

**SHARED TEACHING PRACTICES: INTEGRATING EXPERIENTIAL
KNOWLEDGE INTO PRE-SERVICE MATHEMATICS TEACHER EDUCATION¹**

**PRÁTICAS DOCENTES COMPARTILHADAS: INTEGRANDO SABERES DA
EXPERIÊNCIA NA FORMAÇÃO INICIAL DE PROFESSORES DE MATEMÁTICA**

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ABSTRACT

Recent research literature seems to expose and challenge a tacit and widespread conception that the knowledge required for teaching mathematics in school is situated outside the school's professional and cultural environment, and that the authority over this knowledge rests in groups that do not include school teachers. Taking this scenario into account, the mathematics teachers' undergraduate degree program of the Federal University of Rio de Janeiro (UFRJ) launched a new experience in 2015, in which mathematical content modules are taught jointly by a school teacher and a university lecturer. The experience holds an institutional intentionality: *to integrate professional knowledge emerging from school practice as a formal component of pre-service undergraduate mathematics teachers' education*. This paper focuses on a research project – *Shared Teaching Practices* – developed in the context of this experience. More specifically, we discuss prospective teachers' expectations towards the lectures taught by the two teachers, especially the role of the school teacher in module's conduction. Results indicate a shift in the participants' perception regarding their own professional disciplinary body of knowledge, and the role of this body of knowledge in their formal pre-service education.

Key words: shared teaching practices, teachers' education, teachers' professional knowledge, mathematical knowledge for teaching, school practice.

RESUMO

A literatura de pesquisa recente parece revelar e desafiar uma concepção tácita e disseminada de que o conhecimento necessário para ensinar matemática na educação básica é situado em um lugar externo ao ambiente profissional e cultural da própria escola, e de que a autoridade sobre esse conhecimento cabe a grupos em que professores da educação básica não estão incluídos. Levando esse cenário em consideração, o curso de Licenciatura em Matemática da Universidade Federal do Rio de Janeiro (UFRJ) iniciou em 2015 uma nova experiência, na qual disciplinas de conteúdo matemático são ensinadas de forma compartilhada por um professor da escola básica e um professor da universidade. Essa experiência tem uma intencionalidade institucional: *integrar saberes emergentes da prática escolar como um componente formal da formação inicial de professores de matemática*. Este artigo tem como foco um projeto de pesquisa – *Práticas Docentes Compartilhadas* – desenvolvido no contexto dessa experiência. Mais especificamente, discutimos as expectativas de alunos de licenciatura com respeito às aulas lecionadas pelos dois professores, especialmente quanto ao papel do professor da escola básica na condução da disciplina. Os resultados indicam mudanças nas percepções dos participantes em relação a seu corpo de conhecimento disciplinar e ao papel desse corpo de conhecimento em sua formação inicial.

Palavras-chave: práticas docentes compartilhadas, formação de professores, saberes profissionais docentes, conhecimento matemático para o ensino, prática escolar.

1. Introduction

In recent decades, research literature in Education and in Mathematics Education, both in the international setting (e.g. Shulman, 1986; Tardif, 2000) and in Brazil (e.g. Fiorentini & Oliveira, 2013; Moreira & Ferreira, 2013), has widely discussed the nature

and development of the knowledge needed for teaching, acknowledging its specificity and complexity, as well as its relationship with teachers' practice. These authors have also denounced the alienation between classroom practice and pre-service teacher education: in general, inclusion of practice-related knowledge seems to be poorly taken into account in the conception, structure, and management of undergraduate teachers' preparation programs. As Noddings (1992, p. 202) summarizes, mathematical content knowledge *per se* is not sufficient to account for mathematical knowledge for teaching, and research on teachers' knowledge plays a crucial role not only for carrying out teaching itself, but also for designing teachers' preparation. The author claims that "the expression 'pedagogical content knowledge' is more a political rallying cry than a label for an actual body of knowledge" (p. 198). From our own perspective, the "political cry" put forward by Noddings urges institutions to re-think and re-design prospective teachers' undergraduate degree programs. Moreover, these reforms must be research informed and guided by the principle that teaching mathematics at school is a professional activity involving self-determined knowledge and practices.

However, as Davis & Simmt (2006) advert, experienced teachers' know-how built through practice may not be recognized as a legitimate part of their disciplinary body of knowledge, and it may never be addressed explicitly in their formal education. The authors emphasize that "the subject matter knowledge needed for teaching is not a watered-down version of formal mathematics" (p. 295). In the Brazilian context, Moreira & Ferreira (2013) remark that, despite strong support for a sound mathematical basis for teachers, in general, the aspects that constitute such solidity or its actual impacts in professional practice are never made explicit. In line with such warnings, Nóvoa (2009) advocates a conception of teacher education built within the profession, including a higher authority for experienced teachers in the education of future teachers.

As we have argued in Giraldo et al. (2018b), from our own standpoint, these reflections expose and challenge a tacit and widespread conception that the knowledge needed for teaching mathematics at school is situated somewhere outside the school's professional and cultural environment, and that the authority over such knowledge rests in groups (such as policy makers, textbook authors, university lecturers) in which school teachers may not be included. That is, it would be up to groups whose members do not work at or interact with schools, and may not even recognize the legitimacy of knowledge from practice to dictate how teachers should or should not teach.

