

WHAT IS THE ROLE AND PLACE OF MATHEMATICS EDUCATION IN (SWEDISH) VOCATIONAL EDUCATION?

QUAL É O PAPEL E O LUGAR DA EDUCAÇÃO MATEMÁTICA NA EDUCAÇÃO PROFISSIONAL SUECA?

Received: 28 September 2018

Accepted: 03 December 2018

Peter Frejd

peter.frejd@liu.se

Linköping University

ABSTRACT

This paper presents a discussion of some characteristic features regarding the role and place of mathematics education in vocational education. Based on examples from the Swedish vocational educational system, research studies and curricula, it is concluded that central roles of vocational mathematics education include the connection of informal and formal mathematics as well as facilitation of the use of technology such as artefacts, ICT, and pen and paper. To smoothen the transition from school to workplace, the place of vocational mathematics education relates to different learning environments and activities that connect the classroom and workplaces. For creating a starting point for discussions about how vocational mathematics education might be organized analogies are used. They act as tools to visualize and pin point characteristic features that make impact on teaching to prepare individuals for future work and life.

Keywords: Vocational education, workplace mathematics, analogies, the nature of vocational mathematics education, academic mathematics.

RESUMO

Este artigo apresenta uma discussão de algumas características do papel e do lugar da educação matemática na educação profissional. Com base em exemplos do sistema educacional profissional sueco, estudos de pesquisa e currículos, conclui-se que os papéis centrais da educação matemática profissional incluem a conexão da matemática informal e formal, bem

como a facilitação do uso de tecnologia, como artefatos, TIC e caneta e papel. Para suavizar a transição da escola para o local de trabalho, o local da educação matemática profissional relaciona-se com diferentes ambientes de aprendizagem e atividades que conectam a sala de aula e os locais de trabalho. Para criar um ponto de partida para discussões sobre como o ensino de matemática profissional pode ser organizado, analogias são usadas. Elas atuam como ferramentas para visualizar e identificar características que causam impacto no ensino para preparar os indivíduos para o trabalho e a vida futuros.

Palavras-chave: Educação profissionalizante, matemática no trabalho, analogias, natureza da educação matemática profissional, matemática acadêmica

1. Introduction

An indispensable factor for success in a knowledge-driven global economy is a well-educated working force on all levels of the society (OECD, 2010). A development of individual employers' skills to create an effective and functioning working force requires some type of informal or formal education. One institution with focus on education for the workplace is vocational education. The corpus of research literature within the field of vocational education and training is vast. There are scientific research journals such as *Journal of Vocational Education & Training* and *International Journal of Vocational Education and Training*, international conferences, and handbooks (see for example Maclean & Wilson, 2009; Rauner & Maclean, 2008) that present research literature within the field. The organization of vocational education differs between countries around the globe due to social, cultural, political, economic and historical components that shape the world we live in. However, some goals of vocational education seem to be universal, in the sense that it captures the idea of learning both theoretical and practical knowledge, as illustrated by the following citation:

In vocational education as much as in higher education there is a continuum between studies taught and learned for instrumental purposes, and those pursued for intrinsic interest. Furthermore, in both sectors a study that may be of mainly intrinsic interest to some scholars has more instrumental value to others: studies aren't one or the other for all scholars. (Moodie, 2002, p. 253).

The citation above also highlights that students attending vocational education have their own values and goals with their education, which may not necessarily be the same goals as those held by other actors in vocational education. The goals and objectives of vocational education and the way that it is organized will impact on how and what will be taught. One part of vocational education relates to teaching and learning mathematics. The aim of this paper is to discuss and characterize the role and place of mathematics education within vocational education based on a sample of research literature and curricula.

The paper starts with providing background information about the aim of mathematics education in vocational education to be able to elaborate on the *place* and the *role* of vocational mathematics education. This will generate some characteristics which will be summarised, visualized in a diagram, and finally discussed.

