A 'GLOCAL' LESSON STUDY: THE CASE OF PEDAGOGICAL PRACTICES IN MATHEMATICS

UMA ESTUDO DE AULA 'GLOCAL': O CASO DAS PRÁTICAS PEDAGÓGICAS EM MATEMÁTICAS

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ABSTRACT
This article describes and discusses the education process and the professional learning in the prospective teacher education in the Pedagogical Practices in Mathematics course. The course aimed to know and problematize the teaching and learning practices in the school. The activities of the prospective teachers were developed under the Lesson Study methodology. Prospective teachers have developed a 'glocal' Lesson Study from the choice of topic -relevant to the school curriculum - along with lesson planning, sharing and discussing the lesson proposals, lesson implementation, lesson analysis, and presentation/discussion of results, culminating in the writing of articles. The process and some results of the implementation of the Lesson Study in a pedagogical discipline of the degree course in mathematics will be highlighted. Finally, the continuous opportunities for teacher learning that the graduates had in this formative experience, in a context of reflective and investigative participation in the practices of teaching and learning mathematics in the school will also be focused. That is, the prospective teachers learned from the moment of the choice of the theme, through the socialization and joint discussion of the planning and the execution of the class, culminating in the systematization of the lived experiences.

Key words**:** Mathematics education, *glocal* Lesson Study, Prospective teacher, Pedagogical Practices in Mathematics.

RESUMO

Este artigo tem por objetivo descrever e discutir o processo formativo e a aprendizagem docente dos futuros professores em um Curso de Práticas Pedagógicas em Matemática, durante a formação inicial. O curso visou conhecer e problematizar as práticas de ensino e aprendizagem na Escola. As atividades dos futuros professores foram desenvolvidas sob a metodologia Estudo de aula. Os futuros professores desenvolveram um Estudo de aula 'glocal' desde a escolha do tema - pertinente ao currículo escolar -, junto com o planejamento da aula, compartilhamento e discussão das propostas de aula,

implementação da aula, análise da aula e apresentação/discussão dos resultados, culminando com a escrita de artigos. Será destacado o processo e alguns resultados sobre a implementação do Estudo de aula em uma disciplina pedagógica do curso de licenciatura em matemática. Finalmente serão apresentadas as contínuas oportunidades de aprendizagem docente que tiveram os licenciandos nessa experiência formativa em um contexto de participação reflexiva e investigativa das práticas de ensinar e aprender matemática na escola. Isto é, os futuros professores aprenderam desde o momento da escolha do tema, passando pela socialização e discussão conjunta do planejamento e da execução da aula, culminando com a sistematização das experiências vividas.

Palavras-chave**:** Educação Matemática, Estudo de Aula *glocal*, Formação de professores, Práticas Pedagógicas em Matemática.

1. Introduction

Since 2006, the research group Pedagogical Practices in Mathematics (PraPeM) from the Education Faculty in the University of Campinas (Unicamp), proposed the inclusion of the Pedagogical Practices in Mathematics (PPM) Course in the compulsory curriculum of the Degree in Mathematics. The PPM course proposal for the second half of year 2016 sought to adopt the Japanese teaching methodology *Lesson Study* (LS), which has many advantages in teacher education and research based on the concerns of teachers, from what to plan on. From the LS perspective, there are multiple surveys conducted in several countries, mainly as part of the teacher continuing education, not only in mathematics, as it has been proposed from the first studies of LS in Japan, alluding to its slogan "Teachers learning together". In addition, studies by Bjuland & Mosvold (2015), Fernández (2002), Fernández & Yoshida (2004), Fernández and Chokshi (2002), Sims and Walsh (2009), and Fujii (2014; 2015), among others, discuss favorable results to the use of such methodology, as both researchers and trainers in the classes in which they were researching.

This article aims to describe and discuss this formative process and teacher learning in pedagogical practices in a Mathematics Course, which is characterized by the analysis and problematization of the teaching and learning mathematics practice. In this matter, we assume both roles of instructors to the course and analysts of the results of this formative process under the LS methodology. As follows, we present the theoretical basis of the Japanese LS, some studies developed in the mathematics teacher formation and, then, their implementation in the development of the subject Pedagogical Practices in Mathematics. Later, the learning of the prospective teachers, which happened throughout the Goal-setting steps in the PPM course, will be described, analyzed and discussed.

2. *Lesson Study* as learning and professional development methodology

Lesson Study is a term that translates the Japanese word *jugyokenkyu*[1] (Cf. Fernandez & Yoshida, 2004, p. 7). In Japan, the term denotes the study of teaching practices. It has been used since late 1990s, and it is the subject of study of several international mathematics education research works, both in elementary school and in pre-service and in-service teacher training, as presented in studies by Lewis (2002), Fujii (2014; 2015; 2016), Bjuland & Mosvold (2015), Fernández (2002), Fernández & Yoshida (2004), Fernández and Chokshi (2002), and Sims & Walsh (2009), among others. These studies identify the potentialities of collaborative work and/or the LS in the professional development (teaching and learning) of the mathematics teacher.

The LS is considered, then, a form of joint learning among teachers who plan, teach, discuss and reflect to improve teaching practices in schools. Thus, White et al. (2013), when discussing the existence of different variations of the Japanese LS around the world stated that it "has led to many other designs of professional development programs which, although resonating with some of the aspects of the LS process, cannot be strictly classed as an LS model" (p. 416). In this way, the LS constitutes a global idea of teacher training and professional development, of collective and collaborative action, as well as of joint learning. However, the applications of this model in different cultures/schools make the model local to each one. An idea of it has been developed, where they join the Global to the Local LS, giving way to the *glocal* perspective:

> [...] Local culture needs to be considered very carefully when appropriating successful, global ideas. [...] glocal is understood as the result of the interaction between a global pedagogical model of LS and the local school-culture site. [...] The meaningful integration of local and global forces can help teachers to enhance their teaching practice (Grimsæth & Hallås, 2015, p. 111).

