, p. 115) argued that education should rely on *things* more than on *words*.

⁵ It is important to know the difference between method and process. According to Felisberto de Carvalho (1909, p. 10), “method, in pedagogical sense, is the means to build the object of education. [...]. Teaching processes are secondary, ordinarily mechanical means that are used to ensure the results of the adopted method.” We notice that, from the pedagogical standpoint, method is the way to choose and

the specific case of the teaching of numbers, he indicates a process, “The first lessons will not exceed the number ten and will consist of the *analysis of the numbers*” (THOMPSON, 1895, p. 30, emphasis added). It is clearly noted that the process is analytical. It should be said that, prior to Thompson’s ownership in relation to the Pestalozzian intuitive method, the analysis method would be used for the teaching of arithmetic knowledge. In summary: the appropriation of Pestalozzi’s intuitive method by Thompson showed that the analytical process would be practiced for the teaching of arithmetic in primary school.

The appropriations of Pestalozzi’s intuitive method by Thompson become clearer by splitting the quote into two parts. A Pestalozzian appropriation is characterized with the lamination of the first lessons based on concrete groups of objects up to the number *ten*. According to Pestalozzi ([1801] 1889, p. 178), the first step in teaching numbers consists of having groups of concrete objects before the child, which he or she could observe and/or manipulate, so that each group represents the amount indicated number, by saying *one, two, three* and so forth, up to *ten*. We should note that, as done by Pestalozzi, Thompson also limited the process of counting to the number *ten*.

Following that, Thompson (1895, p. 30) warns that the first lessons consist of the analysis of the numbers. They contain a Pestalozzian appropriation, as well as the ideas that circulated in the intellectual relations and networks of which Thompson was part. Nevertheless, to understand and notice the appropriation of ideas in circulation, the possible links between Reading, Writing and Counting should be considered in methodological terms.

In that same book in which Oscar Thompson publishes the article under analysis, one of the authors of the work, Antonio Rodrigues Alves Pereira, also authored a text entitled “*Leitura Intuitiva*” (“Intuitive Reading”) (p. 15-28). To comment on the advantages of the process adopted for teaching this knowledge, Antonio Pereira commented: “[...] in intuitive reading, children read what they see, while this does not happen in teaching by other processes” (PEREIRA, 1895, p. 15). To read what is understood, the process admitted consists of talking about the object known by the child, which, in turn, could be concrete or be represented by prints and/or drawings: “Students are engaged in a conversation about any object, provided that it is well known by them” (PEREIRA, 1895, p. 16). To discuss something means to build phrases or sentences based on the chain of words and to analyze an entire object by identifying elements through decomposition and comparison while turning them into words – which form phrases or sentences by composition/linking. Therefore, the process admitted in Antonio Pereira’s “Intuitive Reading” is wording⁶. Based on this process, the author stated: “It is admirable to see how fast young children become able to read, fluently and expressively, small declarative, exclamatory or interrogative sentences, [...]” (PEREIRA, 1895 p. 15). The readings of the elements that make up concrete objects should trigger the formation of words and, later, sentences. Such elements would be revealed by analysis. The same methodological process is suggested by Thompson in

prepare the teaching subject; processes are expedients that articulate immediately the way to exhibit the contents of the teaching subject.

⁶ In Brazil and in the name of pioneering, Antonio Pereira points out: “One could argue that reading by wording started among us in the Model School” (PEREIRA, 1895, p. 27). In Europe, this process had already been advocated by Comenius in the 18th century. On the side of the United States, the adoption of this process was encouraged by Horace Mann (SILVEIRA, 1960, p. 107).

teaching numbers: instead of recognizing isolated symbols, the analysis of objects in groups should guide the teaching of this knowledge.

Still, that does not mean that, in Antonio Pereira's intuitive reading, recognition of isolated letters and syllables would not be practiced. On the contrary, this practice would serve other purposes.

Spelling or word *decomposition* exercises appear in intuitive reading not in order to teach reading, because children guided by a good teacher can learn to read without ever spelling by *comparing* the vocabulary, but in order to teach writing. (PEREIRA, 1895, p. 23, *emphasis added*)

There is, at this point, another appropriation of Pestalozzi by both Oscar Thompson (in Arithmetic) and Antonio Pereira (in Reading). According to Pestalozzi ([1801], 1889), another means for children to learn and retain the idea of numbers lay in the division of words into syllables and letters. Based on a phrase or a small text, the methodological procedure to be followed would be, "how many letters are contained in this word? And, then, how many syllables?" (PESTALOZZI, [1801] 1889, p. 178).