2. The aim of mathematics education in vocational education

The main purpose of *vocational* mathematics education, where the term vocational is relating to employment or an occupation, is to prepare individuals for future work or to develop already qualified workers to increase their competence (FitzSimons, 2014). According to Strässer (2000), based on Blum and Strässer (1992), this aim would be described as a *pragmatic aim* to prepare students for some profession. In many countries, other aims of mathematics education in vocational education relate to the goals about why it is necessary to teach and learn mathematics in general and may be described as *formative aims* and *cultural aims* (Strässer, 2000). Formative aims relate to providing students with general mathematical abilities such as problem-solving ability and the ability to communicate mathematically. Cultural aims focus on the rationales of contributing to ideological, political and cultural maintenance and development, as well as to provide students with ‘meta knowledge’ of mathematics and self-reflection. These cultural aims relate to an aim for mathematics education to develop productive, critical and reflecting citizens in a society (Romberg, 1992; Skovsmose, 1994). In line with the overall aims by Strässer (2000), Wedege (2000) described three types of mathematical knowledge that teaching should aim for students to develop in vocational education; mathematical (formal mathematics); practical (the use of mathematics in the workplace); and reflective (meta knowledge of mathematics). To what extent vocational mathematics education gives higher priority on the mathematics used at the workplace or the formal mathematics used in general education is a matter of political, economic and social decisions in a community. There are several stakeholders that come into play regarding what content should be prioritised in the vocational education classroom and how teaching and learning should be organized and implemented. The official curriculum is one source that frames and guides what and how to teach, which, to some extent, determines the aims and objectives of vocational education. What is assessed, from large scale summative assessment to teachers’ guidance in formative assessment, also impact on what is valued in the classroom (Niss, 1993). Niss (1993, p. 27) argues: “What is not assessed in education becomes invisible or unimportant”. Other stakeholders such as the mathematics teacher, the vocational/trade teacher and the workers in the workplace have their own agendas about what should be prioritised in the classroom, which may create tensions about the aim of mathematics education (FitzSimons, 2014).

Another key issue related to the aim of vocational *mathematics* education concerns what is conceptualised as mathematics. A common conception of a large part of the population in our society is that academic or formal school mathematics is *the one and only* view of mathematics

(FitzSimons, 2014). However, workplace mathematics is often characterised as more complex and situation dependent. It includes specific technologies, social, political and cultural dimensions that are not found in educational settings (e.g. Harris, 1991; Noss & Hoyles, 1996; Wedege, 2010). Harris (1991) lists a summary adapted from Hoyles (1991) about the nature of school mathematics and informal mathematics used in out of school activities, illustrated in Table 1 below.

Table 1. Informal vs. School mathematics (Harris, 1991, p. 129)

<i>Informal Mathematics</i>	<i>School Mathematics</i>
Embedded in task	Decontextualised
Motivational is functional	Motivation is intrinsic
Objects of activity are concrete	Objects of activity are abstract
Processes are not explicit	Processes are named and are the object of study
Data is ill-defined and ‘noisy’	Data is well defined and presented tidily
Tasks are particularistic	Tasks are aimed at generalisation
Accuracy is defined by situation	Accuracy is assumed or given
Numbers are messy	Numbers are arranged to work out well
Work is collaborative, social	Work is individualistic
Correctness is negotiable	Answers are right or wrong
Language is imprecise and responsive to settings	Language is precise and carefully differentiated

Table 1 presents a ‘stereotype’ picture of the characterisation of school mathematics and mathematics used in workplaces (Gainsburg, 2003). One may, for example, argue that mathematics never is decontextualised in classroom settings, since the classroom itself is a valid context for doing mathematics, and the motivation for students may not just be intrinsic, because social expectations and the drive for good grades are other examples that may stimulate and motivate students to work with mathematics (Gainsburg, 2003).

In summary, the aims of vocational mathematics education are dependent on political, economic and social decisions in a community, but the overall aim is to prepare individuals for future work and future life. However, aspects identified in the literature that will impact on how the aim will be realised concerns the issue about what is conceptualized as mathematics and to what extent formal academic mathematics are to be prioritised in the classroom at the expense of workplace mathematics.

3. The place of mathematics education in vocational education

Vocational education is organized differently across the world (see e.g. World TVET Database¹) and consequently the place of mathematics education within the different organizations varies. First, vocational education can take place in either formal or non-formal education and be a part of apprenticeship systems. It can be incorporated at different levels: high school, post-high school, further education, and higher education. How vocational education is incorporated at these levels also varies between countries. For example, in some countries such as Germany, a strong upper high school vocational system that includes apprenticeships implies that the post-high school system has focus on further developments of graduate students from vocational education. In other countries such as the United States, with a more limited system of vocational education at the upper high school level, post-high school vocational programmes often offer the first opportunities for students to do career training (OECD, 2010). The intensity and the length of the teaching and learning period also vary between countries and within countries from short periods of time such as short courses, discussion groups, workshops or seminars to longer time periods spanning over several years such as institutionalized vocational programmes.

