FINANCIAL MATHEMATICS OF BANKERS' PRACTICE

CARACTERÍSTICAS DA MATEMÁTICA FINANCEIRA NA PRÁTICA DE BANCÁRIOS

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ABSTRACT

This study aims to present financial mathematics that bankers[1] use in their practice when they are embedded in their workplace. To that end, we observed bankers at two official banks in an inland city of Brazil. Our analysis reveals that financial mathematics actions are structured by banking systems; the actions linked to these systems are characterised by the ways the bankers address the opportunities and limitations of banking systems as cultural tools that serve as a unit of analysis and define specific actions in this context. However, at the same time, it illustrates cases where bankers deal with some limitations of the systems. In other words, while their actions are mediated by these systems, some bankers deliver customer service by considering the customer's specific situation and profile in the decision-making process, thus providing a service that the system alone cannot do.

Keywords: Financial Mathematics; practice; bankers; banking systems.

RESUMO

Este estudo tem como objetivo apresentar características da matemática financeira na prática de bancários. Para isso, observamos bancários em dois bancos oficiais numa cidade do interior do Brasil. Após análise dos dados, verificamos uma prática estruturada pelos sistemas de informação bancários de forma que os fazeres atrelados a ela são caracterizados pelas formas com as quais esses sujeitos lidam com as oportunidades e limitações dessas ferramentas culturais, constituindo uma unidade de análise e definindo fazeres específicos desse contexto. Nossa análise mostra uma matemática financeira estruturada por sistemas bancários, mas, ao mesmo tempo, ilustra casos nos quais os bancários demonstram fazeres para além desses sistemas. Isto é, nas suas ações mediadas por esses sistemas, alguns bancários

[1] The word "bankers" is used referring to the bank employees.

completam o atendimento a clientes, considerando suas situações e perfis específicos para orientá-los em suas decisões, demonstrando algo que o sistema, sozinho, não pode fazer.

Palavras-chave: Matemática Financeira; prática; bancários; sistemas bancários.

1 Introduction

The present study provides an analysis of financial mathematics in the practice of banking. Financial mathematics is understood to mean any practice[2] involving studies, calculations or procedures with the "time value of money" (Abor, 2017; Drake and Fabozzi, 2009). The expression "time value of money" means that the value of a capital depends on the time at which this value is considered. Thus, interest, discounts, capital equivalents, annuities and depreciation are financial mathematics objects related to changes in currency values over time.

Our investigation of the financial mathematics used by the employees of banking institutions is grounded in our interest in developing further insights for teaching financial mathematics in business courses. Initially, we will present a discussion of the literature and theoretical perspectives that will allow us to re-introduce the purpose of this research in more defined terms.

We will analyse some bankers' specific financial mathematics practices in their workplace. According to Lave and Wenger (1991), we can understand these practices in terms of the historical and social context in which they are developed, provided that the interactions between subjects in these social relations are conditioned by shared values. As a result, we can identify different financial mathematics practices according to the contexts in which they were developed and the meanings attached to them. For example, we can discuss financial mathematics as practised in the academic disciplines of financial mathematics, in insurance companies, in banks and so on. This topic will be discussed in the next section.

2 The situated nature of banking financial mathematics and information and communication technology

Generally speaking, studies (Billett, 2011; FitzSimons, 2014; Hoyles et al., 2010; Schmidt and Gibbs, 2009; Swanson and Williams, 2014; Wake, 2014) on the mathematics practised in work environments highlight their specificities when compared with mathematics as practised in educational settings. According to Wedege (2010), the literature on mathematics at work and for work reaches two common conclusions: mathematics is integrated into the environmental practices of the work and often hidden in information technology; and the so-called transfer[3] of school mathematics to the work environment is not a simple matter. The author argues that one of the consequences of the differences between mathematics at work and at school is that workers do not recognise mathematics in their daily practice, partly because

[2] We assume here, provisionally, the concept of practice as an action shared by people in social groups who are acting and interacting according to the customs of that group. This definition, provided by Wenger (1998), is presented below.

[3] There is a debate in the literature about the different conceptions of transfer, which was the objective of study by Lobato (2006). Although this discussion is not within the scope of the present study, we should underline our understanding of transfer as the connections experienced between practices and promoted by subjects who participate in different practices when they cross practice borders.

technology makes mathematics invisible and partly because workers are unable to connect mathematics in their daily work practices with mathematics as a school subject.

