MIDDLE SCHOOL STUDENTS FIRST EXPERIENCE WITH MATHEMATICAL MODELING

EXPERIÊNCIA DE ESTUDANTES DO ENSINO FUNDAMENTAL II COM A MODELAGEM MATEMÁTICA

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Received: 19 April 2017
Accepted: 22 June 2017

ABSTRACT
Internationally, mathematical modeling is garnering more interest because of the many benefits of this approach. There are still many students though that have not had any experience with mathematical modeling. Previous research has shown that students can have difficulties with their first experience with mathematical modeling. This study used the prior research to structure effectively an implementation method to enable middle school students to be successful in their first experience with mathematical modeling. It was ensured that students understood the problem context, group work skills were discussed, cooperative learning was used, and all groups were able to share their work with the whole class. A mathematical modeling activity that had been previously been implemented with quality results was also used. Implications for researchers and teachers are discussed to help students be successful in their first modeling experience.

Keywords: mathematical modeling; middle school; model-eliciting activities.

RESUMO
A modelagem matemática está atraindo mais interesse internacionalmente por causa dos muitos benefícios dessa abordagem. Ainda há muitos estudantes que não tiveram experiência alguma em modelagem matemática. Pesquisas anteriores mostraram que os alunos podem ter dificuldades com sua primeira experiência com modelagem matemática. Este estudo utilizou a pesquisa anterior para efetivamente estruturar um método de implementação para permitir que alunos do ensino fundamental 2 tenham sucesso em sua primeira experiência com modelagem matemática. Assegurou-se que os alunos entendessem o contexto do problema, as habilidades de trabalho em grupo foram discutidas, a aprendizagem cooperativa foi usada e todos os grupos conseguiram compartilhar seu trabalho com toda a classe. Uma atividade de modelagem matemática que tinha sido implementada anteriormente com resultados de qualidade também foi usada. As implicações para pesquisadores e professores são discutidas para ajudar os alunos a ter sucesso em sua primeira experiência de modelagem.

Palavras-chave: modelagem matemática; ensino fundamental; atividades de obtenção de modelos.

1. Introduction

Implementing mathematical modeling in elementary education has been found to be relevant to help prepare students for the competences required by the dynamic, global and technology-based economy of the 21st century. Mathematical modeling develops students' communication, teamwork, and presentation skills (English & Watters, 2005), as well as their mathematical knowledge through different representations (Stohlmann, Moore, & Cramer, 2013).

A number of students have not had experience with mathematical modeling, though, due, in part, to the lack of emphasis on the theme during teacher education (Biembengut & Hein, 2010; Doerr, 2007), so that teachers have incorrect or incomplete understandings of it (Anhalt & Cortez, 2015; Gould, 2013; Tekin, Kula, Hidiroglu, Bukova-Guzel, & Ugurel, 2012). For both teachers and students it is imperative that they have positive first experiences with mathematical modeling. If teachers see the benefits of mathematical modeling, they will be more likely to implement it in their classrooms. In this way, students will see how mathematics can be applicable, enjoyable, and that everyone can do mathematics, which can motivate them in all aspects of their mathematics class.

This study investigated if middle school students could be successful in their first experience with mathematical modeling. Prior research has shown that students have difficulties with mathematical modeling when they handle it for the first time (Biccard & Wessels, 2011; Cheng, 2013; Gould & Wasserman, 2014; MaaB & Mischo, 2011). To improve students' possibilities of success, I used a well-designed mathematical modeling activity (Lesh & Doerr, 2003). I also supported students with positive messages and ideas (Stohlmann, 2017) at different moments of the implementation of the mathematical modeling activity. Finally, the structure of the mathematical modeling activity implementation, including group work and group presentations helped to make students' success more likely (Stohlmann, DeVaul, Allen, Adkins, Ito, Lockett, & Wong, 2016).

2. Models and Modeling Perspective (MMP)

The theoretical framework that guided this research is the Models and Modeling Perspective (MMP). The MMP has been a powerful framework for research on the development of the interaction between students and curricula resources. In this perspective, models are seen as "conceptual systems (consisting of elements, relations, operations, and rules governing interactions) that are expressed using external notation systems, and that are used to construct, describe, or explain the behaviors of other system(s)—perhaps so that the other system can be manipulated or predicted intelligently" (Lesh & Doerr, 2003, p.10).

MMP is based on the idea that students do not only engage their mathematical understandings in solving problems, but also their beliefs, values, and feelings (Lesh, Carmona, & Moore, 2009).