Taking this research framework into account, in 2015, the mathematics teachers' undergraduate degree program of the Federal University of Rio de Janeiro (UFRJ) began a new experience in which mathematical content modules are taught jointly by a school teacher and a university lecturer. The experience aims *to integrate professional knowledge emerging from school practice as a formal component of pre-service undergraduate mathematics teacher education*, especially in modules addressing mathematical content knowledge.

A research project, *Shared Teaching Practices*, was developed in the context of this experience, aiming to analyze its conduction and results. An outline of a pilot study was reported in Giraldo et al. (2016). In Giraldo et al. (2018a), we explore the kinds of knowledge incorporated in the discussions involving the two teachers and the prospective teachers during the sessions. In this paper, we discuss prospective teachers' expectations towards the lectures taught by the two teachers, especially the role of the

school teacher in module's conduction. Results indicate a shift in the participants' perception regarding their own professional disciplinary body of knowledge and the role of practice in the construction of this body of knowledge.

2. Theoretical Framework: Knowledge from Practice and Teacher Education

According to Tardif, Lessard & Lahaye (1991), school teachers must not only know the subject matter they teach, but also hold pedagogical knowledge and develop another kind of knowledge grounded by their daily experience with students. By its own nature, this kind of knowledge is developed *in the practice* and *from the practice*, mobilized and employed in daily practice, originated from the situations of the work of teaching itself, and serves to assign meaning to these situations. Therefore, this is a form of experiential knowledge, which is incorporated into the individual and collective experience in the form of '*habitus*' and abilities, of *know how to do* and *know how to be*. It consists of *teachers' professional knowledge*, which characterizes teaching as a profession, and distinguishes it from other professions and occupations.

Since the 1980's, *professionalization of teaching at school* has gained progressive prominence in the education research literature and has established an academic and political agenda. One particular issue within this agenda is how the academic debate may have an actual impact in teacher education, especially in undergraduate degree programs for pre-service teachers' preparation.

Cochran-Smith & Lytle (1999) identify three radically different conceptions of teachers' knowledge and learning, and their corresponding relationships with teacher education. The conception of *knowledge-for-practice* considers formal knowledge and theory as the source of knowledge teachers need to use in order to improve practice. As Fiorentini & Crecci (2016) remark, according this conception, it is the responsibility of university researchers and experts to produce formal knowledge and theories, which is then studied by teachers to apply to their practices. The *loci* for professional preparation are, therefore, restricted to academic programs designed and directed by these experts. The conception of *knowledge-in-practice* assumes that teachers learn when they have opportunities to probe the knowledge embedded in the work of more experienced teachers. To Fiorentini & Crecci (2016), this conception implies that knowledge for teaching is essentially practical in nature, and therefore cannot be taught, but only tacitly learnt in practice. However, in this case, knowledge may easily become merely naturalized and replicated with little reflection, which may hinder or prevent teacher development and transformation. Finally, Cochran-Smith & Lytle (1999) identify *knowledge-of-practice*, a conception according to which knowledge needed for teaching well is assumed to be generated when teachers treat their own classrooms and schools as sites for intentional investigation, while simultaneously treating the knowledge and theory produced by others as generative material for interrogation and interpretation. Fiorentini & Crecci (2016) observe that, from this perspective, knowledge for teaching cannot be dissociated between theoretical and practical; teachers produce knowledge in the locus of practice, working in communities of investigation in which they theorize from the practice and practice these theories. Therefore, practice is not seen as opposed to theory, since practice is not devoid of theory and theory does not disregard practice.

Tardif (2008) enumerates a set of 14 principles for guiding the implementation of teacher education programs. The author claims that the projects for these programs must be supported by a coherent view, shared by the actors that contribute with teacher education and those that take part of it. This view demands a set of pedagogical and curricular key choices, addressing question such as: What kind of teacher do we intend to prepare? Which educational values and pedagogical commitments do we indent to put forward and defend? Which knowledge and abilities must teachers hold by the end of the program? Which common conditions must guide individual and collective practices in the program, in order to avoid the breakdown of the program into a set of disparate modules and internships, disciplines and knowledge? How will the program be organized with respect to both its structure and its operation? What are the roles of each actor in the team involved with the program? Among the principles ranked by Tardif, we highlight:

- *Strong partnerships with the school* (Principle 3, p. 24). This aims to not only establish a relationship with the prospective teacher throughout the preparation, but a *co-preparation* that also includes the university lecturers and the school teachers involved in the program.
- *Students, as adults and actors of their own education* (Principle 4, p. 26). The education of an adult can only be conceived in terms of a *co-preparation*, that is, a preparation of persons who is assumed to be *authors of their own learning and development*. This implies fully acknowledging not only prospective teachers' competence, but also their responsibility, as future teaching professionals, to intervene in the program team, in order to make sure that their points of view, concepts and convictions will be taken into account.
- *A profession-based education* (Principle 6, p. 31). The center of gravity of the programs must be the *professional action itself*, simultaneously as a subject of knowledge, as an environment for action and education, and as a mechanism for theoretical, cultural and critical reflection.
- *A practice-based education as the center of the program* (Principle 9, p. 35). As stated in Principle 6, the professional action must be the focus of the program. This principle is opposed to the traditional views that establish a hierarchy among three components: theoretical training, seen as fundamental and first component; didactic-pedagogical training, resulting from the first according to the model of applied sciences; and practical training, seen as a place of application of theoretical and applied knowledge.