In addition to these possible methodological links in the Read-Write-Count trinity, it should be noted that Antonio Pereira's recommendations for teaching reading were connected with the North American and European innovations of the time. According to Vidal (2000, p. 504), "in the 1880s, the first scientific studies appeared in Europe and in the United States on the act of reading." In continuation, the authors said that these studies resulted in the understanding that reading of words and phrases stood out "[...] more than by recognition of isolated letters [...]" (VIDAL, 2000, p. 504). In the view of this author, this finding prompted Brazilian educators of the nineteenth century to recognize and defend the teaching of reading by the analytical method, whose lesson was in tune with the precepts of intuitive teaching (VIDAL, 2000, p. 505).

Thus, Antonio Pereira's teaching proposal for an "Intuitive Reading" was, in a way, in tune with the methodological advances of the time. This finding reveals that the Europe/Brazil and the United States/Brazil communication circuits were in full operation, aiming at the organization of the primary school unit in terms of teaching methods and knowledge to teach. The method through which these pedagogical ideas circulated in this circuit and arrived in Brazil relied on the mental capacity of those who appropriated them. It should be considered that the characterization of appropriation lies in the reuse and interpretations built on the ideas in circulation. One should not forget that appropriation still comprises a creative practice (CHARTIER, 2014, p. 46). With that in mind, we could say that both Antonio Pereira as Oscar Thompson created their own methods on the pedagogical ideas of their time, whether for those in circulation in other countries or those disseminated in Brazil. At this point, we should continue with the analysis of Thompson's text to see which meanings he built on the intuitive method to address the knowledge of numbers and calculation in São Paulo primary schools.

Following his defense of the study of Arithmetic by concreteness in primary school, i.e., what should be taught first, Thompson offers teachers a proposal with two lessons under the title: "*Preparação da classe*" ("Class Preparation") (Thompson, 1895, p. 30). Class preparation rituals for the concrete study of Arithmetic would be: teacher and student standing by a table; teacher stands at one of the ends; several concrete objects and

pictures are placed in the middle of the table – “so the class will be prepared for the lesson’ (THOMPSON, 1895, p. 30). The indications for the first lesson would then be:

Invite the children to take one tablet from the middle of the table.

- João, how many tablets do you have?

- I have one tablet.

- If you take one tablet from one tablet, how many are there?

- How many tablets does one tablet have?

- One and one tablet, how many tablets are there?

- One pencil taken from one pencil, how many pencils are there?

- One taken from one?

- How many apples does an apple have?

- How many ‘ones’ does ‘one’ have?

- One pear, how many pears are there?

- One ‘one’, how many ones are there?

The result should be as follows:

- One from one is none.

- One ‘one’ is one.

- ‘One’ has one ‘one’.

The child is thus able to count, add, subtract, multiply and divide by *one*. (THOMPSON, 1895, p. 30-31, *emphasis added*)

In general, it is understood that the first count and calculation practices would be given simultaneously, i.e., within the same activity. In particular, the lesson is that, initially, the teaching and learning of this arithmetic knowledge would be oral, or rather through the conversation process developed by the teacher and the student based on the concrete thing. Conversation is a type of *game* that is configured between questions and answers. According to Pestalozzi, one of the ways to exploit knowledge from concrete objects is by establishing “a conversation *with* the child” ([1818-1819] 2012, p. 120, *emphasis added*). With that in mind, what “rule” sets the beginning of the conversation? Pestalozzi states: “Instead of listening and repeating, one should observe and think” (PESTALOZZI, [1818-1819] 2012, p. 120). Such is the Oscar Thompson’s proposal: to make the children look at the object and answer the questions made based on it – which implies observing and thinking. From this perspective, to make the child think to count and calculate is to have concrete objects at his or her eyes and then practice a dialogue. As regards the child, the mental operation is essentially inductive⁷ and analytical⁸ at the same time. That is the foundation of Thompson’s recommendation that the materialization of the study of numbers should be analytical (1895, p. 30).