The Japanese LS has been consolidated as Global proposal in Mathematics Education that "provides a collaborative process for teachers to make sense of educational goals and standards and to bring them to life in the classroom" (Lewis, 2002, p.8). However, the proposal has been adapted to the *local* conditions of each country, each region, and even each school, always trying to maintain the LS principles of "learning together" or "doing together". We present below some results of the LS and three model proposals (Lewis, 2002; Fernandez & Yoshida; 2004, Fujii, 2014) from the global LS process, and how they were adapted to propose the *glocal* LS used in the Pedagogical Practices in Mathematics course (PPM course).

The LS is a Japanese teaching methodology with favorable results in teaching. Particularly in mathematics, different factors make the LS a successful methodology, not only as part of the recognition of teaching difficulties, but also of the importance of good planning prior to the lessons, as Lewis acknowledges:

> Lesson study recognizes the central importance and difficulty of teaching. [...] Lesson Study invests time and resources in planning, studying and refining what actually happens in the classroom. It is a system of research and development in which teachers advance theory and practice through the careful study of their own classrooms, constantly testing and improving 'best practices' (Lewis, 2002, p.12).

[1] The compose word means: Lesson (*jugyo*) and Study/Research (*Kenkyu*).

In this sense, teaching presupposes the reason for studying the LS, so that a set of teachers work together through student learning and long-term development. Teachers who plan together are also invited to discuss and reflect together after applying the Lesson Plan. Considering the Research Lessons are the centerpiece in the LS model, such methodology is cyclically proposed in the handbook of the 2002 LS, as shown in figure 1.

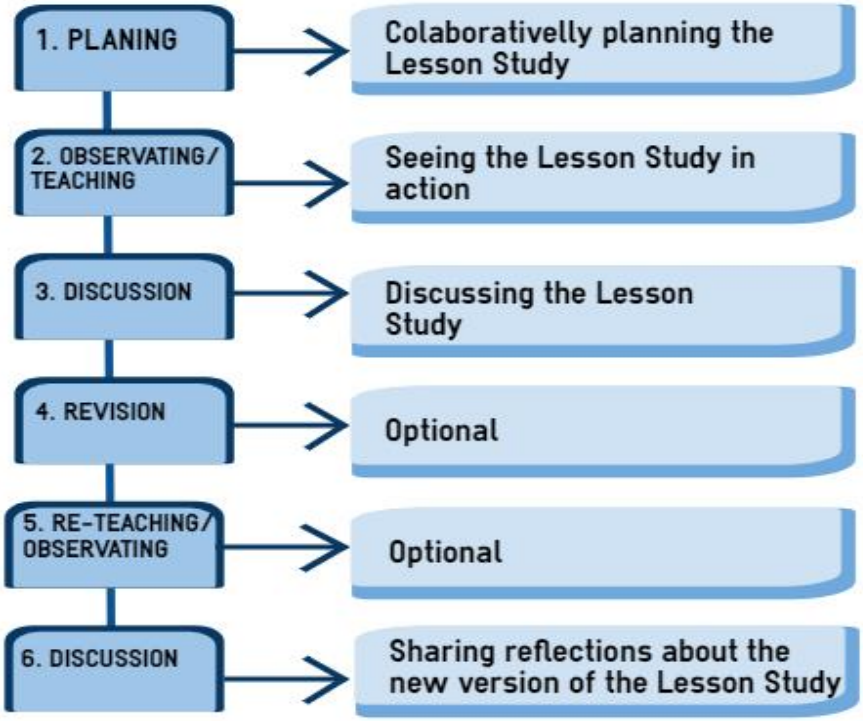
Figure 1- Lesson Study Cycle in Lewis

Source: Lewis (2002, p.3)

This LS proposal was an adaptation of the original Japanese model, and later applied in the U.S., in the 1990s. Initially, the author defines the Goal-setting, as identity goals for student learning and long-term development. That is, Goal search is a search for teaching thinking of long-term development. In the same step, the Planning is built collaboratively and refers to the initial designed plan to bring to life these goals, including the "research lesson" that will be observed. Later, in the Research Lesson, one planning team member teaches a classroom lesson, while other team members collect data on student thinking, learning, engagement, behavior, etc. After application of the Lesson in the process of Lesson Discussion, the group meets to share and analyze the data collected at the research lesson and tries to answer the questions: what evidence aiming at student learning and development was fostered? What improvement to the lesson and to instruction should more generally be considered?" (Lewis, 2002, p. 3). Finally, the cycle ends with the consolidation of learning. In this case, if the group decides to discuss the lesson, reformulate it and improve it, it is applied again. It is advisable that the group of teachers do write report that includes lesson plan, student data, and reflections on what was learned.

Later, we find the LS processes proposed by Fernandez & Yoshida, which highlight six steps, as shown in Figure 2. The processes follow the Planning, in which a group of teachers elaborates a collaborative planning, sharing their ideas about “how to best design the lesson by drawing on their past experiences, observations of their current students, their teacher’s guide, their textbooks, and other resource books” (Fernandez & Yoshida, 2004, p. 7), finally obtaining the Lesson Planning.