3.1. Vocational Mathematics Education in Sweden

As described above, vocational education can take many shapes and forms. The complexity of trying to give an overall description of the place of mathematics education within this rich flora of vocational education across the world is difficult (or even impossible). To illustrate the complexity and identify the place of mathematics education in vocational education, I will use some examples from the Swedish vocational educational system with both formal and non-formal education at different levels.

3.1.1. Vocational Education at upper high schools

In Sweden, after 9 years of compulsory school, students may choose between vocational or general streams within upper high school education. There are 12 vocational programmes, which normally last for three years and include at least 15 weeks of workplace-based learning. All students at vocational education at upper high school study mathematics in at least one course (Mathematics 1a) while some students, depending on study programme, also do a second course (Mathematics 2a). These courses are the same for all the vocational programmes and described in national curriculum documents (Skolverket, 2012). The curriculum documents emphasise that “Where appropriate, teaching should take place in environments that are relevant and closely related to praxis” (p. 1), and the students are expected, after a completed course, to be able to “use concepts to solve mathematical problems and problem situations in subjects typical of a programme in familiar situations” (p. 5). In addition, all students have the right to study more mathematics courses in the general stream to be eligible to access tertiary education. The

¹ <http://www.unevoc.unesco.org/go.php?q=World+TVET+Database>

mathematics teachers teaching at this level are trained at the university and there are no formal requirements that they need experiences from the ‘workplace’ in order to teach.

3.1.2. Apprenticeship education in high school education

An alternative to the vocational education programmes is *apprenticeship education*, where students spend at least 50% or more of their time at a workplace. The responsibilities for apprenticeship education are shared by the school and the workplace, but the school is responsible for assessing the knowledge requirements. In mathematics these requirements are the same as for vocational education in upper high school (Skolverket, 2012). Mathematics, as well as other ‘core subjects’ such as Swedish and English, that students study at upper high school is therefore similar to the mathematics taught and learnt in vocational education at upper high school.

3.1.3. Municipal Adult Education

Based on adult students’ specific needs and preconditions, they may choose to study one or more courses at municipal adult education. The courses are the same as upper high school education and it may lead to various vocational qualifications based on the completed courses. Also, apprenticeship education is available for adult students.

3.1.4. Higher Vocational Education

Upper high school graduates may be enrolled in higher vocational education with a focus on professions that require certification or specialised knowledge. Examples of professions relate to administration and sales; manufacturing and technology; and social work. Higher vocational education varies in length; from six months to two-year full-time studies and it combines work-based and school-based learning. There are no courses offered called mathematics; instead, the mathematics learned and used is integrated in other courses, depending on the aim of the program. How ‘explicit’ the mathematics is in these programmes may be discussed. The teachers are rarely mathematics teachers, but have a background in the specific profession.

3.1.5. Professional higher education

Higher level job-specific training at universities or university colleges may include courses named mathematics (e.g., teacher training programmes for kindergarten and primary education, engineering programmes) or not (e.g. nursing programmes). Teachers at professional higher education have a background as mathematics teachers, mathematicians or a background in some specific profession. In engineering education at universities the teacher is most frequently a mathematician.

3.1.6. *Non-formal and informal TVET systems*

Liberal Adult Education (Sw. folkbildning) is one type of non-formal learning, where each school and adult education association decides on the organisation of their education and on the content of their courses. They offer a rich choice of courses with aims, such as to prepare students to become a church musician, journalist² and sign-language interpreter. The mathematics taught at Liberal Adult Education with focus on vocational education is integrated within the programmes.

In summary, the place of mathematics education within vocational education varies between the different institutions. Workplaces, practical training classrooms and regular classrooms are all learning environments that are used for teaching mathematics. The mathematics taught may either be organized in specific courses, integrated within other courses or a combination of both. The teachers have backgrounds as mathematics teachers or as experienced workers from some profession.