This same methodology is indicated for the second lesson, which would work, alongside the child, the number *two* in the art of counting and calculating concrete objects. The teacher always teaches this knowledge by exploring the conversation: the child sees the objects, hears the question, intuitively and mentally analyzes them, and, then, responds. This is the dynamics of materialization and intuition of the teaching of

⁷ The word comes from Latin *induction*, meaning a driving action, which comes from the Greek translation *epagôgè*. According to Blay (2006, p. 426), “induction is the intellectual process that is founded upon generalizations about observable facts, [...]” Specifically, induction is a “mental operation consisting of passing the findings of one or more facts to the law of all the facts of the same nature” (BLAY, 2006, p. 426).

⁸ The word comes from Greek, resulting from the combination of *ανα* (aná) with *λυσίς* (lusi). In a more precise sense, the term analysis is the decomposition (lusi) of a whole in equal parts (aná) (ECHAVARRÍA et al, 2010, p. 24-25).

numbers and calculation. The question makes the child think, allowing his or her higher faculties⁹ to remain active.

3. The teaching of arithmetic knowledge by Sampaio Doria's intuitive analytical method

Active in educational issues, Sampaio Doria¹⁰ became a member of the Education Society, established on November 23, 1922. São Paulo intellectuals such as Oscar Freire, Almeida Junior, Renato Jardim, Fernando de Azevedo and Lourenço Filho were also members of this Society. On August 10, 1923, the Society published a journal carrying its own title. As was the case with all other printed journals, the *Revista da Sociedade de Educação* disseminated pedagogical conceptions of the members of the group that published it. After all, as Ana Nery (2009, p. 25) said, “pedagogical journals are the main outlet for circulation of the ideas advocated by the groups responsible for the publications.”

Revista da Sociedade de Educação “reflected a project for teacher training and improvement of national education, apparently more structured than previous journals. In short, the proposal [of this journal] was to provide its readers with a pedagogical guidance” (NERY, 2002, p. 3). Following an analysis of the summary of each of the nine issues of the journal, it could be argued that this proposal became a reality.

At the Education Society, Sampaio Doria took different positions such as Secretary General and Vice President. He was also the author of several articles on *Revista da Sociedade da Educação*, which had a relatively short duration: from August 10, 1923 to December 10, 1924, publishing nine issues in three volumes.

In the edition of this journal published on October 10, 1923, Volume I, Number 2, then Dr. Sampaio Doria wrote an article entitled “*Aplicações didáticas: método no ensino da Matemática*” “Didactic Applications: Method in Teaching Mathematics”. I believe that the analysis of this article reveals the arithmetic knowledge teaching recommendations proposed by Doria. This belief is confirmed in the first lines of the text, when the pedagogue assumes three educational standards for the teaching method advocated by him – the analytical intuitive method:

- 1 – the students should observe the realities in which they learn;
- 2 – the teacher should establish which and in what order the realities to be taught succeed;
- 3 – the teacher should proceed to the students’ observation, so that they may softly acquire new knowledge by the laws of analysis. (DORIA, 1923, p. 160).

By continuing with the exposure of methodical guidelines to be applied didactically in school knowledge, Doria announces two degrees of intuition for its analytical intuitive

⁹ Pestalozzi ([1827] 2003) highlights the child’s lower and higher faculties and how they can be developed through application of the intuitive method.

¹⁰ On April 29, 1920, Sampaio Doria took over as Director of Public Instruction of the State of São Paulo, at the invitation of then President Washington Luis. His tenure did not last long, as on May 2, 1921, he delivered “his letter of resignation to the Secretary of the Interior” (MEDEIROS, 2002, p. 1).

method: immediate intuition, which is characterized by the actual contact of the student with the concrete objects; and mediated intuition, comprising a methodological way evoked by the teacher to allow students to recognize the impressions acquired by the immediate intuition, so that the combination of impressions generates the idea of an unknown object – this process should not rely on the support of the concrete thing, but rather the word (DORIA, 1923, p. 160).

It is not possible to infer the author(s) through Sampaio Doria's appropriation of these two concepts of intuition, as the article provides no references. Even so, Doria's position allows us to raise a few names. The first is John Locke. In his monumental book "An Essay Concerning Human Understanding," written in 1690, Locke argued in Chapter II of Book IV that the immediate intuitive knowledge should be one that is acquired without the intervention of any faculty other than one's own intuition. He referred to this form of gaining knowledge as immediate intuition because "the mind does not have to struggle to prove or examine [its perception]" (LOCKE, 1999, p. 217). He called referred knowledge that is mediated by the juxtaposition of ideas acquired by immediate intuition as demonstrative. That type of knowledge is characterized when the "mind cannot gather its ideas through immediate comparison" (LOCKE, 1999, p. 218).