Similarly, Nóvoa (2009) defends a *teachers' preparation built up from inside the profession*. For the author, to prepare teachers means *to introduce someone in the culture of the profession*. For that purpose, the author indicates 5 working proposals intended to inspire both pre-service and in-service teachers' preparation programs (Nóvoa, 2009, p. 1):

- To assume a remarkable practical component that has to be focused on pupils learning and the study of specific cases, having schoolwork as a reference.
- To be carried out *inside* the profession, by means of the acquisition of a professional culture and granting teachers with more experience a central role in the training of the youngest.

- To dedicate a special attention to the personal dimensions of the teaching profession by focusing on the capacity of relation and communication which defines the pedagogical component.
- To value teamwork and the collective practice of the profession, reinforcing the importance of school educational projects.
- To be characterized by the principle of social responsibility, promoting public communication and professional participation in the public space of education.

Some researchers have presented strongly practice-oriented in-service teachers' preparation proposals. For instance, Davis & Simmt (2006) propose the model of *concept study* as a structure of collective investigation, in which teachers share experiences from their practices in order to discuss and (re)build their mathematical knowledge for teaching. The fundamental organizing principle of the concept study methodology is an understanding that the articulation between *categories of knowledge* (mathematical objects, curricular structures), usually treated as more stable, and *categories of knowing* (classroom collectivity, subjective understanding), usually treated as more dynamic, is crucial for mathematics teaching. The authors consider that, for teachers, knowledge about established mathematics is inseparable from knowledge about the ways by which mathematics is produced. For Davis & Renert (2009, p. 45), teachers are not peripheral agents whose role is to passively spread established mathematics; on the contrary, they are key actors for the shaping of *cultural mathematics*, that is, not only formal mathematics, but also a diversity of culturally situated practices, perspectives and applications. From this standpoint, mathematical knowledge for teaching is essentially dynamic and emergent, and cannot be described by pre-established categories or taxonomies.

One key political agenda is how to achieve a structure for teacher education that actually embodies practice-based knowledge and a professional culture, as Tardif and Nóvoa advocate, and assign experienced teachers actual authority to this knowledge and culture. The concept study model for *in-service* education, developed by Davis and colleagues, propose that mathematics for teaching is collectively developed, from a cultural perspective, using experiences from the participants' practices as a groundwork. However, in the case of *pre-service* education, this possibility does not apply straightforwardly. It cannot be assumed that prospective teachers hold extensive classroom experience themselves. Additionally, undergraduate degree programs for teacher education faculties often do not include a majority (or at least a reasonable proportion) of lecturers who are or have been school teachers themselves. Finally, the participation of an experienced school teacher is often constrained to well defined spaces, such as school internships, with little articulation regarding other curricular components. Therefore, it is an ongoing challenge to find ways to integrate knowledge from practice into pre-service mathematics teacher education. The present research is situated within this rationale.

3. Research Setting, Aims and Methodology

3.1. Institutional setting

The Federal University of Rio de Janeiro (UFRJ) is a major public university in Brazil, being the federal tertiary education institution with highest number of lecturers and students. The mathematics teachers' undergraduate degree program in UFRJ is run by the Institute of Mathematics (IM-UFRJ), which is a leading institution in research in Mathematics, hosting a graduate program assessed with the highest score by the country's Ministry of Education. Moreover, IM-UFRJ has a long-term tradition in Mathematics Education, including a graduate program in the field. In IM-UFRJ, as in most of Brazilian universities, there are separate undergraduate degree programs for the education of future mathematicians and of mathematics teachers². The syllabus of the former consists only of modules on pure and applied mathematics and related sciences (such as physics and computer science), whilst the later includes the addition of modules on general pedagogy, such as psychology of education and general didactics, history of mathematics, and mathematics education. Despite the abovementioned academic quality standards, the mathematics teachers' undergraduate program³ has been assigned the lowest score in recent assessments.

Motivated by the context of the institution and informed by the research literature in teacher education, a group of lecturers and graduate students from IM-UFRJ's graduate program in Mathematics Education gathered a study group (in which the authors of this paper are included), intended to design and test proposals for the mathematics teachers' undergraduate program. One of those proposals suggested offering modules taught jointly by a school teacher and a university lecturer. The proposal was explicitly intended to *integrate professional knowledge emerging from school practice as a formal component of pre-service undergraduate mathematics teacher education*. Moreover, the intention was to maximize the integration between mathematical academic knowledge and mathematical knowledge emerging from school practice. We understand that this design can put at stake specificities of mathematical knowledge for teaching, enabling contributions from the lenses of the two actors. Thus, two methodological choices were made:

1. Modules addressing mathematical content, which would allow such intertwined approach, were chosen for the application of the proposal.
2. The school teacher and the university lecturer would be expected and recommended to share all the aspects of the module's conduction, including not only teaching classroom lectures, but also planning the general structure, designing assessment, marking written exams and other assignments, and so on. They were also recommended to contribute with these discussions with the viewpoints from their own professional environments.

Furthermore, a political stance was taken: it was important not only to integrate knowledge from school practice in pre-service teacher education, but also to formally acknowledge it as a component of the undergraduate program. Therefore, the proposal was formally registered as part of the program's syllabus. A pilot application was carried out during the first academic term (March to June) of 2015. In each subsequent academic term, the proposal was applied in up to 8 modules, addressing contents such as arithmetic, algebra, geometry, functions and history of mathematics.

² Respectively called in Brazil *bacharelado* and *licenciatura*.