We use the word "practice" in the present study in accordance with the theoretical definition proposed by Wenger (1998), who refers to practice as "a way of talking about the shared historical and social resources, frameworks, and perspectives that can sustain mutual engagement in action" (Wenger, 1998, p. 5) and defines it as a concept that denotes an action "in a historical and social context that gives structure and meaning to what we do" (ibid, p. 47).

The two aspects cited by Wedege (2010) as common characteristics of mathematics **at** and **for** work are of direct interest to us. Regarding the matter of mathematics being hidden in technology, we propose a different focus; we aim to show what is visible and what is mediated by the use of technology, using the banking context as our research environment. Regarding the transfer of school mathematics to the working environment, we will use the present study as a starting point for reflection on the issue, because in parallel studies we analyse alternative methods for teaching financial mathematics in business courses. That is, we will present the kind of learning bankers have to engage in as a path to inspire different focus on the teaching and learning processes.

The central element of banking financial mathematics is the information systems that bankers use in their professional environment (Hoyles et al., 2010). These systems are the specific computer programs to each bank but are connected to other banks. According to Hoyles et al. (2010), the introduction of information technology (IT) into work environments is one of the most important trends in recent decades.

When a person participates in a particular practice, his/her actions in it are synchronised with actions developed by others and therefore by shared understandings about this action. As an example, consider individuals who participate in practices developed by a financial mathematics department as part of a university course and in those developed in a banking institution. Their actions are not isolated; rather, they reflect the way they participate in a shared practice in each context.

In short, in the words of Wenger (1998), "participation is a process of mutual recognition" (p. 56). This theoretical statement describes the synchronisation between actions and attributes an active nature to this concept. Consequently, we will describe some banking financial mathematics actions by studying the ways that subjects (in this case, bankers) participate in the workplace, with the understanding that these forms of participation are linked to the use of technology in this environment.

Our understanding of the studies by Lave and Wenger (1991) and Wenger (1998) allows us to suggest that, given that knowledge is situated, its nature is distinct in different practices. However, knowledge can be transformed as a result of the ways in which subjects participate, because they bring their experiences into the various social practices in which they participate. In other words, people bring with them the learning trajectories (Lave and Wenger, 1991; Wenger, 1998) that are formed in various practices and that will influence the new practices in which they participate. Therefore, while a new technology can transform a practice, its use can also be transformed when subjects cross boundaries between different practices.

Zevenbergen (2011) observed that calculations performed by hand or with mechanical cash registers approximately twenty years ago are far removed from automation systems that we see in the retail industry today. This author notes that the participation of younger people, whose ability to use the technology inherent to their world conditions their ways of thinking and acting, has also transformed the workplace. For example, the attention that was once devoted to practices that required several calculation procedures is now given to performing estimates, solving problems, collaboration and improved customer service. Similarly, we can imagine, for example, that the consolidation of Banking Information Systems (BISs) as mediating tools has transformed the practice of banking.

Hoyles et al. (2010) focus in the "invisibility of meanings" associated with the mathematical models in BISs. However, in our study, we consider that such meanings are not pre-existing; they are not inherent in the mathematical models used in BISs, but are negotiated by a social group during social interactions.

According to Wenger (1998), meanings are not pre-existing, nor are they simply constructed; they are negotiated in a historical and dynamic way within a context. However, this concept of negotiation is not restricted to common sense, such as reaching an agreement; rather, it implies the idea of continuous interaction based on social relationships. Voigt (1994) considers mathematical meanings to be products of social interaction that emerge between individuals rather than being inherent or existing independent of individuals. Voigt (1994) adds that mathematical meanings are negotiated even when participants do not have different views; however, in cases of conflict, negotiation becomes more problematic. Although Voigt's study predates that of Wenger (1998), the idea is similar. Thus, we can conclude that *meanings* are inter-subjective understandings about objects that are derived from continuous interactions among subjects and are socially and historically created through negotiation and renegotiation in the contexts from which they emerge.

Thus, we can understand that the meanings that are not visible to bankers, according to Hoyles et al. (2010) studies, are the ones that did not involve negotiation with or by this social group. Instead, those meanings were negotiated by others, for example, in the case of a bank, by the system designers.