Brazilian author Felisberto de Carvalho, in his book "*Tratado de Metodologia*" ("Treaty on Methodology"), published in 1909, introduced two types of intuition capable of being applied in teaching: immediate intuition and mediated intuition. He said that intuition is "*immediate* when it is obtained by the presentation of the object itself and *mediated* when it is achieved by a representation of the object [...] or by imagining" (CARVALHO, 1909, p. 118, *emphasis added*). From the writings of Locke and Carvalho, we can assume that Sampaio Doria appropriated the pedagogical guidelines of at least one of these books. With that assumption, let us return to the analysis of Doria's article, built under the aegis of his analytical intuitive method.

By taking three topics from arithmetic knowledge and one from geometric knowledge, Doria shows how such knowledge would be contemplated in his method. Let us take the example of the teaching of the Multiplication Table. We could perhaps assume that Doria had no special reason for choosing the Multiplication Table, as this pedagogical instrument/knowledge is present in primary school environments and brings an evil with it: learning by rote. Seeking to inhibit the progress of this evil and announce other learning practices for the Multiplication Table, Doria says: "As is the case with everything, multiplication tables are taught by a single teaching method, which is analytical intuition" (DORIA, 1923, p. 161). But how could the methodological action of the teaching method devised by Sampaio Doria be implemented? According to Marta Carvalho (2013, p. 53), "the analytical intuition method was, in his view [i.e., in the opinion of Sampaio Doria] fully in accordance with children's evolutionary psychology, understood similarly to him or him as a passage from the syncretic vision to the synthetic vision, mediated by analytical processes."

Even so, the question is: how should the teacher use the analytical processes in teaching the Multiplication Table? and how should the student analytically learn the Multiplication Table? The answers to both questions can be drawn from Doria's recommendations, considering their teaching proposal for the Multiplication Table. The proposal is as follows:

It consists of teaching that three times five is fifteen.

The teacher should begin by presenting to the students pins, corn kernels, coffee beans, and other small objects. The children organize three groups of five pins and count them. They note that 3 groups of 5 pins each make 15 pins. They repeat the observation with coffee beans, pebbles, and beans. Three groups of 5, of whatever it is, always make 15. That is, 3 times 5 is 15 (DORIA, 1923, p. 161).

The quote shows that the teacher should first equip him- or herself with concrete objects and arrange them to the child. In this case, the teacher's "task" would put in motion the immediate intuition defended by Doria: the student in real contact with objects. In turn, the student/child should take the group of fifteen objects and decompose it into three groups of equal parts. Guided by comparison, the child would observe that each of the three groups contains five objects. This observation would be confirmed and clarified in the counting of objects. Still, the practice of counting would reveal not only the total quantity but also the process of composition of the group: as pointed out by Doria, by bringing together the objects of the three groups through the practice of counting, the child would have the opportunity to verify, intuitively and analytically, that 3 groups of 5 pins equal 15 pins.

The teacher's participation would not only comprise the "offering" of objects at the child. By taking the third standard presented by Doria, the teacher could refer to other observations by the child, allowing him or her to develop what he referred to as mediate intuition. If in the example announced above, what was in question was the basic idea of the number fifteen, using multiplication, based on 3 groups of 5 objects, the teacher could then exploit that 5 objects multiplied by 3 would result in the same amount. We notice that using the same object distribution structure would be enough to reverse the order of contemplation of parts so that the child invokes unknown combinations: he or she would already know 3 times 5, as this was the initial process; now, 5 times 3 would be a way to invoke unknown combinations.

In addition to the methodological action proposed by Doria in the teaching of the Multiplication Table, we clearly notice that the use of the analytical intuitive method in the perspective announced by this pedagogue would be featured in the passage from the concrete to the abstract: using 3 groups of 5 pins was the most intuitive way to teach 3 times 5. Instead of numbers (digits) and signs (+, -, x, ÷), concrete objects would be used. Continuing with the analysis of Doria's teaching proposal, we read that, with this process, the study of the Multiplication Table does not need to consider the sequence of practices of calculation: first, addition; then subtraction; then, multiplication; and finally, division. For him, it would be teacher's duty to give explore alongside the child, during the teaching of the Multiplication Table, the practice of addition, subtraction, multiplication, and division (DORIA, 1923, p. 162).