A specific type of mathematical modeling activity has been developed based on the MMP called Model-Eliciting Activities (MEAs). MEAs are client-driven, open-ended realistic problems in which students can develop mathematical understandings. MEAs are developed based on six principles (Table 1) to ensure that students apply their mathematical knowledge successfully.

Table 1. *Principles for Guiding MEA Development*

Principle	Description
Model Construction	Ensures the activity requires the construction of an explicit description, explanation, or procedure for a mathematically significant situation
Generalizability	Also known as the Model Share-Ability and Re-Useability Principle. Requires students to produce solutions that are shareable with others and modifiable for other closely related situations
Model Documentation	Ensures that the students are required to create some form of documentation that will reveal explicitly how they are thinking about the problem situation
Reality	Requires the activity to be posed in a realistic context and to be designed so that the students can interpret the activity meaningfully from their different levels of mathematical ability and general knowledge
Self-Assessment	Ensures that the activity contains criteria the students can identify and use to test and revise their current ways of thinking
Effective Prototype	Ensures that the model produced will be as simple as possible, yet still mathematically significant for learning purposes (i.e., a learning prototype, or a "big idea" in mathematics)

(Lesh, Hoover, Hole, Kelly, & Post, 2000)

Teachers can use MEAs with an implementation model that helps to maximize student learning. First, students read an opening article or watch a video that helps them to become familiar with the problem situation. Then, they are requested to answer readiness questions meant to highlight important aspects of the reading or video that relate to the problem that they will solve. The teacher then leads a whole class discussion on the readiness questions and the problem statement that describes about what students will develop their models. Next, students work in groups on the problem, and then present their solution to the rest of the class. Finally, groups have time for revisions and reflection on the mathematics that they used, and how well they did working in a group. Besides studies done with MEAs, other research on mathematical modeling has stated the importance of students working in groups and sharing their ideas with the whole class (Albarracin & Gorgorio, 2013).

3. First Experiences with Mathematical Modeling

Research has shown that students have difficulties with mathematical modeling in their first experience. MaaB & Mischo (2011) scored German middle school students' work on two modeling problems based on six steps of the modeling cycle with a scale of 0 to 5 points for each step. Students scored an average of 1.99 on the six modeling steps. In the U.S., a study with 7^{th} to 9^{th} graders found that the students frequently created models that either oversimplified or overcomplicated, and had difficulty in choosing the most important variables and assumptions for their models. The students worked on a problem to decide which gas station is the best to buy gas from (Gould & Wasserman, 2014). In this study, students did not share their solutions with the whole class. Also, the problem statement could have been discussed more for students to better understand the realistic situation. In Singapore, a study done with one class of students age 14-15 found that it was hard for students to apply their mathematical knowledge in the real world modeling problem, in part due to their difficulties in working in a group. The problem had students design the layout of car park spaces. The implementation of this modeling problem was not done well, as the teacher did most of the talking. It was also suggested that a better designed modeling task might work better (Cheng, 2013).

Two studies done with MEAs showed that not all groups might develop successful models, but modeling abilities improve over time. Aliprantis and Carmona (2003) implemented the Historic Hotels MEA with U.S. 7^{th} grade students, in which students have to determine the best price for hotel rooms to maximize profit. About half of the groups had an acceptable model, and all students understood the context and what the problem was asking. In South Africa, one class of students worked on 3 MEAs. Students initially displayed weak competencies in all areas of modeling, but these developed slowly and gradually (Biccard & Wessels, 2011).

The MEA used in this study was modified from a MEA previously done with middle school students: the Big Foot MEA. In using the Big Foot MEA, Lesh and Doerr (2003) found that average ability U.S. students can progress through multiple modeling cycles, and progress through stages of development of constructs that have been observed over periods of several years. However, these results were seen after students had already participated in several MEAs. I wanted to investigate in this study the best structure to support students to be successful in their first experience with mathematical modeling. Learning from past research, I ensured that students understood the problem context, that the teacher discussed group work skills, employed cooperative learning, and that groups were able to share their work with the whole class. I also selected a well-designed modeling activity that had been successful previously, and supported students with important messages and ideas at different parts of the modeling implementation.

4. Methods

This study was conducted with 19 middle school students (age 11-13) that voluntarily enrolled in a Saturday STEM program at a large research university in the Southwestern part of the United States. The students were from a large urban school district. The purpose of the Saturday STEM program was to provide a series of inquiry experiences designed to provide interesting and exciting opportunities in STEM education. The program lasted five Saturdays and the results from this study are taken from the first Saturday of the program. The students did not have prior experience with mathematical modeling before participating in this program.