Depending on the environment in question, technology can become an indispensable ingredient of practice. A good example of this is the banking environment. Currently, it is almost inconceivable that an employee dealing with a customer who wants a 48-month loan, for example, would have to manually calculate the 48 lines of the amortisation spreadsheet. In addition to the need to optimise service time in this environment, a miscalculation could harm the bank or the customer. A similar idea also appears in the previously mentioned study by Zevenbergen (2011): changes in the retail industry influenced a transformation in the way workers participate, which shifted towards a focus on estimates, problem solving and social relationships among co-workers and with customers.

According to Wertsch (1991, p. 12), human action uses meditational means, such as language and tools, that shape actions in such an essential way that when referring to the agents of that action, it would be more appropriate to refer to "individual(s)-acting-with-mediational-means" rather than just "individual(s)". Thus, by way of analogy, the participation of banker-plus-mathematics-plus-technologies allows us to discuss a particular way of performing banking financial mathematics. If we compare banking practice before and after the arrival of IT systems, we will of course say that certain knowledge has been eroded. However, the introduction of new information and communication technologies has led to new insights that were not previously possible. Thus, it may be more appropriate to treat these new interactions as another type of social practice, in order to identify the changes in those actions that are associated with

IT. This can be an ongoing process, because new technological tools are routinely introduced in workplaces such as banks. For the purposes of the present study, we examine the financial mathematics negotiated by bankers in this bankers-with-systems context. To explore this issue, we will follow the work of employees at two bank branches in an inland city of Brazil, as we describe in the following section.

3 Context and methodological procedures

In the present study, we use a qualitative approach (Denzin and Lincoln, 2005), because we seek an understanding of the financial mathematics practised by bankers in terms of the actions and meanings they negotiate that are conditioned by the use of technology.

Observation (Angrosino, 2005) was selected as the data collection procedure to allow direct contact with banking practice. The observations in the studied banks were conducted by the first author of this study. The data used in this work were collected at two banks in an inland city of Brazil. For each bank, the observation of banking practice took place over three days, for approximately 6 hours each day, being concluded on January 2011. The researcher’s notes were recorded in a field notebook; when appropriate, we shall refer to excerpts of these notes in our presentations of the data.

The managers who authorised data collection expressed concerns about bank secrecy, but the researcher reassured them by explaining that, because of the study’s objective, there was no interest in specific customer information. The researcher also made the field notes available to the managers and maintained the anonymity of the banks and their employees and customers. Because the bank employees were research subjects, we have used pseudonyms to maintain their anonymity.

Because of banking security issues, no audio or video recordings could be made during data collection. Therefore, it was only possible to record data in a fieldbook. Doing so allowed us to select information relevant to the research and avoid the exposure of specific customer information.

Observations were made during both public opening hours and internal business hours. It was thus possible to observe some bankers (research subjects) as they provided customer service and also during internal business hours, when they were under less pressure.

Non-structured interviews (Fontana and Frey, 2005) with some participants were conducted in the context of the observation to ask about their practice, to improve our understanding and to allow them to freely discuss the issue at hand. These respondents were not selected in advance; they were chosen in the context of observation according to the researcher’s needs and the subject's availability. The bankers chosen for interviews were selected according to the following criteria: bankers who were managers (of the personal banking and corporate banking) and therefore had more access to certain information than their assistants; bankers who had greater availability.

In both banks, the employees' practice in various areas was observed, including services to personal banking customers, service to corporate customers, work in the credit room and housing section, and cashier services. The cashiers were observed from the outside, as it is forbidden for anyone other than bank employees to be present in that area.

Analysis began with encoding the data after excerpts from the fieldbook were examined line by line (Charmaz, 2005). The selected excerpts were based on observed incidents among the bankers' actions that would be applicable to our objective. According to

Charmaz (2005), these codes enable the selection, separation and organisation of data for analysis.

Based on the initial codes, some excerpts were labelled according to their similarities. The data segments were thus categorised according to the participants' actions, and then the codes were organised into categories, which were refined according to the objective and theory adopted, before finally being named according to the observed phenomenon. For each category, we describe the banker actions that formed the category. These categories guided the presentation and analysis of these data in the next section.