Sampaio Doria also used his intuitive analytical method to make teaching recommendations for other subjects of primary school, such as the Reading and Writing. Also on *Revista da Sociedade de Educação*, in the issue of August 10, 1924, Volume III, Number 7, he wrote the article entitled "*O ensino da Leitura*" ("The Teaching of Reading"). In this text, he points out that the teaching of Reading should be addressed based on the analytical intuitive method, and that the starting point would be the reading of sentences – that is, reading full sentences instead of starting by reading words, syllables, and letters (DORIA, 1924, p. 23). Further, he makes the following

reflection: “Writing follows reading” (p. 32). One wonders: How does Writing follow Reading? The answer lies in the method and teaching process. For Doria, it is by writing that one learns to read. In terms of method and teaching process, he says, “one should not begin, for example, by copying dashes, letters and syllables. [...]. As [the child] mentally analyzes sentences, [he or she] can keep writing, as desired, syllables and letters” (DORIA, 1924, p. 34).

Through the two articles by Sampaio Doria, both published on *Revista da Sociedade de Educação*, the possible articulations of the Read-Write-Count triad become apparent, based on his analytical intuitive method were. It also became apparent that this articulation is characterized in terms of method and not processes, as he indicated the sentencing process for the teaching of Reading and Writing and proposed the immediate and mediate intuition processes for the teaching of arithmetic knowledge.

4. Not to conclude, a few remarks...

Given what this text presents, it could be argued that the recommendations for addressing the arithmetic knowledge were based on the teaching method. These recommendations also provided a link between the Read-Write-Count triad.

From the standpoint of Arithmetic, we saw that, by appropriating Pestalozzian pedagogical concepts, Oscar Thompson recommended the teaching of numbers and calculation based on the intuitive method. From the analysis of Thompson’s article, it was noticeable that the teaching of numbers would be addressed alongside the teaching of calculation. Those would be in line, as both sets of knowledge should initially be studied by the *materialization*, i.e., with the aid of concrete objects. It should also be simultaneous, as numbers and calculation should also be addressed in the same activity – this implies that, in the intuitive teaching proposed by Oscar Thompson, more of one set of knowledge could be taught simultaneously, as was the case with these sets.

Oscar Thompson’s article revealed that the Pestalozzian intuitive method already presented a teaching proposal based on groups of objects. The recognition of each object in this group should occur by the analysis process. This finding shows us that, based on Thompson’s appropriation, the Pestalozzian intuitive method had an analytical teaching foundation, even prior to the existence of the discussion surrounding the analytical intuitive method.

In turn, the two articles by Sampaio Doria do not explicitly show his appropriations. Even so, it was revealed that the analytical intuitive method advocated by him resulted in the development of methodological steps for teaching the Multiplication Table. In the methodology presented by Doria and analyzed here, we saw that fifteen concrete objects comprised the unit and, above all, the starting point of the analytical method for intuitive visual recognition of the concept of the amount of 15; the distribution of this unit into groups revealed that the analytical procedures that characterized this amount based on the regrouping of the objects present into groups in terms of calculation, i.e., in terms of multiplication. It can be inferred that the first calculation learning would not lie in memorization of the Arabic symbols (1, 2, 3, 4, etc.) or the fundamental signals (+, -, x, ÷, =), but rather the recognition of a set of concrete things resulting from an addition,

a subtraction, a multiplication, or a division of objects. The teaching of calculation, or at least multiplication, by the intuitive analytical method, had a methodological proposal: it began with the exposure to a set of objects; this amount was then decomposed in equal parts; it continued with a conversation on each group in order to confirm equal quantities by counting, comparison and symbolic representation of the amount; finally, the quantities were regrouped by indicating the ordering of the symbols in terms of number: in the example provided, the number fifteen consists of three groups of five objects. This number also consists of ten plus five units.

Partial results provided here reveal the need to know better how teaching methods and processes are characterized in the education of Brazilian children during the times of the intuitive space. Still, these results show us that we should not admit a prior definition of both method and process in the case of primary school subjects. In the specific case of Sampaio Doria's intuitive analytical method, we noticed that the same teaching method designed to guide the Read-Write-Count triad admitted distinct processes. Apparently, school subject teaching method and process characterizations rely on the appropriations of his principles. This highlights the need for studies on other items considering the methodological guidelines for teaching. These studies certainly reveal more closely the efforts performed in the articulation of primary school methods, processes, and subjects.