Before the students started any part of the MEA, the teacher went over a list of messages and questions that are important at different parts of mathematical modeling (Table 2). The teacher wanted to prepare students for the mathematical modeling experience so that they would be more likely to be successful and to provide support to them. Students kept this information with them throughout the modeling activity, and the teacher reminded students of the messages at each stage of the modeling implementation.

Table 2. *Messages or questions for students when doing mathematical modeling*

Before mathematical modeling
There is more than one right answer to this problem. There is not one type of person that is the best at mathematical modeling. Everyone can contribute. Make sure everyone in your group understands your solution. Use multiple ways to demonstrate your solution: pictures, graphs, symbols, words, or equations.
During mathematical modeling
Keep in mind what the problem is asking you to do. Make sure everyone in your group understands your solution. Does your solution make sense in the realistic situation? Can your solution be improved? Is your mathematics correct?
Before group presentations
Listen carefully to each group and think of a question to ask them. Try to see if there is anything from a group that you can use in your solution. Look to ensure that each group's mathematics is correct.
After group presentations
After hearing from other groups' ideas, can our solution be improved? Is there any feedback we received to improve our solution? Was our solution clearly explained?
After mathematical modeling
What mathematics did my group use in our solution? How well did I understand the mathematics that was used? How well did I do working in the group?

(Stohlmann, 2017)

The MEA that the students completed for this study was the Bigfoot MEA, which was a modification of the Big Foot MEA (Lesh & Doerr, 2003). The general structure of the MEA was kept the same but the realistic context was changed. In the modified MEA, students watch a video that describes the many different facets of how footprints are used in different fields by forensic scientists, professional trackers, and scientists. Students answer the following readiness questions:

(1) What clues or evidence can scientists and professional trackers get from footprints?
(2) Can scientific knowledge change over time? Explain
(3) How is mathematics used to give us a better understanding of the natural world?

Students then read the problem statement in which the client for this MEA, the Northern Minnesota Bigfoot Society, wanted help to determine the possible height of Bigfoot or Sasquatch based on footprints that they have found. Table 2 contains the problem statement. Materials were made available for the groups to use, including rulers, string, scissors, graph paper, graphing calculators, and laptops.

Table 3. *Bigfoot MEA problem statement*

Problem Statement
The Northern Minnesota Bigfoot Society would like your help to make a “HOW TO” TOOLKIT; a step-by-step procedure, they can use to figure out how big people are by looking at their footprints. Your toolkit should work for footprints like the one that is shown on the next page, but it also should work for other footprints.

Complete MEA available at https://unlvcoe.org/meas/

Before beginning the MEA, students watched a video on how to make effective group decisions to support students’ social skills when working in a group (FlowMathematics, 2012) as suggested by cooperative learning research (Johnson, Johnson, & Smith, 2007). The cooperative learning strategy of numbered heads was also used. In this strategy, the teacher randomly picks one student from each group to present, so that all students are more likely to be engaged and prepared.

This study follows naturalistic inquiry (Patton, 2002) with the lens of the Models and Modeling Perspective (Lesh & Doerr, 2003). The data included students’ written work, audio recordings of each group and the whole class discussions, and photographs of students’ work on whiteboards. All audio recordings were transcribed. Memos (Corbin & Strauss, 2008) were written based on the data to describe each group’s solution development.

5. Results

Each group’s final solution and solution process will be described in this section. For each group, the students in the group will be designated as S1- student one, S2-student two, S3-student 3, or S4-student 4.

5.1 Group one

This group came up with a general notion of how to approach the problem, but took some time to figure out what specific steps to take. Initially, they had some discussion around looking at the Bigfoot footprints and then decided to measure the length of one of the footprints. They got several different answers for this, ranging from 15 inches to 15 and one fourth, to 15 and one half. The students tried to use two of the same rulers to help get a more accurate measurement. They also discussed if one of the footprints that they were given was different from the other one, but decided they were equivalent. They ended up using 15 and one-fourth inches as their measurement, though the foot length was 15-and-a-half inches.