4. The role of mathematics education in vocational education

Assuming that the aim of vocational mathematics education is twofold, including both to prepare students for future work and to become critical participants in a society, one of the roles of vocational mathematics education is to serve as an interface between academic mathematics and mathematics used in the workplace (Wake, 2014). In other terms, one could say that one of the roles of vocational education is to connect horizontal and vertical mathematisation (Treffers, 1987). It means linking activities originated in some realistic contexts (horizontal), such as workplace situations, with formal academic mathematics (vertical). This implies another role for vocational mathematics education, which is to facilitate interdisciplinary collaborations to support ‘authentic’ learning environments (Coben & Weeks, 2014). Interdisciplinary collaboration is one of the cornerstones in vocational education, which makes the mathematics teacher collaborate with other educators and/or workers to gain insights of challenges and demands of a particular workplace with its own social and cultural norms (FitzSimons & Björklund Boistrup, 2017). Collaboration across disciplines is useful for designing authentic and appropriate problems and to identify situations outside the mathematics classroom, where students may apply their mathematical knowledge safely, without any risks for human injuries, property damages, or loss of money. The research findings of LaCroix (2014) illustrate the tight synergy between workplace mathematics and workplace practice activity, suggesting that students in vocational education should benefit “to have their attention drawn explicitly to the mathematically pertinent details of the objects, tools, norms, and goals of workplace production prior to, or coincident with, focusing on specific workplace mathematics practices” (p. 173). This

² There also exist formal vocational educations to become journalists at professional higher education.

is aligned with Jablonka (2009) who writes that “graduates of school do not automatically apply the more general mathematical techniques they have been taught, but instead use the methods that are transmitted at the work-place as a part of the cultural resource of the profession. [...] We find workers who perfectly manage their jobs, but as soon as exposed to formal training of the mathematics they actually use, they feel incompetent” (p. 302). The role to enable and smooth the transition from vocational mathematics education to the workplace is therefore central.

A frequent finding within the research field of workplace mathematics is that mathematics at the workplace is hidden in technology (e.g. Chevallard, 1989; Noss & Hoyles, 1996; Triantafillou & Potari, 2010; Wake, 2007; Williams & Wake, 2007a, 2007b). Many working decisions are based on input and output values of technology and to develop students’ abilities to understand and critically analyse the underlying mathematical models used for decision making is, according to Skovsmose (1994), a fundamental aim in mathematics education. Another role of vocational mathematics education is therefore to make the mathematics used in technology at the workplace more visible. Technology and mathematics often go hand in hand, in particular at the workplace (SIAM, 2012), and it is difficult (or even impossible) to separate the two notions, at least if one uses a broad description of technology and see mathematics as a tool to solve problems. Williams and Goos (2013) argue that the mathematical activities related to out-of-school situations are complex, and it is not possible to distinguish between technology and mathematics. Instead, they describe a fusion of technology and mathematics, claiming that “mathematics is always mediated by the technology” (p. 554). That connects to what Noss, Hoyles, and colleagues describe as techno-mathematics or techno-mathematical literacies (e.g., Hoyles et al., 2002; Noss, Bakker, Hoyles & Kent, 2007; Noss & Hoyles, 2011). The fact that a large part of the mathematics used in the workplace is hidden in artefacts might be one reason for why most observers of different workplace practices, including both in- and pre-service mathematics teachers (Nicol, 2002), argue that not much mathematics is actually used in the working practice. It should also be noted that some researchers (e.g., Pais, 2013) problematise the usefulness of academic mathematics in vocational education and question the possibilities of transfer between workplace and school. However, to address these issues in more detail would require some space and may be a topic for another article.

More roles or specific suggestions for aspects to be emphasised in vocational mathematics education are sometimes listed in research articles (e.g., Hoyles et al., 2002; LaCroix, 2014; Wake & Williams, 2007). An example of such a list is presented below (Coben & Weeks, 2014, p. 266).

- Using relatively low-level mathematics in quite complex situations and contexts, [...]
- Encouraging experiences of a diversity of conventions and methods
- Having students experience activities where the mathematics is embodied in context and to use artefacts with which they have become familiar

- Preparing students to transform their existing mathematical knowledge to make sense of activities in unfamiliar workplace situations
- Having students design spreadsheet programmes for modelling and for recording, processing and analysing data
- Making students aware that there are many and varied ways to solve any problem

The list above may be used to discuss and summarise the role of vocational mathematics education. The first and the fourth proposals in the list relate to the role of vocational mathematics education as a producer of opportunities for students to use their academic mathematics knowledge in workplace situations. The second and the last proposal indicate that vocational mathematics education should facilitate the use of a variety of teaching and learning methods in different contexts. Proposal three and five address the issue of technology and mathematics hidden in artefacts. One of the roles of vocational mathematics education is to make the mathematics in workplace contexts visible. To explicitly link academic mathematics with mathematics used in the workplace requires interdisciplinary collaborations across boundaries to enable a smooth transition from school situation to work place practice. However, what is not captured in the list is the role of vocational education to develop productive, critical and reflecting citizens in a society (Romberg, 1992; Skovsmose, 1994).