³ Licenciatura em Matemática.

The team involved in the proposal design had also an expectative that it would have a complementary relationship with the curricular internship: whilst the internship implies in taking prospective teachers to the school environment, the proposal is about *bringing experiences from school into other components of prospective teacher education*.

3.2. Research setting: The *Shared Teaching Practices* project

The results reported in this paper are part of a broader research project, *Shared Teaching Practices*, developed in the context of the proposal described in the previous subsection. This broader project aims to analyze the experience in three main strands:

1. participant prospective teachers' expectations concerning the lectures taught by two teachers, specifically with respect to the role of the school teacher;
2. dynamics of the interactions among the actors (school teacher, university lecturer, and prospective teachers);
3. professional knowledge mobilized during the experience.

A discussion about the pilot study conducted in the first term of 2015 is reported in Giraldo et al (2016). The main study was conducted in a module named "Foundations of Arithmetic and Algebra", during the second academic term of 2015. The module addressed number systems, including natural, integer, rational and real numbers, and their teaching in elementary and secondary school. The module was 60-hour long, conducted over 15 weeks in weekly 4-hour sessions (on Tuesday evenings). The term started in August 2015 and was supposed to finish in mid-December of the same year, but due to a strike in the University, it lasted until March 2016.

The module was taught by Victor, a university lecturer (1st author of this paper), and Fábio, a school teacher (2nd author of this paper). Victor is a lecturer at UFRJ, where he works in the undergraduate program for teacher education and in the graduate program for Mathematics Education. Fábio is a teacher at elementary and secondary public schools in Rio de Janeiro and, when the main study took place, he was a student in the same graduate program. Both have more than 20 years of experience in teaching (at university and school levels, respectively) and are involved with research in Mathematics Education. They were chosen due to their long-term teaching experience and to their propensity to reflect on teacher education.

The module was attended by about 30 undergraduate students, all of which were prospective teachers. All the prospective teachers had some experience in teaching at elementary and/or secondary school, varying from few experiences in private or small groups tutoring to some years as classroom teachers. From now on, we will refer to the undergraduate students who participated in the study as *prospective teachers* (to avoid misunderstandings with other students that may be mentioned).

Consistent with the project's 3 strands, several instruments were used in data collection:

1. before the beginning of the module, focus groups with the participant prospective teachers;
2. during the module:

- a. audio and written records of Fábio and Victor's joint discussions about the module's planning;
 - b. Fábio's and Victor's individual daily logs, produced immediately after each lecture;
 - c. written reports of two independent observers, produced during each lecture;
 - d. video records of the lectures;
 - e. written assignments submitted by the prospective teachers;
3. after the end of the module:
- a. group interviews with participant prospective teachers;
 - b. written questionnaires completed by participant prospective teachers.
 - c.

Results concerning strand 3, professional knowledge mobilized in the experience, are reported in Giraldo et al. (2018a).

3.3. Research aims and data production

This paper addresses a particular aspect of the broader research project *Shared Teaching Practices*. We focus on the prospective teachers' expectations towards the lectures taught by the two teachers, especially the role of the school teacher in module's conduction, and on shifts in these expectations throughout the experience (strand 1). We do not focus in individual participants, but instead on the prominent views of the whole group of prospective teachers.

Our data analysis was inspired in the model proposed by Powell, Francisco & Maher (2004), which prescribes seven stages for the analysis of data from video: careful observation, description with situations coding over time, identification of critical events, transcription of critical events, mathematical ideas coding, plot construction, and narrative composition. The authors characterize critical events as events that indicate significant or constant change from previous conceptions, which may confirm or contradict research hypotheses. From the perspective that a methodological model should provide frameworks for research, rather than chain it up, Powell & Quintaneiro (2015) propose a variation to this model, based on descriptions articulated with transcriptions.

Our inspiration in this model considers that its contribution may transcend the types of media, being applicable also to written logs and reports. Thus, we apply the idea of critical events to data from different sources. We organize our analysis in three moments, in which we draw on some of the instruments listed above, namely:

1. Before the experience: We identify, from the focus groups (instrument 1), prominent expectations about the lectures with the two teachers, which we illustrate with representative excerpts from the transcripts.
2. During the experience: We report critical events that may be related to shifts in the initial expectations. The narratives of these events were constructed combining data from the teachers' logs, observers' reports, and video records (instruments 2b, 2c, 2d). A more detailed account of the construction of these narratives is reported in Giraldo et al. (2018a).

3. After the experience: We identify, from the group interviews and written questionnaires (instruments 3a, 3b), relevant shifts from the initial expectations, which, again, are illustrated with representative excerpts from the transcripts.

4. Results

In this section, we report results from the data analysis, as described in the previous section. We refer to the prospective teachers whose experts are quoted by the code PT#. However, following individual responses is not relevant to our analysis, since our focus is in the group of prospective teachers as a whole, and not in individual participants.

4.1. Before the experience: Expectations

During the focus groups, prospective teachers were asked about the possibility of having school and university lecturers teaching a module together in the undergraduate program. One of the questions addressing this issue was stated as follows: “*Do you consider that two teachers, one from the school and the other from the university, could work together, teaching the same module?*” At first, the question seemed to cause some awkwardness, and the participants asked the interviewer to repeat, which suggests a difficulty to even consider the scenario. After the question was explained, the prospective teachers declared that the idea could be interesting, but the role of the school teacher seemed obscure. For some of them, he would act only as a “translator” or a “tutor” for the university lecturer, as the following expert illustrates:

PT1: I agree with PT 2 that it'd be nice to have the guy to translate, isn't it?