During this discussion, student one started to put forth an idea of what they could do. *"What if we measure our own foot size and then we measure how tall we are and then see if it is the same thing. Then if it is the same thing"* (student did not finish the thought). At this point, none of the other students responded and continued to discuss measuring the Bigfoot footprints. After a few minutes, student two tried to put forth an idea. *"What if measure our height and if it is the same amount wouldn't it be. Like so, say your foot is 10 inches or no."* After this, student three went back to the problem statement and read it aloud. *"We need to make a step-by-step procedure to figure out how tall people are by looking at their footprints."*

Student one, then, directed the group on what to do, though was met with some resistance on how this would help.
S1: "We should measure our feet and then measure our height."
Two students then responded:
S4: "How are we supposed to use the information to figure out how tall Bigfoot is?"
S3: "I have no clue."
Student one went ahead with the measurements, but it was clear not everyone still understood what they would do with these measurements.
S4: "I still don't get how we are supposed to use feet to measure height."
S1: "You are basically just going to say what it is based on the footprint.
S3: "He is 15 and ¼ feet?"
Student three thought that the foot length in inches would translate then to the height in feet.

Student one, then, got the foot length and height of student 4, but had to think for a few minutes on what to do with this information.
S1: "What is the equation again?"
A few minutes passed.
S1: "So basically I want to get a fraction. I remember how to get the equation. You guys know how to do this right?" (sets up a proportion)
S4: "So cross multiply"

The final solution that this group described is shown in Figure 1. When they presented, they had not yet figured out the height of Bigfoot from their proportion but added later that Bigfoot would be around 15 feet tall. Though calculators were available, this group solved the proportion by hand and incorrectly multiplied 59 inches by 15.25 inches to get 137.25. They knew a correct method for solving the proportion, but did the math incorrectly.