Figure 2- The Lesson Study process in Fernandez and Yoshida



Source: Fernandez and Yoshida (2004, p.7)

Like in Lewis's proposal, the second process described here, Teaching/Observing, refers to the observation implemented in school class. However, this implementation is of a public nature. Observers (may be other teachers) participate by attending the classes in the classroom and only observe (not teach) and analyze if the Lesson Planning is executed in its entirety; and make records of what was not contemplated in the plan to make further improvements.

In the third phase, *Discussion*, the teachers and the teachers/observers gather to reflect on what happened in the classroom, report reactions of students that caught their attention, and offer some suggestions on the development of the class. The processes of Revision and Re-Teaching/Observing are optional, as they depend on the application and data spaces in schools where Lessons run. The Revision corresponds to the preparation of a new plan to be applied in the classroom and, again, bring their thoughts to the last moment, the Discussion. This is a linear model that ends with discussions and reflections on the application of a new version of the Lesson.

Figure 3- The Lesson Study process in Fujii

Source: Fujii (2014, p.113)

In 2014, and after the writing of different articles on the LS, Fujii presents the cyclic model of Figure 3, composed of five processes to the methodology. In the first one, Goal Setting, long-term goals for the student learning and development are considered and, finally, the research theme is formulated. In the Lesson Planning, Fujii adds to the proposals of Lewis, Fernandez and Yoshida on the collaborative planning:

> "a document that describes the research theme, content goals, connections between the current content and related content from former and later grades, rational for the chosen approach, a detailed plan for the research lesson, anticipated student thinking, data collection, and more" (Fujii, 2015, p. 412).

In the Research Lesson process, the analytical observation and collected data are described. In the next step, Post-Lesson Discussion, the observer shares data about learning, disciplinary content and lesson and unit design, and broader issues in teaching and learning. Finally, Reflections is the moment to consolidate professional learnings, and raise new questions for the next cycle of LS.

In the three models the processes of LS are presented, being common to find the time to questioning and identification of Lesson (Goal Setting), and basically answer the question: what to teach? This question, in turn, is based on other ones that invite us to answer: in which subject of the current curriculum (of the grade that I teach) could the students have the greatest learning difficulties? Why is the chosen topic of interest of all teachers that will meet to plan? Why do I need to make a different and planned lesson with more dedication than normal? Do other teachers have the same difficulties teaching this Goal?

The next step is the planning (Lesson Planning), in which they discuss some basic issues of Goal teaching. Now, the question would be: how to develop the Lesson? This leads us to other questions, such as: what are the possible difficulties for students to learn this Goal? What are the possible interactions (questions/answers) among students in the classroom as we develop this Lesson?

Subsequently, the moment of Lesson application occurs, to which the question is: which part of the planning is happening and which is not? From what can be seen, what interactions were not foreseen in Lesson Planning? Is there something planned that is not happening? In a moment of later discussion, when what happened in the classroom and the effectiveness of the proposed plan are evaluated, it is suggested that this question is answered: what could be improved from the initial Lesson Planning? In some of these models, it is proposed to apply the Lesson again with some improvements, in another group. Although these moments of the process are standardized in each proposal, they are not used in the same way everywhere, because learning is a social, historical and cultural fact.

We can emphasize, among other research works including LS that has been developed, different variations of the original Japanese model for *glocal* proposals, as in the case of Canada, "that research possibilities and challenges of implementing LS in teacher education" (Chassels & Melville, 2009), and the Indonesian LS, "that research observable changes in relation to the academic base of the lessons, the structure of the lessons, as well as in the pupils' reactions, when LS was implemented" (Saito, Harun, Kuboki, & Tachibana, 2006), as well as the ones developed in the U.S. by Lewis (2002; 2016). Based on this review, we believe that it is important to investigate implementations in Pedagogical Practices in Mathematics course, with prospective teachers without preparation for the teacher-researcher life.

3. Research design and methodology

3.1. Pedagogical Practices in Mathematics course

Since 2006, the research group Pedagogical Practices in Mathematics (PraPeM) from the Faculty of Education proposed the inclusion of the Pedagogical Practices in Mathematics (PPM) Course in the compulsory curriculum of the Degree in Mathematics. The course is offered semiannually for the students during the last terms of the Degree. It is considered a course that seeks to (re) configure itself every semester in which it is offered. Particularly in the second half of 2016, the course was developed in a 30-hour length distributed in fifteen classes of two hours a week. The PPM Course sought to promote studies and research of pedagogical activity in mathematics at school, that is, the lessons of the subject were developed from three perspectives: *i) academic*, which meant theoretical and epistemological studies produced from research in/from the mathematics classroom, or conceptions of learning in the classroom; *ii) professional*, which proposed the use of reflective/investigative narratives written by practicing teachers and which were brought for analysis of students along the course; and, *iii) school practice*, through participation, observation, recording and analysis of mathematics teaching-learning classes. These three perspectives helped us analyze math classes mainly by considering the concepts of situated learning and LS.