PT2: The guy would have this role of translator.

PT3: The school teacher could serve as a tutor during the lectures.

PT4: Damn, are you lessening the guy?

PT3: He is translating what the university lecturer says. That's not lessening, I'm not undervaluing people who work as tutors, as I've never done it myself. Damn, I think I may be showing some sort of prejudice, you see? But the school teacher will be a tutor, but I understand that's because he is not prepared to teach at the university, maybe because he finished undergrad and full stop, and the other went beyond, he has a graduate degree.

This episode reveals an alienation between prospective teacher education and school practice in different ways. Firstly, the awkward presence of a school teacher seems to be not even imagined by the participants, within the possibilities for their own undergraduate education. Secondly, the description of the role of the school teacher as a “translator” suggests that his only function would be to present the knowledge held by the university lecturer in a simplified manner. Therefore, the school teacher would not hold any kind of knowledge of his own that would be relevant to the prospective teachers' professional preparation. Underlying this view, there may be a conception that the reference knowledge for teaching at school is the academic mathematical knowledge, which must be “simplified” in order to be taught at school level; the university lecturer is the authority over this reference knowledge, whereof the school

teacher is only a user, and not a producer; and there is no relevant knowledge produced from school practice. Thirdly, the description of the role of the school teacher as a “tutor” indicates a hierarchical perception, according to which he would be “less prepared”, since the university lecturer “went beyond” with his education. Once again, academic knowledge is overvalued against any knowledge produced through school practice. These results may emerge from the widespread view that knowledge needed for teaching is situated outside the school’s professional and cultural environment, and that the authority over this knowledge rests in groups that do not include school teachers, as we have observed in Giraldo et al. (2018b). On the other hand, the discussion in the focus groups also indicates that, for some of the participants, the school teacher knowledge is not entirely disregarded, but its usefulness is restricted to a “know how to do things”. This view is illustrated by the following excerpts:

PT2: I think that he [the university lecturer] will demand more formalism from us. The school teacher will be closer to practical applications, and the university lecturer will make a point of explaining the reason of things.

PT4: That’s something that, at my point of view, actually happens with all undergraduate programs, not only ours. Some friends of mine study marketing. They say the lecturers have never worked in a firm, and even so they want to say how things work. I think it’s the same thing we go through, a guy from the university, who never taught at school, never got in a school classroom, will come to us and say “look, you will have to teach algebra to your students this way”. Then, a guy who actually lives there, who actually experiences that many times, will be more important to us. But that would be in the Maths Education modules, in the other ones it’d be cool. But for me it’s extremely important that someone who has practice, who will talk about the practice, do you understand? Someone who won’t talk about theory, who will say “look, I do this, and it works”, then it should work for you too.

Thus, these participants do recognize the importance of the contributions from someone who is “closer to practical applications”, or who would tell them that “if this works for me, than it should work for you”. This “practical knowledge” seems to be seen strictly as “applied knowledge”, or “know how to do things”, as opposed to a “theoretical knowledge”, which, in its turn, would belong to the university lecturer. For instance, PT2 attributes a concern with formalism to the university lecturer, while, on the other hand, the school teacher would be more worried with its applications at school. PT4 recognizes practice as an important aspect of her pre-service professional education. However, she refers to this knowledge from practice as something disconnected from any theory, whose goal is to find ways to execute things effectively, based on experimentation – “if it has worked for me, should work again for you” – without “talking about theory”.

In summary, our analysis from the focus groups with the prospective teachers, which took place *before* the shared teaching practice experience, indicates two prevailing views for the kind of knowledge that is relevant in their pre-service education:

1. as *simplified theoretical knowledge*, which is originated in theories produced by university lectures, and must be “simplified” by school teachers, in order to be applied at school;
2. as *practical experimental knowledge*, which is disconnected from theory, produced by experimentation in practice, and learnt by repetition from more experienced teachers.

Thus, these views seem to correspond to the conceptions described by Cochran-Smith & Lytle (1999) as *knowledge-for-practice* and as *knowledge-in-practice*. No evidence of the *knowledge-of-practice* conception was identified. Moreover, these views seem to be associated with an expectation of hierarchical roles to be played by the university and school teachers, according to which the school teacher would be restricted to act as a “tutor” of the university lecturer, as a “translator” of his knowledge, or to present “practical knowledge”, disconnected from theory. Therefore, the participation of the school teacher in the undergraduate program is expected to be secondary or peripheral.

4.2. During the experience

Critical event #1 – Questions about practice

The first three lectures followed a similar pattern: The mathematical content approach was mainly led by Victor, with occasional insertions from Fábio, based on his experience in school. It was clear that the prospective teachers did not view them equally. The 4th lecture was conducted only by Fábio, without Victor present. He proposed that the prospective teachers assess and discuss answers given by school students to problems involving algorithms for the operations with whole numbers. The discussion throughout the lecture developed around this activity.