5. The nature of vocational mathematics education

As a tool to illustrate the nature of (the role and place of) mathematics education in vocational education, I will use an analogy as a language of description to make relationships and abstractions visible. Consider the following analogy between electric circuits and mathematics education in vocational education.

An electric circuit is a path in which current from a power source flows from one point to another. Depending on the aims, it may be constructed in many ways to make multiple pathways in different branches by which the current can flow. To control the current, different components such as resistors and switches may be used. For example, in a parallel circuit, the flow divides into separate branches so that there can be more current in one branch than there is in another, which is due to the amount of resistance in the branch; less resistance, more current. The person that regulates the current to flow by making connections is the electrician. To what extent the electrician fulfils the regulation of the current depends on factors such as the available tools, working experiences, knowledge, time etc. How the electrician works may vary from following pre-defined manuals to being creative and designing own circuits to meet certain criteria.

Vocational mathematical education may also be described as a path, an educational path, to acquire mathematical knowledge to get access to the labour market, and, if possible, to provide a good foundation for future life as a critical citizen. The electromotive force (EMF) that creates the energy in the vocational educational system is the students driven by their internal and

external motivation as well as teachers' engagement, attitudes and knowledge. The vocational educational path may be constructed in many ways depending on, for example, the aims of the education; the group of students; teachers' attitudes and knowledge; classroom resources; the curriculum; norms and values; and social, historical and political reasons. To control students, learning components such as avoiding, restricting or emphasising opportunities to learn, may be used. One may argue that having less opportunities to learn makes it more difficult to learn. A person whose actions and opinions influence and assist students to learn is the teacher (mathematics teacher, craft teacher, or qualified worker). However, to what extent the teacher may assist the students may, on the one hand, depend on the available resources, teacher experiences and knowledge, time, among other things; and, on the other hand, on the students' abilities and motivations. How the teacher works to assist the students may vary to a large extent from being in the workplace and teaching on site, to being a lecturer in a classroom.

The analogies are several. Students as well as currents move in multiple pathways to meet the goal. During the process they encounter more or less resistance (or more or less opportunities to learn) in order to continue their path towards the goal. Both the electrician and the teacher are the main characters that have the authority to influence and control the different pathways. However, both are restricted by factors such as working experience, knowledge, available time and other external restrictions.

Based on the discussions in the earlier sections, central issues identified about the role and place of vocational mathematics education will below be visualised metaphorically by an electric circuit diagram, as a tool that may be used in the discussions about the organisation of mathematics teaching in vocational education.

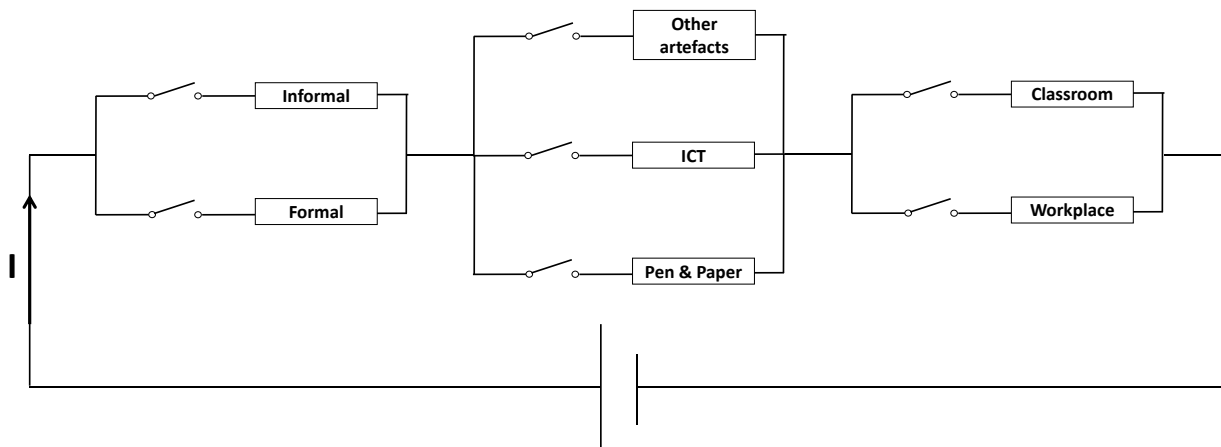


Figure 1. An electric circuit diagram with components connected to the role and place of vocational mathematics education.