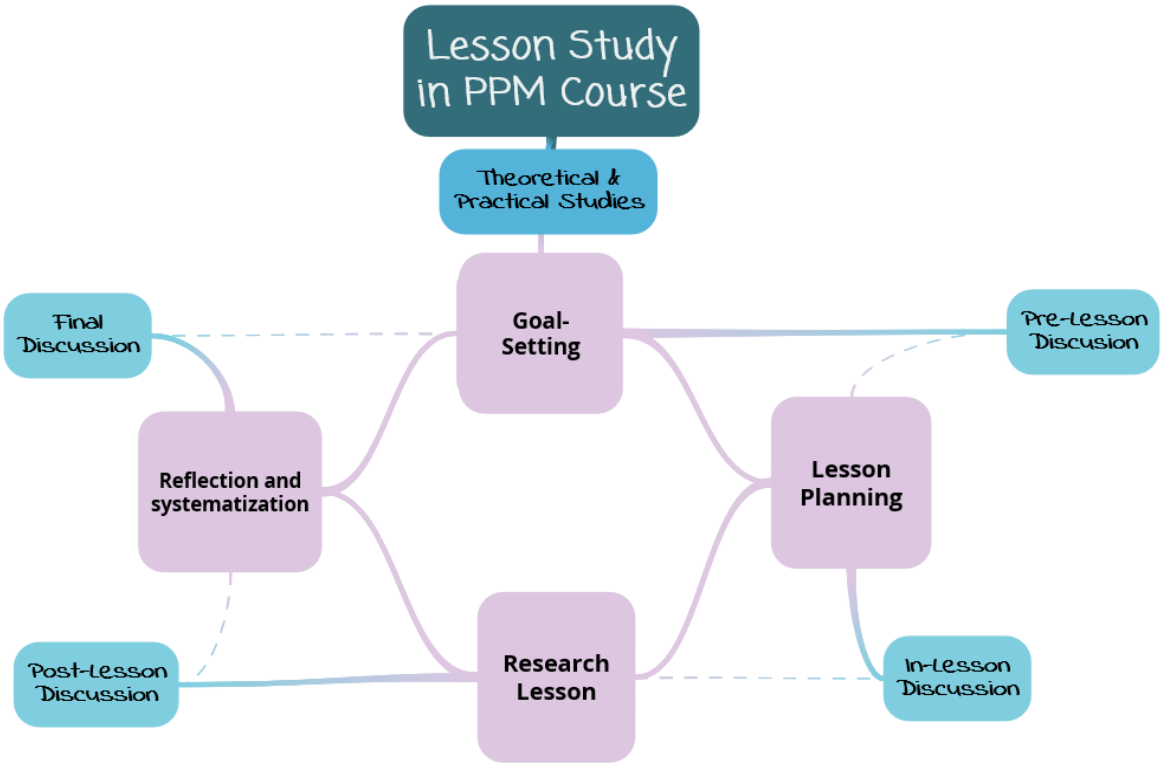
One of the proposed approaches to the *Pedagogical Practices in Mathematics* (PPM) course is the Japanese LS. Therefore, in the LS, the learning develops and consolidates itself in the practices of being a teacher. In this regard, the Lesson Study, in the initial training of prospective teachers, allows the fostering of teacher professional learning to the math teacher according to Shulman (1987), Ball (1996) and Carrillo (2013), not only for future practice, but in current school practice, and mediated by the research practice

investigation (Lave & Wenger, 1991; Cochran-Smith & Lytle 1999). Similarly, the LS, in each of the moments in which it runs, also allows preparation for the teaching practices of *teaching-learning*. According with Carvalho and Fiorentini (2013, p. 11), from the initial study of tasks/exploratory-investigative activities of school mathematics, through the questioning process and deconstruction of the exercise paradigm (Skovsmose, 2000, p. 66), prospective teachers come to very interesting analysis and reflection processes.

In this sense, we can affirm that the development of the PPM course used a collaborative cycle, according to the meaning of Grimsæth & Hallås (2015) of *glocal* LS, based on the models proposed by Lewis (2002), Fernández and Yoshida (2002) and Fujii (2014), but adapted to the teaching conditions for the Mathematics education program of Unicamp, and compatible with the course schedule. In Figure 4, we present different moments of the *glocal* LS cycle adopted by the PPM Course.

In the Figure 4, highlighted in blue and located outside of the diagram, there are the points that have been developed in/with the group of 23 prospective teachers, such as: Theoretical and practical studies, and discussion moments (Pre-Lesson Discussion, In-lesson and post-lesson Discussion).

Figure 4- The process of LS in the PPM Course



Source: made by the first author

Also, within the figure and in pink, there are the moments attended by each of the six groups, which correspond to the specific steps of the original Japanese LS: Goal Setting, Lesson Planning, Research Lesson, and Reflection and Systematization. Below in the Table 1, we describe briefly each part of the *glocal* LS cycle in the PPM Course, participants and the time used for their development over a fifteen-class course at the University of Campinas.

Table 1- Processes description in the *glocal* LS

Process	Description	Lessons quantity	Meeting place	Time (hours)
Theoretical and practical studies	Initial meeting and conducting theoretical and practical studies on the concepts to be developed by the students during the course, such as: situated learning, LS, discussion of narrative exploratory-investigative practices and activities of mathematics lessons.	5 Lessons	Unicamp	10 hours
Goal-Setting	After the observations in the classroom, the prospective mathematics teachers identified the possible goals for student learning. In addition, goal setting was chosen in a collaborative and negotiated form.	2 Lessons	Unicamp	4 hours
Pre-Lesson Discussion	Each group present and discuss its proposal to the general group of the PPM course. The groups received suggestions from their colleagues and course professors about materials and alternatives on tasks and activities in the classroom.	1 Lessons	Unicamp	2 hours
Lesson Planning	In most cases, a lot of self-study was needed to complement their studies on goal-setting. The group was meeting to record the lesson into the Lesson Plan offered by the professors with: activity title, objectives, course, contents, activity description, methodology and assessment.	1 Lessons	Unicamp	2 hours
		0 Lessons	Extra-scholar meeting	6 hours
Research Lesson	The activity resulting from planning was applied in real conditions with the teacher in the School chosen.	1 Lesson	School	1 hour[2]
In-Lesson Discussion	Within the development of the activity, the prospective teachers felt the need to change some of the steps previously planned due to the needs of students, the unforeseen questions, the unexpected results, or simply the fact that the form of registration proposed did not let them see the potential of the activity; or, as a different activity compared to what they were used to, the students did not feel able to start solving the task or failed to			