In all the lectures, there was significant participation of the prospective teachers in the collective discussions. However, the 4th lecture represented a defining moment, concerning both the content of the discussions and the attitudes towards the two teachers, which lasted for the rest of the module. The discussions regarding mathematical concepts and teaching at school were significantly more intertwined. Moreover, the prospective teachers started to address questions directly to Fábio, without Victor’s previous consent. These questions concerned issues more closely related with school practice, such as: *How can we assess answers different from the expected? Is reading parts of textbooks with school children a good practice? How can we balance depth of explanations to students with very different levels of learning in the same classroom?*

This event illustrates an inflection on the prospective teachers’ attitude towards Fábio. They started to recognize his authority over a specific kind of knowledge, as well as the specificity of this kind of knowledge and its relevance to their education. They seem to perceive the two teachers as having different roles, but with balanced importance to their professional education.

Critical event #2 – Complementary kinds of knowledge

In the 7th lecture, during a discussion about using examples from students’ daily lives, a prospective teacher asked Fábio how to deal with a common mistake of the school

children: using zero as the denominator of a fraction. After Fábio discussed the issue, Victor exclaimed, “*I didn’t even know this was a common mistake!*” Some prospective teachers related episodes from their own experiences, showing they were aware of this common mistake. Fábio, Victor and the prospective teachers proceeded to discuss how the impossibility of division by zero could be addressed at school, considering the algebraic structure of the number sets and the adequate level of abstraction of students in elementary school.

This event illustrates how one issue can be discussed simultaneously from different perspectives, the school practice and the formal mathematics. It also illustrates how the prospective teachers perceive how Fábio and Victor can contribute with both perspectives, but each one has more authority over a specific kind of knowledge, which is different, but equally relevant to their education. In particular, the prospective teachers reacted somewhat restless when Victor said he did not know about an aspect Fábio was fully aware of. Indeed, some of them later declared in the group interviews that it was a great surprise that a university lecturer, holding a doctorate degree, could not know something, publicly admit his ignorance and, moreover, recognize that a school teacher would know more than him about that particular aspect.

In our interpretation, there is evidence this event was crucial for a change in the way the prospective teachers see Fábio’s role, shifting from Victor’s tutor to someone with authority over a specific kind of knowledge.

Critical event #3 – Focus on the practice

In the 8th lecture, Fábio and Victor proposed to the prospective teachers to prepare how they would approach the first introduction to integers at elementary school in small groups and then present to the whole class. It was recommended to “focus on how to start, what kind of speech to use, and which examples and nonexamples to use”. At first, the prospective teachers showed some worries about the public presentations, but shortly after, they started gathering in small groups and engaging in the activity. Both the work in small groups and presentations to the classroom were very intense, involving the two teachers and the prospective teachers, and addressing the mathematical content and its teaching at school.

In two additional lectures following this event, Fábio and Victor assigned tasks involving preparation and presentations of approaches for teaching a mathematical topic at school. In all three occasions, the lectures ran similarly. The engagement of the prospective teachers on these assignments and the discussions that followed provide evidence of the importance they attributed to school practice, and show how they considered it as a relevant aspect of their education. Thus, the central subject matter of the module was not seen as the mathematical content, but as the reflections based on its teaching at school.

4.3. After the experience: Shifts in perceptions

The analysis of the group interviews and the written questionnaires, which took place *after* the shared teaching practices experience, indicate shifts in the prospective teachers’ expectations previously revealed in the focus groups. In general, they assessed

the experience as positive. They claimed, among other things, that the participation of the school teacher provided a view of the school classroom that they assumed the university lecturer would not have experienced; that both teachers showed mastery of the module content matter; and that the responsibilities in the lectures were equally shared by them. For instance, when describing the positive aspects of the experience, one of the prospective teachers declared:

PT3: The most relevant aspect of the experience was that university stage was shared with the school practice.

This comment by the participant illustrates that, from their perception, the university granted space to the school practice, in “a shared stage”, that is, in even levels of hierarchy. The participation of a school teacher in the university landscape, which was seen before as awkward and a hard to guess scenario, was now highlighted as a positive aspect of the experience, due to the integration of a view from the school classroom, which shared importance with the academic mathematical knowledge. When asked about the importance of the participation of the school teacher, the prospective teachers mostly refer to the specificity of their own education, and the balance between the knowledge of the two teachers:

PT5: Yes, due to the great importance of relationships between academic mathematical content and school teaching practice, almost nonexistent in the undergraduate program, for the prospective teachers. It's precisely the goal of our undergraduate program. It doesn't make any sense to present the mathematical content purely, as it is presented to the guys in the pure maths program.

PT4: Definitely! The presence of the two teachers will from now on serve as a reference to the whole undergraduate program, as we, students, are very much in need of examples related to our profession. All the time when Victor raises some new topic, then Fábio shares his experiences with things that make us relate mathematical contents discussed in the lectures with school practice.

PT6: At no moment could I label something as exclusively mathematical or theoretical. All the time, the relationships between the content that was been discussed and the ways it could be taught at school were prominent.

Thus, the “relationships between academic mathematical content and school teaching practice, almost nonexistent in the undergraduate program” seems to have become more evident to the prospective teachers, from the moment when “Fábio shares his experiences with things that make us relate mathematical contents discussed in the lectures with school practice”. These results suggest that knowledge from practice, shared by Fábio, was integrated with the academic mathematical knowledge mobilized during the shared teaching practice experience, establishing an articulation between school and university in pre-service education environment.