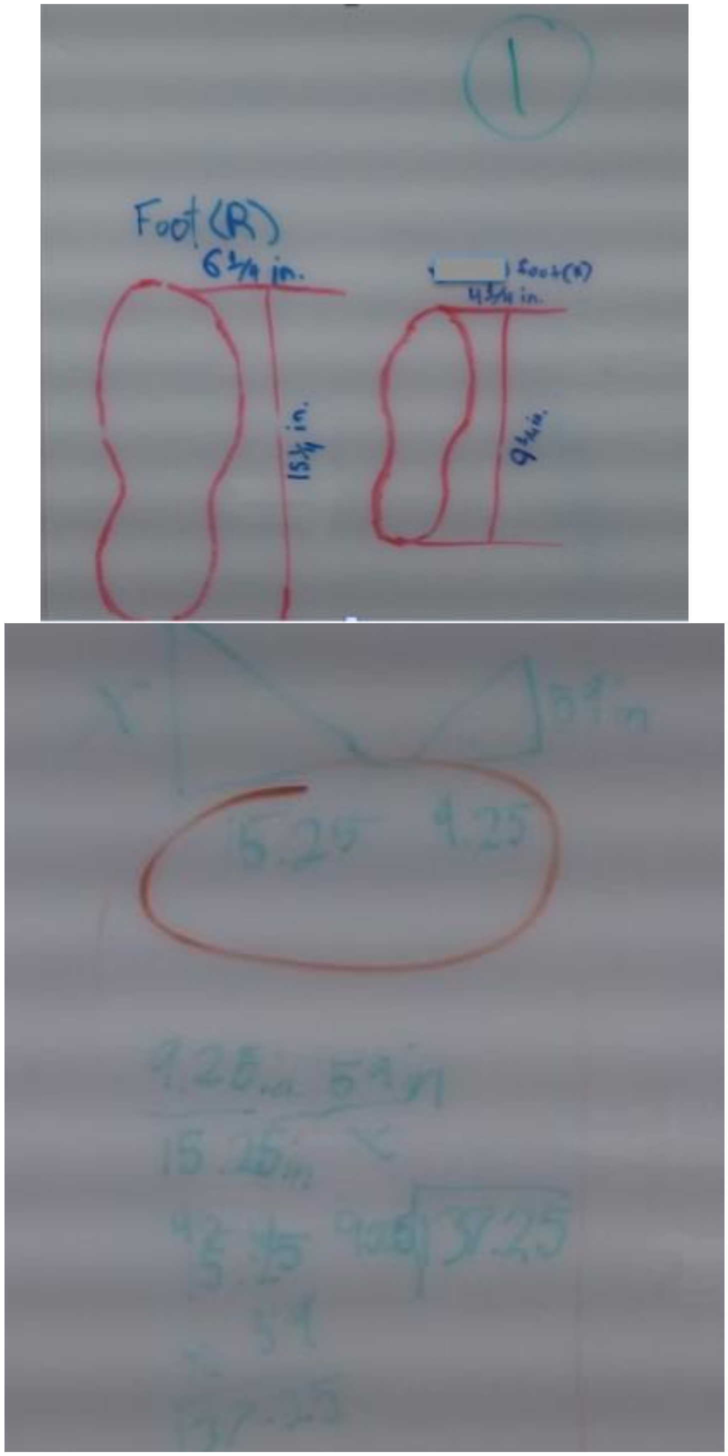

Figure 1. *Group one's final solution*

5.2 Group two

Group two went right to the Internet to help them with their solution, and were able to quickly identify a method that they were able to check was appropriate. They found the equation x divided by .15. Initially, they had measured the Bigfoot footprint at 15 inches, but when asked by the instructor to measure again, they came up with 15-and-a-half inches. After the students got the equation, student one questioned, "*What if the maximum is the width?"* Student two and three quickly responded that the website says to use length, and not width.

The instructor questioned the students to see why they would divide by .15 in the equation, and the group provided a few responses. They tried their own foot lengths in the equation to the see if the height was accurate, and also referenced information given to the students before working on the problem, that Bigfoot has been estimated to be between 6 and 10 feet.
S2: "Because it says so on this website. And it seemed legit and it actually worked out.
S3: "The paper says it is between 6 and 10 feet."
Group 2's final solution is shown in Figure 2.

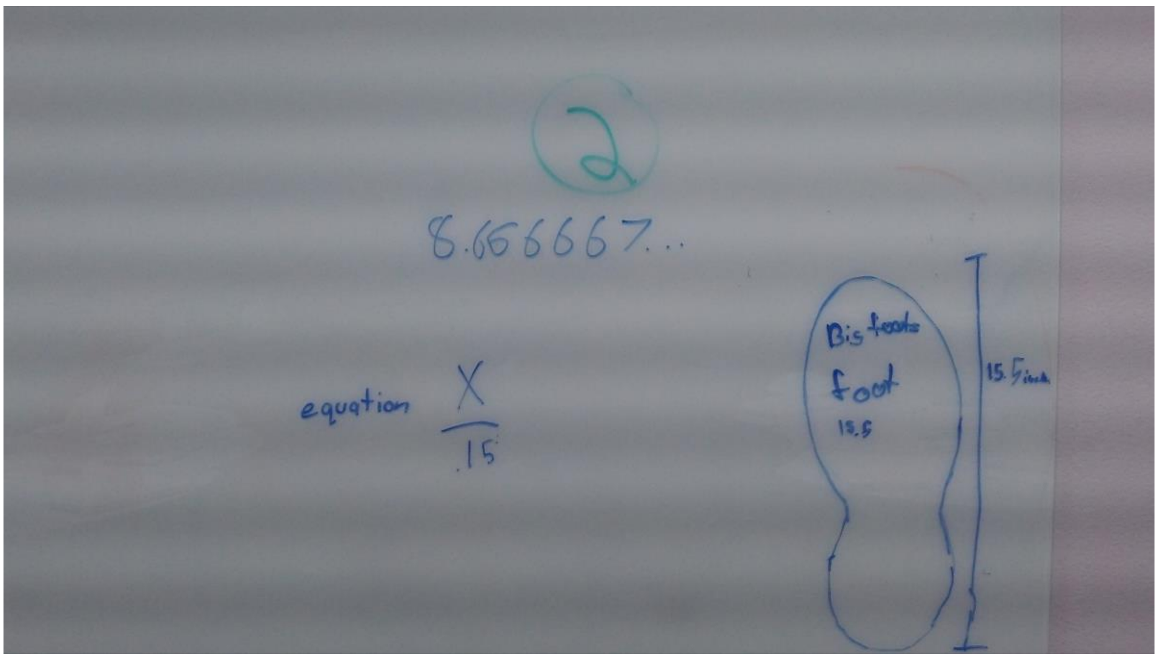

Figure 2. *Group two's final solution*

5.3 Group three

Group 3 used the Internet to assist in their solution development and determined two possible heights for Bigfoot that made use of ratios. The group started by having a discussion of their own heights and doing general Internet searches on Bigfoot. They then had a discussion on the length of the Bigfoot footprint that was provided to them because they got different measurement lengths. They ended up deciding on 15.5 inches.