[2] Depending on the need to improve the class, some groups needed to return to the schools after better adjusting the planned tasks. This was the case of the G3 and G6 Group.

	engage in the activity; or, also, because they could not understand the instructions of the assignment.			
Post-lesson Discussion	To assess whether the objective proposed in the Lesson Planning and the Research Lesson has been reached. It is at this stage to verify what instructions and tasks or activities allowed the proposed objective to be achieved and, above all, look at the professional learning of prospective teachers, especially when faced with real situations.	1 Lesson	Unicamp	2 hours
Reflection and Systematization	A few episodes to share and discuss with all colleagues of the course were selected, in order to question the practices and receive contributions to the final article writing, highlighting indications of improving teaching and learning for all involved. Into the Group, they discuss together their reactions, observations, questions, comments and reflections on the implementation of Lesson Planning. For the systematization, the proposition was to write a paper[3].	2 Lessons	Unicamp	4 hours
Final Discussion	To every paper written by a Group, another Group was assigned as critical reader, so that after the presentation of each paper, being two per class, it was for the group designated as critical reader to organize an evaluation and discussion of the work, with some suggestions to improve the paper.	2 Lessons	Unicamp	4 hours

Source: Constructed by the first author

In this article, we present an interpretative case study centered in the course. The reasons for choosing the PPM course as a case study is because this is not common into the perspectives of mathematics education training as strategy for the prospective mathematics teacher education. In other words, the PPM course had differentiated and atypical prospective teacher experiences in the Mathematics education program into the University of Campinas (Unicamp-Brazil).

[3] For most prospective teachers, this was the first experience of writing an article, which required the use of a post-Lesson Discussion class to explain to them the model and general guidelines for writing.

3.2. Research participant

The course, which lasted about four months, was constituted by twenty-three students of the Mathematics Institute [4] at Unicamp (IMECC/Unicamp). They were grouped voluntarily within the groups mentioned in Table 2. As a condition for grouping, the criterion was that at least one of the prospective teachers should be practicing as a teacher, trainee or a scholarship holder (PIBID[5] or PIBIC[6]) in a public or private school. In this table, we present the goals-setting chosen and the school grade to teach.

Table 2- Groups into the PPM course

Group	Participants	Goals-Setting for student learning	Scholar grade
G1	4	Area, perimeter and volume in geometric solids with origami constructions	7th
G2	4	Area and perimeter using recoating with triangles explorations.	9th
G3	4	Experiments for probability of events with coins explorations.	Non-scholar course[7]
G4	4	Area and perimeter in circumference with *pi* explorations.	9th
G5	4	Trigonometric relationships in similar rectangular triangles with similarity concept explorations.	9th
G6	3	Combinatorial analysis with hat positions.	Non-scholar course[8]

Source: Constructions by authors

In the Table 2, we can highlight the interest of the prospective mathematics teachers to explore the geometrical concepts, probability, combinatorial analyze and trigonometric relationships.

3.3. Data collection

We had only written records as sources to analyze in this first PPM course as *glocal* LS. In the Table 3, we describe the sources, the context and the audience in each source.

[4] The students were from the Mathematics Institute, someone from the Mathematics Education Program (Teaching Degree in Portuguese) and others from the BA area (Bachelor Degree in Portuguese).
[5] PIBID is an undergraduate program to become a teacher.
[6] PIBIC is an undergraduate program to become a researcher.
[7] Small course, with public students from different places, that prepares students for the national exam to enter public universities in Brazil. Generally, these students are from public schools and/or students without resources to pay for a certificate course to enter the national exam.
[8] Mathematics Reinforcement Project for High School Students.

Table 3- Data collection in the *glocal* LS

Source	Description	Context and audience
Memorial records	Individually, the prospective teacher writes a short record after reading the lectures and before the Lessons in the *theoretical and practical studies* process. In total, each prospective mathematics teacher delivered 3 memorials.	23 students as recorders 2 professors as reviewers
Lesson Planning	Each group writes the Lesson with the *Goals-Setting for student learning* in the Lesson Plan model given by the professors.	6 Groups 2 professors as critical reviewers
Initial Papers	Each group delivers and discuss their first written version of the paper, which should bring the first attempt to systematize the experience lived.	1 Group as presenter 1 Group as critical reviewer 2 professors as reviewers
Final papers	With the suggestions and their own reflections, the group writes the final version of the paper and delivers it to the professors.	6 Groups 2 professors as critical reviewers

Source: Constructed by the first author

Is important to explain that, in the initial papers, each group had one group as peer reviewer and critical reviewer. Although only one group was assigned as peer/critical reviewer, the other groups also had the opportunity to participate in the discussion and contribute to improving the texts. To write the academic article, the trainers proposed a template with ABNT technical standards. For most prospective teachers, this was their first experience writing an article, which required the use of a Post-Lesson Discussion class to explain to them the model and general guidelines for writing.