The (metaphoric) electric current I in figure 1, relates to students [**I**]nvolvement in teaching and learning activities connected to; informal/ formal mathematics; the use of technology (pen and paper, ICT, and other artefacts); and different learning environments (mathematics classroom and workplaces). The role of the teacher is to connect the switches and regulate the amount of

current (opportunities to learn) in each branch. The first two parallel circuits in the diagram visualises the *roles* of vocational mathematics education as a link between academic practice and workplace practice (connections between informal and formal mathematics) and the role of giving opportunities for students to learn about technology used in the workplace (by connecting the switches in the branches relating to pen and paper, ICT, and other artefacts). The *place* of vocational mathematics education is depicted in the diagram as a connection between the classroom and the workplace. If there are no switches connected in one or several of the three parallel circuits there will be no current, which means that if there is no formal or informal mathematics taught or learnt it will not be called mathematics education. In line with Williams and Goos (2013), technology is intertwined with mathematics, making it impossible to separate them, which means that without technology there would be no mathematics education. If there are no learning environments such as classrooms or workplaces, there will not be any place where teaching may take place and there will be no mathematics education. At the other extreme, if all switches are connected, the resistance will be minimal, or in other words, the opportunities for learning will be at the top. How to adjust the resistance within the parallel circuits, or in other words, the balance between informal/ formal mathematics, artefacts /ICT/ pen & paper and classroom/workplace is another analogy for how to organise vocational education, which will be discussed in the next section.

6. Discussion

A central role of vocational mathematics education is to give students opportunities to develop adequate core academic knowledge as well as workplace related knowledge. This role suggests a need for connecting work-based learning with classroom learning. Sometimes, for example in the vocational education at Swedish upper high school, mathematics is taught and learned mainly connected to the classroom environment. However, workplaces are also suitable learning environments. Firms have often the newest technologies and usually people with adequate knowledge of how to use them that may facilitate students' learning. Also, many cognitive abilities, such as problem solving, and social abilities, like conflict management and communication might be taught and learned more effectively in workplaces than in classrooms (OECD, 2010). The place of vocational mathematics education, based on the arguments above, is to be integrated in different learning environments to prepare the transition from school to workplaces. The role of vocational mathematics education as means for making the mathematics visible in relation to the technology used at workplace requires a teaching workforce that can offer a balance of mathematics skills, teaching abilities as well as knowledge and experience from the workplace.

The diagram in figure 1 was constructed to visualise some central aspects related to the role and place of vocational mathematics education. In particular, it highlighted the aspects of informal and formal mathematics; the use of technology in vocational mathematics education; and learning mathematics in different environments. In addition, figure 1 may be used as a tool for

supporting further discussions about the organisation of mathematics teaching in vocational education. It may address how the electric-circuit inspired diagram would be designed to optimise vocational mathematics education in terms of controlling the current (opportunities to learn) between and within the branches. Or one may argue that it should be alternating current, indicating that students' and teachers' motivations and engagements change over time, and other components might be integrated, such as transistors used as amplifiers as an analogy for learning tools that increase students' motivations and opportunities to learn. The law of electric power ($P=VI$), as another analogy, indicates that vocational mathematics education would generate a high power if the opportunities for learning in all branches are high (a large value for I) and if students' motivations and teachers' engagement are high (large value for V).

The question is *how* vocational education can be developed to strengthen all the aspects discussed above. There are no trivial answers, but according to FitzSimons and Björklund (2017), collaboration between the subject area of mathematics and the vocational subject area is central for achieving an effective vocational mathematics education. In figure 1, the balance between formal and informal mathematics, the use of artefacts and where teaching could take place may be established through respectful discussions between mathematics teachers and professionals in vocational subject areas. To use meaningful contexts for students to explore also motivate students to learn (Coben & Weeks, 2014; FitzSimons & Björklund, 2017), and possibly engage mathematics teachers if they familiar with the vocational knowledge object of learning (FitzSimons, 2014).

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