In our interpretation, the experience induced a shift in the prospective teachers' expectations about the roles of the two teachers throughout the module. The school teacher was no longer seen as a “tutor” or a “translator”, but rather takes on a main role in the lectures, with respect to the recognition and appreciation of knowledge emerging

from practice, which are shared and integrated with academic mathematical knowledge. The presence of the school teacher in the university landscape is no longer seen as awkward or uncommon, but becomes a reference to the prospective teachers, as *they acknowledge his authority over a kind of knowledge that is a relevant aspect of their formal education*. Some of the participants even refer to his presence as “inspiring”, since it shows that school teachers can successfully teach at the university, engage in research activities, and acquire graduate degrees – possibilities that seemed to be impossible for some of them. Therefore, his presence had an effect in the acknowledgement of the profession in the eyes of the prospective teachers, as they now perceived it as being embedded in an academic research field of its own, and not simply subordinated to the academic field of mathematics.

These results also indicate a shift in the prospective teachers’ initial expectations with respect to the recognition of the nature of the knowledge from practice, its relevance and value as an aspect of their own pre-service education. In the focus groups, the knowledge relevant to their education was identified as a *simplified theoretical knowledge* or as a *practical experimental knowledge*, which we associated with the conceptions referred as *knowledge-for-practice* and as *knowledge-in-practice* by Cochran-Smith & Lytle (1999). Our analysis suggests that, after the shared teaching practices experience, the prospective teachers recognized the relevance of a kind of knowledge that articulates aspects from academic mathematical content and from school practice, and that they acknowledged Fábio’s key role in articulating these aspects. Knowledge from school practice was no longer seen as a simplification of the mathematical theory, nor as a kind of knowledge based on experimentation. The dichotomy between the university, as the place to develop theory, and the school, as the place to apply theory into practice, was broken, or at least put into question. Rather, *mathematical knowledge needed for teaching is seen as both theoretical and practical, and built up from the articulations between university and school*. This can be associated with *knowledge-of-practice* from Cochran-Smith & Lytle (1999), since practice is not opposed to theory, and seen as a terrain to produce knowledge.

4.4. After the experience: Reflections about pre-service education

In the group interviews, the prospective teachers raised discussions on the undergraduate program. Despite this not directly expressing shifts in their initial expectations, and therefore not directly linked with the main goal of this paper, we consider that these discussions highlight reflections on their own professional preparation, which are likely to be provoked by the shared teaching practice experience.

Most of the prospective teachers suggested that more modules with shared teaching structured should be offered in the mathematics teachers’ undergraduate program, aiming to approximate prospective teachers to the profession of school teaching. Some of them also recommended the shared teaching structured be applied to the first year of the undergraduate program, as a mean of integrating newcomers in the program and getting them acquainted with the profession from the beginning. According to the prospective teachers, the modules are mostly oriented to the pure mathematics undergraduate program, with the mathematics teachers’ undergraduate program treated as secondary, and, as a consequence, students “don’t know why they are doing things” until a very advanced stage of the program. For instance, one of them said:

PT7: We enter the mathematics teachers' undergraduate program, supposedly, because we want to be teachers. Then, the university expels us, saying "you have to be a pure mathematician!"

This same participant used the expression “a mathematics teachers’ undergraduate program oriented to mathematics teacher education”. In our interpretation, this claim – which may sound obvious or redundant – expresses the prospective teachers’ feeling that it is necessary to rethink mathematics teachers’ undergraduate programs, based on the recognition of teaching as a professional activity.

5. Concluding Remarks and Perspectives

5.1. Results summary

Our results suggest that the shared teaching practices experience was associated with shifts in the prospective teachers’ expectations on the role of the school teacher in the module, as well as in the underlying conceptions on the nature of the knowledge from practice and its relevance to their pre-service education. The initial expectations that the school teacher would act only as “tutor” or a “translator”, whose role would be subordinated to the university lecture, shifted to a recognition of his key role in articulating mathematical academic knowledge and knowledge emerging from school practice. Thus, the prospective teachers acknowledged his authority over a kind of knowledge that is a relevant aspect of their formal education. Consonantly with the principles suggested by Tardif (2008) – especially *a profession-based education*, and *a practice-based education as the center of the program* – the experience has offered participant undergraduate students a view of teachers as protagonists of future teacher education, as Nóvoa (2009) defends.

Moreover, the prospective teachers’ views about the knowledge from school practice shifted from a simplified version of theoretical knowledge or a practical knowledge based on experimentation, which has parallels with *knowledge-for-practice* and *knowledge-in-practice*, to a kind knowledge that is both theoretical and practical, and is built up from the articulations between university and school, which can be associated with *knowledge-of-practice* (Cochran-Smith & Lytle, 1999). In our interpretation, these shifts are intrinsically interrelated: on the one hand, the school teacher is seen as someone who has authority over the knowledge developed within his own activity, rather than someone whose function is to adapt knowledge produced elsewhere; on the other hand, the mathematical knowledge for teaching is seen from the perspective of its own epistemology, associated with its own academic field, rather than subordinated to the academic field of mathematics.

Therefore, the school teacher was now seen as someone who has authority over the knowledge of his own professional activity and who can engage in research and graduate studies in an academic field inherent to this activity. This perspective is related with the acknowledgement of teaching at school as a profession, as advocated by Tardif (2008) and Nóvoa (2009); and of teachers as actors in the production of cultural mathematics, rather than mere spreaders of stablished mathematics, in the sense of Davis & Renert (2009).