3.4. Analytic procedures

We developed the analysis with narrative analysis of the data collected, according to Riessman (2002) and Bamberg (2012). We selected, organized and interpreted the data by ordering experiences and making sense to a particular audience. The analysis was developed in two stages. First, we collected the records from the prospective mathematics teachers (memorial records), and from the six groups (Lesson planning, initial papers, final papers) in the Pedagogical Practices in Mathematics course. In this first stage, we could identify significant contributions to the professional learning in the prospective mathematics teacher in each stage of LS application. Because of these results, we decided to encourage the second stage to organize the data analysis in four parts, answering to the temporal development. That is to say, the temporality refers to our chosing "among several options to make a story cohesive" (Bamberg, 2012, p.81). This is to say, the data analysis attends to answer the next five questions:

- What was identified by the prospective mathematics teacher as problematic or necessary into the teaching-learning practices in one *glocal* LS?
- How does the prospective mathematics teacher problematize the teaching-learning practices in one *glocal* LS?

- What reflections emerge from the teaching-learning practices during the participation teaching-learning practices in one *glocal* LS?
- How is the data collected used to systematize the experience in one *glocal* LS?
- How do the LS discussion moments contribute to the professional learning of the prospective mathematics teacher?

Finally, to describe and discuss the participation processes in one *glocal* LS, in the Pedagogical Practices in Mathematics Course characterized by the analysis and problematization of the teaching-learning practices.

4. Narrative analysis

Next, the narrative analysis of the collected data will be presented, aiming to describe and discuss the four internal processes of our *glocal* LS (Goal-setting, Lesson Planning, Research Lesson and reflection and systematization). The Discussion moments (Pre-Lesson, In-Lesson, Post-Lesson and Final discussion) will be analyzed in a transversal way to the development of the four processes. For the narrative analysis, we will use some episodes, by textualization, that demonstrate the importance of the processes in the development of LS, as well as their contributions to the learning of the prospective mathematics teachers.

4.1. Goal-setting

In the seventh lesson, after reading and discussing the articles, narratives and lessons on exploratory-investigative classes, the prospective mathematics teacher identified the potential for meaning of mathematical learning whenever a favorable learning environment is included, such as the inclusion of exploratory-investigative activities. In order to approach the teaching and learning practices of mathematics at school, prospective mathematics teachers observed classrooms in which one of the members of the group was a teacher (pre-service, in-service or PIBIC / PIBID scholarship holder). After approaching the school practices of teaching and learning, by observing the classes directly, the prospective mathematics teacher identified the possible mathematical contents to teach through planning of LS. Goals-setting into the school practices of teaching-learning were problematized. That is, after "shar[ing] their ideas for how best to design the lesson by drawing on their past experiences, observations of their current students" (Fernandez & Yoshida, 2004, p. 7), they identified the possible goals for student learning.

Groups G3 and G5 chose the theme because of the students' needs to learn and / or deepen the concepts studied. These two groups asked students about their needs by using the brainstorming technique, which is based on the dialogue between the students of the school, the teacher and the members of the group as mediators. They come to a common agreement on the theme. In contrast, groups G1, G2, G4 and G6 made the choice among its members, according to the criteria of the need to deepen the themes. However, in all the groups, the Goal-Setting for student learning was chosen in a collaborative and negotiated way. It was interesting to see that the major focus was on geometry themes (G1, G2 and G4), and the other groups focused on the teaching of probability (G3), trigonometry (G5) and combinatorial analysis (G6). As consequence, the prospective teachers approached the planning.

4.2. Lesson Planning

To begin this process, the prospective mathematics teacher looks for articles, didactic and curricular materials. This was the first time that they think and write activities, methodologies to one lesson plan in their Mathematics education program. For this moment, some materials (textbooks, articles or teaching materials) were taken to the classroom with some suggestions of tasks that could be adapted to become exploratory-investigative activity. For this process, Fernandez and Yoshida (2004) proposed the Lesson Planning with details, such as possible answers and difficulties from the students in the Lesson class. Our model differs from theirs, as well as from the models presented by the *glocal* LS. In general, although the prospective teachers have taught groups or individuals, most had not yet written a Lesson Planning. Thus, to prepare them for this step, the trainers gave them some suggestions of Lesson Planning, guiding them on what to register in each space, according to the model. The reason is that they only had their first approximation to Lesson planning as (future) teacher activity during this Pedagogical Practices in Mathematics course. This is to say, we suggested that the students proposed the exploratory-investigative activity, in general terms, into the one table with basics aspects, for example: activity title, objectives, course, contents, activity description, methodology and assessment. Although the prospective teachers planned on the shape created and suggested by the first author, they did not keep in mind the possible answers, unexpected answers and possible errors of the students when carrying out the proposed activities. In spite of this, the activities presented were changing, as the socialization and discussion of the activities in the PPM course helped them to include aspects that were not contemplated in the initial plan.

The Lesson planning was an important process for the prospective mathematics teacher. According to their reflections, in most cases, they reaffirm the importance of planning in advance. The G5 group made a complete and detailed planning of the classroom activities, dedicating a number of out-of-school hours to it. In their reflections, students refer to success in designing a plan in which they "anticipate all the possibilities (of response) we can imagine that could occur during the lesson." However, even though they had contemplated the possible answers, they had unexpected responses due to miscalculation or inappropriate use of the ruler to calculate the measurements. Other reflections were raised in group G4 compared to the use of the proposed planning format. The prospective mathematics teachers identified that the proposal they constructed was not as complete as the one made by the teacher. In fact, the G4 still emphasizes in the reflections that the plan "was incomplete and little systematized, we only sequentially list what we would do in the activity, without thinking about the different possibilities of students' responses and actions for each situation".