On a website they found an average height to foot ratio and student one explained how they could use this. *If the height to foot ratio is 6.6 to 1 then the foot size would be 1 and the height would be 6.6 so write 6.6 to 1 ratio. So then you would multiply 6.6 times 15.5…So* *that is 102.3 and then this is in inches. You divide this by 12. You get 8.525. Bigfoot is 8 feet tall."*

Student two then questioned, *"How tall is the tallest man in the world?"* This led student three and four to investigate this. They were able to determine that the tallest man ever was Robert Pershing Wadlow at 8 feet and 11 inches. The group decided to use the height to foot ratio for this person to help determine the height of Bigfoot as well.

S1: *8 times 12 is 96 and 96 plus 11 is 107. So it would be 107 to 17.5. The tallest man, wait never mind let me simplify this…So it is about 6.1, so the actual. Listen to me. We are going to erase.*
S3: *You have to keep all your evidence.*

Student three convinced the group members to keep all of their work and they calculated what the height of Bigfoot would be, based on the height to foot ratio of Robert Wadlow. Figure 3 has this group's final solution. When this group presented, student 4 incorrectly stated the height to foot ratio as 8.525, and student two corrected him and finished the explanation. After all the groups had presented, this group added in the table that had their own heights and foot lengths as well.

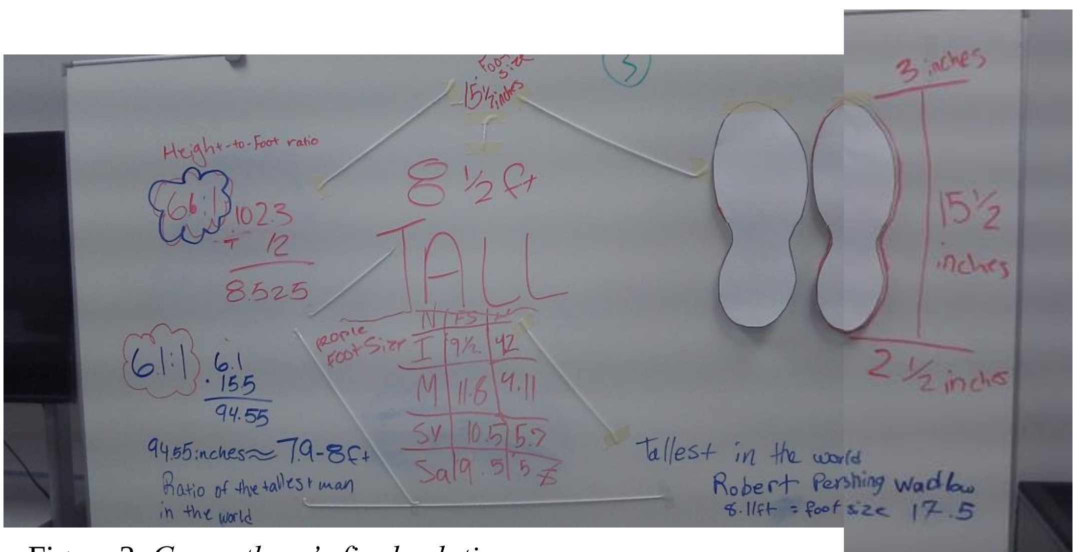

Figure 3. *Group three's final solution*

5.4 Group four

This group struggled for a while to figure out what to do, but were able to browse the Internet to find an idea. Initially, this group measured the length of the Bigfoot footprint and got 15.5 inches. Student three, then, thought about using an additive method, *"15.5 plus 50 that is about 66. That is not true."* There was no explanation for selecting 50 to add and the answer did not seem to fit for the actual height of Bigfoot. The group, then, measured the width of the foot, but was unsure what to do.

As the group tried to think of ideas, student one kept pushing for an idea, but had trouble knowing exactly how to explain it. *"We could think about our foot size and our height. I* *have tiny feet so I am a tiny person."* Later, she stated, *"How about we measure our height and then measure our foot length and then if we add this foot length and multiply our foot length.* Student three responded that this would not work because *"A lot of people could be his height and have big shoes and a lot of people could be his height and have small shoes."* Student one responded that it does work *"because usually tall people have big feet. I am guessing we can figure it out by multiplying."*

The group proceeded to look on the Internet for several minutes to see what they could find, but were unsure what to do, until student 4 was able to find information on a proportion for foot length and height. *"Divide the length of each person's height by their foot so we are going to set up a proportion."*