This perspective is also consistent with the remark that “the subject matter knowledge needed for teaching is not a watered-down version of formal mathematics” (Davis & Simmt, 2006, p. 295), and goes towards a research-informed political agenda that strives for a conception of teacher pre-service education built up from inside the profession, as Nóvoa (2009) advocates. We have referred to this conception as an *affirmative perspective* for teacher education (Giraldo et al., 2018): a perspective oriented by *the integration professional knowledge emerging from school practice as a formal component of pre-service undergraduate mathematics teacher education*.

We consider that the experience with shared teaching practices reported in this paper may contribute to the research related with such agenda. The experience is oriented towards a *profession-based* and *practice-based* education – guiding principles 6 and 9 for pre-service teacher education programs proposed by Tardif (2008). Moreover, it represents a movement where the university gives space to the school to share an authorship role in the education of future teacher. In our view, this also opens ways to Tardif’s guiding principle 3 – *strong partnerships with the school*.

5.2. Sharing teaching

The idea of the shared teaching practice proposal is not to merely include a new character – a school teacher – in the scenario of the university lectures, as passive observers, who could easily take to role of “assistants” of the university lecturers. The aim is to grant school teachers an authority role over a kind of knowledge that is proper of their professional activity, and whose legitimacy is institutionally recognized. Thus, the syllabus of the modules is not *a priori* taken for granted, but the active participation of the school teachers is expected to transform its usual approach, classroom dynamics and content matter, through the multiple interactions among all the actor involved – school teacher, university lecturer and undergraduate students. For those reasons, we consider to be fundamental to institutionally recognize the proposed structure as a formal component of the teacher education undergraduate program.

The aim is not, either, to include the participation of school teachers with the same kind of role they could have alone in another curricular component, such as internship supervision. The structure of *shared teaching* may establish an environment of collective interaction that stimulates reflections on the academic mathematical knowledge from the standpoint of its social and historical production processes, and, reciprocally, reflections on the mathematical knowledge produced at the school social context, recognized as a legitimate kind of knowledge, from an academic standpoint. Thus, the aim is to constitute, in the prospective teacher education programs, the cultural environment of the profession. We align with the stance that to prepare teachers is not to assure they know any prescriptive list of topics or categories of knowledge, but to assure they build up a critical understanding on the relevance of the different kinds of knowledge to their professional activity and, as Nóvoa (2009) states, *to integrate them in the culture of the profession*.

From this perspective, we consider *shared teaching* itself to be an aspect as important as the *integration of knowledge from school practice* in prospective teacher education. Sharing teaching does not regard only the two teachers lecturing together, but the whole environment of sharing knowledge and experiences from all the actors participating –

teachers and students. This is consonant with Tardiff's principles 4, students as actors of their own education, and with one of the Nóvoa working proposals for prospective teachers' preparation, teamwork and the collective practice. The relativization of the figure of the single teacher, as the sole holder of one single dimension of knowledge, transforms the whole topology of the classroom, and establishes an environment where all the actors are simultaneously teachers and learners.

However, this different topology may face challenges, such as the following one, raised in the group interviews, about the generality of the experience: despite that prospective teachers generally assessed the experience as "positive", for some of them, there was a considerable dissent whether it would "work so well" in other modules, with other teachers. For some of them, other modules, such as Calculus 3, are "too far apart from school", and it would be hard to establish links. Other prospective teachers claim that other university lecturers may not be open to engage in a structure so different from the "usual", as they consider Victor's and Fábio's profiles as "unusual".

This aspect can be interpreted as limitation of the proposed structure, as it seems to be strongly dependent of the profiles and attitudes of the teachers involved, and therefore not easily generalizable to other cases. Indeed, we see that Fábio and Victor were both engaged with research in Mathematics Education and that they were aware of the research project rationale and aims as determinant to the experience's results. However, this was not incidental. On the contrary, we consider the fact that the two teachers responsible for conducting the experience were also part of team that designed the research project, and therefore aware of its guiding framework and premises, as part of the project conception itself.

5.3. Perspectives and reflections

Taking the discussion on generalization into account, we place our goals for future outcomes of this project within the need to rethink pre-service teacher education (which has been acknowledged by recent Brazilian legal prescriptions). An important accomplishment towards this direction has been achieved: the shared teaching structure has been institutionally approved as a curricular modality in UFRJ's mathematics teacher education undergraduate program, and is adopted in modules focusing the discussion of mathematical content from the perspective of knowledge for teaching. Nevertheless, further research may shed light on shared teaching dynamics in other environments, and involving actors with different backgrounds.

In this direction, the experience reported in this paper had also an influence in our research team's practices. Most of the team members have incorporated shared teaching structure in their practices, and are currently conducting such experiences with actors in different roles, in different settings: from elementary school to graduated programs. Some of these experiences will lead to new research projects. For instance, our team is currently designing a proposal of sharing teaching in pre-service teacher education involving also *elementary and secondary school students* (besides university lecturers, school teachers, prospective teacher). Thus, our ongoing research focus shared teaching in different educational settings, and may contribute with reflections upon teacher pre-service and in-service education, and professional culture.

Other theoretical reflections that may arise from this project – and that also demand further research – concern the nature and potential range of teachers’ collective work. Several authors (e.g. Fiorentini & Crecci, 2016; Davis & Simmt, 2006) focus on teachers working collectively, from different theoretical standpoints, but with a common overall structure: teachers discuss and reflect together on lessons each one of them has taught or will teach on their own. In our research, collaborative work reaches a broader scope: *teachers discuss and reflect together on practices they share*. This places the debate on teachers’ knowledge and practices in a different cultural perspective.

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