On the other hand, groups G1, G2 and G6 identified in planning an opportunity to predict what might happen in the classroom. However, the imagination of what happens in the classroom conflicts with school reality. As G1 says, we did not "think much about the obstacles that could arise in the classroom" or, as G2 stresses "we could not correctly deduce the time for the activities, we could not carry out deep analysis that are necessary for the application of an exploratory-investigative activity and we could not predict many situations that would happen". In addition to the time factors and (didactic) obstacles in the classroom, the G6's prospective mathematics teachers refer to the planning, not only prior to the development, but the materials and the classroom to begin the Research Lesson. In this sense, G6 went to the classroom "unprepared, because they did not print the problems that they would propose and did not anticipate

that the students could have difficulties to understand the theme" due to problems of visualization or oral understanding of the exercises that had to be printed. Finally, despite the differences between this *glocal* LS and the traditional strategy, the prospective teachers thought in general situations that could happen in the classroom, until they feel comfortable to teach. This writing approach, to which teachers are subject in their routines, it is another way to be prepared for teaching.

4.3. Research Lesson

This process implies the teaching and the observation from the prospective mathematics teacher, in real teaching-learning practices. To problematize the participation, the prospective mathematics teacher tried to perform as teachers in the classroom, observing each other's practices. Although the Lesson Planning considered some actions, reactions and interactions of students and teachers, some setbacks happened in real time application, which required an intra-group discussion, demanding some decision making. We call this In-Lesson Discussion, which will be covered later.

Therefore, the real conditions in the classroom made changes happen to the previous lesson planning, which also changed the way of planning, as with G1 and G5. They used exploratory investigative activities, but sometimes the (prospective) mathematics teachers need a new approach. Thus, as G4 says, the implementation of an exploratory investigative class "requires a change in posture, not to respond immediately to students' questions, but take them to their own answers instead".

In cases reported by most groups, they refer to the need to know beyond the 'right' responses of activities, but from unexpected responses, unforeseen records in the notebook, or unexpected reactions that are motivated by the different type of activity. An example of this type is proposed by the G4 in which it states that: "results that we did not expect have emerged, such as the group that concluded that the measure of the length of a circumference is greater than that of the diameter of the same circumference". These discoveries led prospective mathematics teachers to problematize planning and redirect the course of the lesson. Likewise, a more rigorous planning would lead to a greater control over the knowledge of the (possible) answers, the timing, the students' attitudes towards the class, and the same preparation of the teacher in front of the class. In particular, G3, G5 and G6 manifested some difficulties in pretending that the students "get by themselves" to the answers. Another reaction found in developing the Research Lesson is waiting for answers from students who do not normally participate in the classroom. The G6 emphasizes that in addition to everything planned, it is necessary to "have patience and follow the students' time so as not to hasten exploration and so the student can be the protagonist of their teaching". G2, in turn, states that because the concept is different, prospective mathematics teachers found it "difficult to make students understand that they should seek different methods to develop the activity and that the teacher would be there to encourage them and direct them impersonally, that from their investigative sense, they would proceed to discover the expected formulas". In this sense, when changing course, some activities gained conceptual wealth, as in the case of G2 and G4, while other activities referred to the loss of the purpose, becoming a more basic activity than the one proposed, as the case of G1.

Finally, also throughout the class, the prospective mathematics teachers have been able to make records in writing or by video or audio recording, which were used to analyze, systematize and present to the PPM Course colleagues, especially during the time of Post-Lesson Discussion.

4.4. Reflections and systematizations

In this process, it is important to assess whether the objective proposed in the Lesson planning has been reached. This was a moment to organize and discuss all the experience with LS, highlighting the successes and learning and, also, the failures and difficulties. It is at this process to verify what instructions and tasks or activities allowed the proposed objective to be achieved and, above all, look at the professional learning of prospective teachers, especially when faced with real situations. In the PPM course, the prospective teachers, from what happened in Lesson Research and in the In-Lesson Discussion, selected a few episodes to share and discuss with all colleagues of the course, in order to question the practices and receive contributions to the final article writing, highlighting indications of improvement of teaching and learning for all involved. In general, the prospective mathematics teacher found it difficult to decide what to put in that short space of eight pages, which included an introduction, literature review and theoretical background, experience context, methodology of teaching and research, reflections, discussion of the results and conclusions, ending up with the references.

Only groups G3 and G6 prospective teachers had the opportunity to go back to the classes and make a second application, now including, on one side, their reflections about the first class and, on the other side, contributions and suggestions from the prospective mathematics teachers' colleagues. The reflections presented by the students emphasize, mostly, the rights and wrongs in the planning process as a learning opportunity. The G2 highlights that during the reflection "we find methods to improve or even extinguish errors and thus, as a reflex, we perceive the first possibility to improve the way we act in the classroom". The G5 learnings refer to LS as an "extremely important activity because, as future math teachers, we can confirm the value of good planning for classes." Other learnings referred to by groups G2, G4, G5 and G6 are the 'wealth' of the use of exploratory-investigative activities. In particular, the (prospective) mathematics teachers of the G5 refer to this type of activity as an opportunity to "have the opportunity of experiencing in practice the difficulties and benefits of this type of practice that stimulates knowledge so much", since we not only learn from success in teaching and learning practices.

The G4 highlights the choice to talk about "our learning as students and teachers throughout this cycle (LS), especially in the application of the activity (Research Lesson), and this was the most problematic moment for the group", bearing in mind that they skipped the Lesson Planning document and did not plan as fully as they hoped to have. In particular, G4 emphasizes the success of LS lies in "analyzing one's practices, and, in particular, in group. That's why we have a more critical attitude towards the facts that occurred during the activity". These reflections were written seeking to question the practices of teaching-learning mathematics in school, but also to understand the reality experienced by the mathematics teachers in today's school. It was a space in which –from the analysis planned and what happened throughout the process– the prospective teacher managed to bring different situations to the discussion. The duty of registering the experience in the limited space of an article made them select relevant parts of their work, as well as photos, video transcripts and some notes that demonstrated learning as a product of what they had planned previously.