5. References

- Blay, M. (2006) *Dictionnaire des concepts philosophiques*. Paris: Larousse.
- Carvalho, F. (1909) *Tratado de metodologia*. 3. ed. São Paulo: Livraria Francisco Alves;
- Carvalho, M. M. C. (2013) Estratégias editoriais e territorialização do campo pedagógico: um livro de Sampaio Doria sob a pena do editor da biblioteca de educação. *Revista História da Educação* (Online). Porto Alegre, v. 17, n. 39, p. 39-56, jan./abr.
- Chartier, R. (2014) *A mão do autor e a mente do editor*. 1. ed. Tradução George Schlesinger. São Paulo: Editora UNESP.
- Doria, A. S. (1923) Aplicações didáticas: methodo no ensino da mathematica. *Revista da Sociedade de Educação*. São Paulo, v. I, n. 2, p. 160-173, out.
- Doria, A. S. (1924) O ensino da Leitura. *Revista da Sociedade de Educação*. São Paulo, v. III, n. 7, p. 14-54, ago.
- Echavarría, J. D. L. et al (2010) *El Método Analítico*. Colômbia: Universidade de Antioquia.
- Locke, J. (1999) *Ensaio acerca do entendimento humano*. Tradução Anoar Aiex. São Paulo: Editora Nova Cultural.

Medeiros, V. A. (2002) Antonio de Sampaio Doria nos debates educacionais: A longa duração de uma “presença ausência”. *II Congresso Brasileiro de História da Educação: História e memória da educação brasileira*. Natal, RN: p. 1-11, nov.

Monarcha, C. (1999) *Escola Normal da Praça: o lado noturno das luzes*. Campinas, SP: Editora da Unicamp.

Nery, A. C. B. (2002) *(In)Formando, Divulgando e Educando: uma década de imprensa periódica em São Paulo*.

Nery, A. C. B. (2009) *A Sociedade de Educação de São Paulo: embates no campo educacional (1921-1931)*. São Paulo: Editora Unesp.

Pereira, A. R. A. (1895) Leitura Intuitiva. In: THOMPSON, Oscar *et al.* *A Eschola Publica: Ensaio de Pedagogia Pratica*. São Paulo: Typographia Paulista, p. 15-28.

Pestalozzi, J. H. ([1818-1819] 2012) *Cartas sobre educación infantil*. 3. ed. Tradução de José María Quintana Cabanas. Madrid, España: Editorial Tecnos S. A.

Pestalozzi, J. H. ([1827] 2003) *El canto del cisne*. Tradução de José María Quintana Cabanas. Barcelona: Editorial Laertes.

Pestalozzi, J. H. ([1801] 1889) *Cómo Geetrudis enseña a sus hijos: fines y métodos de la educación del Pueblo*. Cartas dirigidas a Gésser. Tradução José Tadeo Sepúlveda (versão chilena).

Silveira, J. (1960) *Leitura na Escola Primária*. São Paulo: Empresa gráfica da “Revista dos Tribunais”.

Souza, R. F. (2000) Inovação educacional no século XIX: A construção do currículo da escola primária no Brasil. *Cadernos CEDES*. Campinas: ano XX, n. 51, p. 9-28, nov.

Thompson, O. (1910) *Anuario do Ensino do Estado de São Paulo*, referente aos anos 1909 e 1910. São Paulo: Typographia do “Diário Oficial”.

Thompson, O. (1895) *Arithmetica Elementar*. In: _____. et al. *A Eschola Publica: Ensaio de Pedagogia Pratica*. São Paulo: Typographia Paulista, p. 29-54.

Thompson, O. (1935) *Escola Nova*. *Revista de Educação*. São Paulo, ns. 9 e 10, vols. IX e X, mar.-jun., p. 196-203.

Valdemarin, V. T.; Pinto, A. A. (2010) Das formas de ensinar e conhecer o mundo: lições de coisas e método de ensino intuitivo na imprensa periódica educacional do século XIX. *Revista Educação em Questão* (Online). Natal: v. 39, n. 25, p. 163-187, set./dez.

Vidal, D. (2000) *Escola Nova e processo educativo*. In: LOPES, E. M. T.; FARIA FILHO, L. M.; VEIGA, C. G. (Orgs.). *500 anos de Educação no Brasil*. 2. Ed. Belo Horizonte: Autêntica, p. 497-517.