Student four proceeded to explain what she had come up with to the other three group members.
S4: *A proportion is a over w equals p over 100 and since it is 15% of his foot size, what would a and w be though? I don't know his height, wait no just kidding.*
S1: *Why would you put 15 and not 15.5?*
S4: *Because it is 15%*
S1: *Okay I get it.*
S4: *So he is 8 foot 6. Do you understand how I got it? You know what proportions are right?*
S2: *nope*
S4: *So proportions are basically ways to find percentages. A is something. Basically it is a over w equals p over 100. The p equals percent. These two are just different numbers that you can put down.*
S1: *So it doesn't matter what numbers?*
S4: It does. If you give them the numbers it does.
S1: *Is it height, width?*
S4: *No. The percentage of your*
S1: *foot to your entire body*
S4: *It is 15%. 15 over 100 right? That equals*
S3: .15 right?
S4: *It equals that. With this one, if you don't have two numbers and you have one, put that number on a and leave w. So you would multiply that it would be 1,550. Then you divide by 15.*

While student four could have given a better explanation on how to decide which number is a or w, this group was able to use the proportion correctly and come up with an estimate for Bigfoot's height. Figure 4 shows this group's final solution.

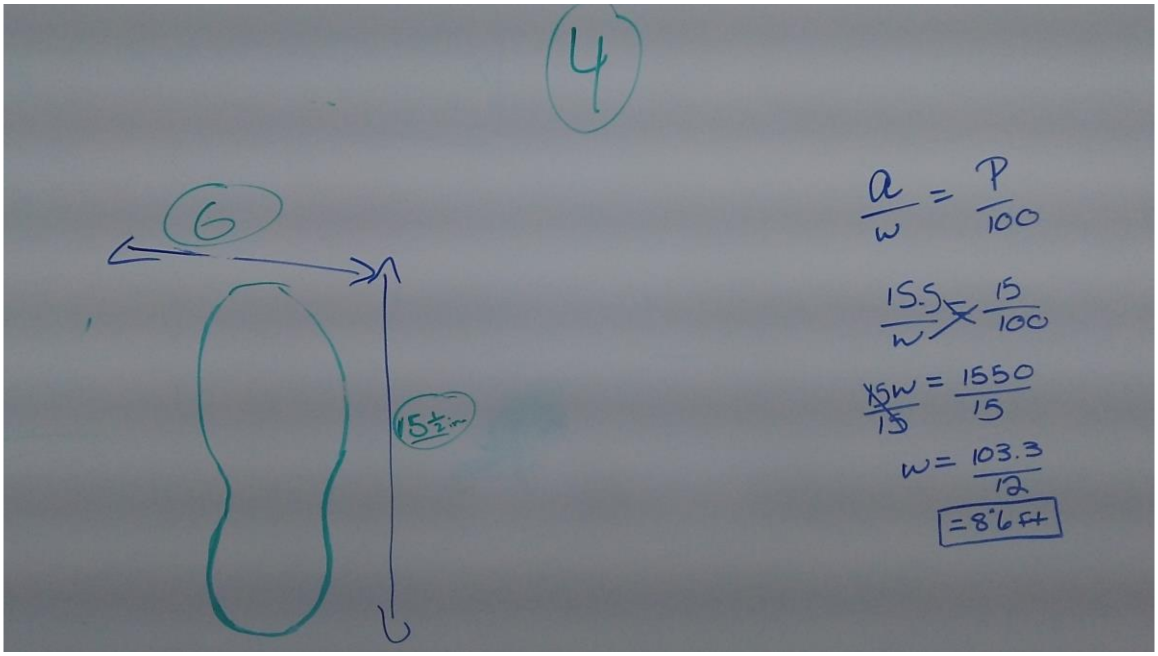

Figure 4. *Group 4's final solution*

5.5 Group five

Group five had limited conversation because they decided on an idea right away and just went to work on it. Student one proposed an idea with what the other two group members agreed. *"I think we should measure our feet and try to figure out a proportion."* The group measured their feet and recorded their heights. They also measured the Bigfoot footprint and noted that an estimate of Bigfoot's height was between 72 inches and 120 inches.

In this group's final solution (Figure 5), it was unclear how they ended up with 27 over 4. After the students had obtained their measurements, they started working at the whiteboard, which was away from their table, and the audio recorder did reach their conversation. In presenting their solution, it seemed that they had used their height and foot measurements to find this ratio, but after checking the math on this, it did not work. It might have been a calculation mistake. They used this ratio, though, and multiplied by 15.5 or 31/2 to obtain the height of Bigfoot as 8 feet and 5/8 inches.