4.5. The Discussion moments

Many questions emerged, even concerns and problematization about *teaching-learning* mathematics in school. In general, the Pre-Lesson Discussion became a time in which the contributions of colleagues, active class participation and suggestions from trainers made the lessons be designed collaboratively, and, in the sense of Wenger (1998), to negotiate the meaning of mathematics class planning and observation-participation in those classes. In particular, the Pre-Lesson Discussion process was emphasized by all groups, highlighting the elaboration of the exploratory-investigative task and the recording of a lesson plan and activities projection and (some) responses. Later, the In-Lesson Discussion process, G4 and G5 highlighted the importance of the dialogue and decision-making among prospective mathematics teachers during the Research Lesson, to complete the development of the previously planned task. The fact that three of the groups were in ninth grade of middle school made the group's participation interesting during the episode presentations. In the Post-Lesson Discussion process, G3 and G6 highlighted it as a 'strong basis' for a new Research Lesson application. That is, in presenting the results to the class, they concluded that they should return to class and go deeper into other probabilities and combinatorial experiments respectively. Finally, the prospective teachers also reported different conditions of their classes in schools, by bringing questionings and concerns regarding the times and the conditions for the conduction of the Research Lesson at school.

5. Conclusions

The development of the Pedagogical Practices in Mathematics course sought learning located in school and university settings. In this regard, the LS was an invitation for prospective teachers to problematize different teaching practices in mathematics and analyze the possible learnings (of the future teacher and student) that these practices could promote in school and university settings. As a result of *glocal* LS implementation in Pedagogical Practices in Mathematics course classes, we found different reactions from prospective teachers when analyzing critically the different moments in the development of the course, like the Lesson Planning.

In the different processes, we can notice the way in which the PPM course was developed. The prospective teachers were positively affected, in the sense that they developed activities that had not been seen in other courses of the Degree, besides teaching as pre-service, tutors or as junior scientist. That is, the course exceeded the expectations of the prospective teachers, and we can say that they had rich individual and collective learning, such as: identifying school content and school mathematics; identifying educational needs from another reading of the classroom; learning to do a lesson planning; making favorable decisions pre/in/post-Lesson; identifying the professional teacher performance; mediating with students during the Research Lesson; making and involving students in exploratory-investigative activities with students; collaborative perspective with teachers and classmates; and, finally, writing of an article.

It is a work that can and must continue, especially if there is cooperation among teachers of the Mathematics Institute and the Education faculty at the University of Campinas, because, for now, this collaboration is not happening. In this sense, this proposal is different from the original Japanese LS. Likewise, "in Japan, teaching is a professional occupation with life-long goals to be accomplished. This is the reason why LS is a purpose-orientated and continuing life-long practice" (Fujii, 2013, p.12). In the

final assessment to the PPM course, the prospective teachers stressed the need to include more educational activities of this type in the Mathematics education program courses. However, the time invested personally in the discipline and in the group meetings deserves more than 2 credits in the curriculum load, because these courses must be as important as the others in the Institute of Mathematics.

The LS is based on a model of continuous work and teacher education in the long term and from practice, which is a condition for the improvement of teaching and learning in schools. As stated by Slavit and Nelson (2010), interactions between teachers help them negotiate meanings into collective's issues. This article tries to clarify the *glocal* process of Lesson Planning and the role and function of the LS, based on case studies of Brazilian Pedagogical Practices in Mathematics Course.

The LS is a process that requires a joint learning of prospective teachers through a process of participation in activities typical of teaching, mediated by their reflection and research. In this regard, the LS helps develop a learning *from*, *in* and *for* the *practice* of the mathematics teacher, as to Cochran-Smith and Lytle (1999). However, this is a long and time-consuming process of teaching learning and professional development. In consequence, the teachers learn together from the actions, interactions and reflections into the research process (White et al., 2013). However, it is a consistent and continuous professional teaching based on study and research in teaching communities, with real needs.

The teacher develops their professional autonomy and authorship, being able to improve continuously their practice and their community, and update the curriculum, connected with other communities, for example, the academic community (Fiorentini, 2013). In this regard, White et al. (2013) agrees that the "LS can provide a process that enables and encourages collaborative professional learning and sharing between teachers and Mathematics Teacher Education" (p. 217).

Finally, we can recognize the richness of each of the desired moments in LS from the initial unease, the questioning of the practices, to the written observations, discussions and reflections. Observation, reflection, recording and analysis of teaching-learning classroom situations of basic school were important to the dynamics and development of the course. We emphasize here the importance of the four Discussion moments (Pre-Lesson, In-Lesson, post-Lesson, Final Discussion) and joint and collaborative participation of prospective teachers from the group as much as from the PPM course group. In this way, we can identify the Discussion as spaces for Learning together, as other voices, in the sense of Bakhtin (2003), contributed to the questioning of teaching-learning practices in basic school, in addition to mobilizing dialogues in the formation of groups and identifying common needs, which reflected in the construction of the questioning of a common situation in the field schools of prospective teachers. Learning was consolidated from the perspective of situated learning, in teaching practices (at school), as well as learning (in PPM course). As Acevedo and Fiorentini (2016) claim, teachers learn to teach (*for practice*) much more *in practice* and with other teachers in schools than in initial and continuing education courses taught by university professors.

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