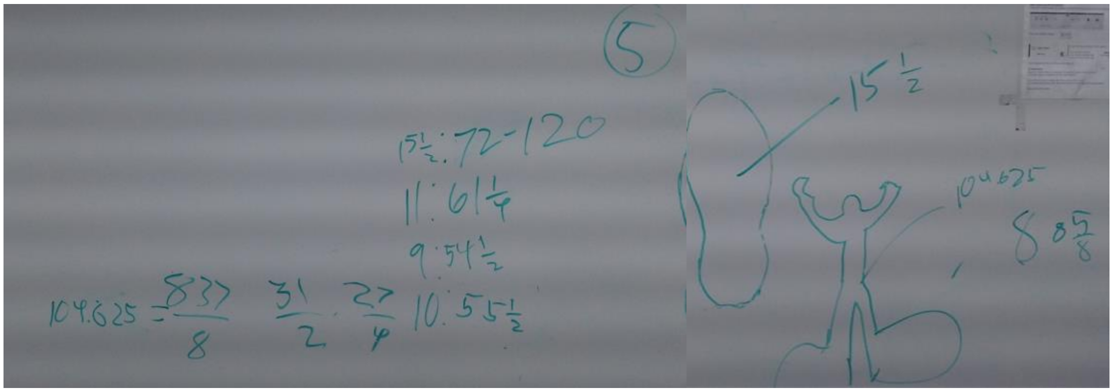
Figure 5. *Group five's final solution*

6. Discussion

This study investigated if middle school students could be successful in their first experience with mathematical modeling. By using a well-designed mathematical modeling activity (Lesh & Doerr, 2003), supporting students with metacognitive messages and questions, using cooperative learning, and the Model-Eliciting Activity (MEA) implementation structure, it was more likely that the groups would be successful.

These structures were put in place because, in the past research, students who were participating in mathematical modeling for the first time did not get to share their solutions with the whole class (Gould & Wasserman, 2014), had difficulties working in groups (Cheng, 2013), and struggled progressing effectively through the steps of the modeling process (MaaB & Mischo, 2011).

The groups, at times, had some difficulties or followed wrong directions in their solution development, but used the Internet, their group members, and other groups for assistance. During early stages of working on MEAs, groups can sometimes have inaccurate or unproductive ideas, but throughout the work time, groups are able to integrate and reorganize ideas, sometimes rejecting early ways of thinking. Group 4 struggled with developing their method and group 1 had an idea, but found it difficult to use it at first. They were able to progress to productive models, though.

Students were able to use their other group members and the Internet to develop solutions to the Bigfoot Model-Eliciting Activity. All groups understood the problem context and were able to come up with a method that worked to estimate Bigfoot's height, although there were some mathematical errors in their procedures. In the future, after group presentations, it would be good for instructors to ask the other groups explicitly to check each group's mathematics to ensure that is accurate.

The use of the Internet to help find relevant information is a paramount skill for contemporary education. Letting students have access to the Internet during mathematical modeling can support them in their solution development. Three of the five groups used the Internet to assist in their solution development. A fourth group was also going to use the Internet, but one of their group members told them that they could not use it. Prior research with MEAs has shown that not all groups have developed successful models (Aliprantis & Carmona, 2003; Biccard & Wessels, 2011). It is not known in these studies if students had access to the Internet.

Mathematical modeling has a natural connection to Smith and Stein's (2011) five practices for orchestrating productive mathematics discussions. The five practices can be used by instructors in mathematical modeling, because students will often come up with different solution strategies. The five practices are:
"1. **Anticipating** likely student responses to challenging mathematical tasks;
2. **monitoring** students' actual response to the tasks (while students work on the tasks in pairs or small groups);
3. **selecting** particular students to present their mathematical work during the whole-class discussion;
4. **sequencing** the student responses that will be displayed in a specific order; and
5. **connecting** different students' responses and connecting the responses to key mathematical ideas" (p.8).
The results of this study make the anticipating stage easier for teachers, because it details students' solutions. The instructor in this study also monitored groups while they worked, and ensured that they kept the problem statement in mind. For mathematical modeling, it is important that all groups get the opportunity to share their solutions. In the connecting stage, the instructor in this study had students think about how group 2 and group 4 used the same idea, having just written their solution in a different way. The instructor also followed up the activity with explicit work on ratios and proportions.

This study demonstrated that middle school students can be successful in their first experience with mathematical modeling. More research is needed on the development of mathematical modeling activities that enable students to work with "big ideas" in mathematics. These studies can detail students' solutions to help other teachers implement these activities successfully. Mathematical modeling has many benefits, and it is vital that all students are given opportunities to develop valuable 21st century competencies and mathematical knowledge through mathematical modeling.

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