In the next step, we selected banker actions that were fit into multiple categories because of their similarities, in accordance with the theoretical framework. We thus developed some analytical structures that correspond to the bankers' actions, which will be addressed in the discussion.

4 Data presentation and analysis

Based on the data collected from the observations at the two banks, we verified the centrality of the BISs being used. That is, all bank transactions were observed through an information system. In their first contact with the researcher, the bankers themselves reported that everything was done through the system, as the fieldbook excerpts confirm:

> Gustavo [...] informed me immediately that almost everything is done through the banking system. (Fieldbook, p. 3, line 6-8)
>
> The GM [General Manager] of Bank A, [...] soon made a point of saying that everything was done through the system (Fieldbook, p. 101, line 5-7).

Consequently, the categories also demonstrate the relationships between the practice subjects and the system. These categories were based on the codes, which demonstrated how financial mathematics is manifested in the bankers' practice, namely:

- the bankers inputting data into the system;
- the bankers interpreting financial mathematics in their relationship with the system;
- the financial analyses that the bankers perform in conjunction with the system for customer service;

We shall present these categories in the following subsections.

4.1 The bankers inputting data into the system

In their relationship with the system to serve the customer, the bankers use financial mathematics by inputting data into the system.

In addition to inputting such information as the capital value of loans and financing, check values, and discounts and terms for such operations, the bankers must also input customer-specific information, such as addresses and the Natural Persons Register

(Cadastro de Pessoas Físicas - CPF)[4]. The CPFs allow the system to perform customer information history searches of both the customer's relationship with the bank itself and his or her relationship with other financial institutions, as the systems are interconnected.

Thus, the system generates a classification for the customer using the codes AA, A, B, C, D or E. These codes represent, in the order presented, a scale of lowest to highest risk to the transaction. A customer with an E rating is unable to perform the desired transaction. In the case of a D rating, the transaction may or may not be approved, at the banker's discretion. The system also generates the interest rate for the transaction based on these codes, as risk is one factors that contributes to the calculation of these rates.

At Bank A, the rates are always generated by the system according to this classification. However, in the case of check discounting, Marcos, the manager of the corporate section, reported that there is a window in the system where the value can range from 0 to 200, and the bankers can input a value based on their evaluation of the customer's relationship with the bank. This window has a default value of 200, but it can be changed by an employee. A change made by the banker, in this case, automatically changes the discounting interest rate according to the input amount, as the following excerpt from the fieldbook shows:

> In addition [to the risk code and rate], [the system] gives a range (from 2 to 200) that is internal information and represents a margin of negotiation between the employee and the customer, according to his relationship with the bank and the bank products he can acquire. A change in the band value automatically changes the check discounting rate. The lower the value the employee enters in the range, the lower the rate will be (Fieldbook, p. 25-26, line 26, 1-9).

In this case, as bankers enter that value, the system allows them the ability to negotiate rates according to their evaluation of the customer. As the employee himself noted, this evaluation also depends on the products that the customer wants to acquire. Thus, for example, the customer can improve his or her evaluation by applying for a particular amount.

In payroll loans, the bank, in agreement with the customer's employer or the National Institute of Social Security (Instituto Nacional do Seguro Social - INSS)[5], in the case of retirees, grants the loan according to a margin, known as the consignable margin. This margin represents the maximum instalment amount that the agreement allows for that operation and that will be taken directly from the customer's paycheck. The customer therefore enters into a contract with the bank for the loan or for financing, and the instalments are deducted from his or her salary or the paycheck itself. Because the instalment amount is being deducted from the customer's pay, the risk of the transaction is lower, and the contracted interest rates may therefore be lower. In this type of loan, the consignable margin limits the value of the instalment and hence the loan amount that the customer can seek within a specified period. This characteristic also affects the data the banker inputs into the banking system, as the following passage shows:

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[4] The Natural Persons Register (Cadastro de Pessoa Física - CPF) is an official document issued by the Brazilian government that includes individual identification numbers used, for example, in financial and tax operations.

[5] The Brazilian social security system.

> Jane was making changes to the customer simulation by changing the value of the loan to be granted until the instalment value that the consignable margin of the customer's paycheck allowed appeared on the screen (Fieldbook, p. 58, line 3-8).

The bankers used a *trial and error* strategy of inputting capital values for payroll loans until the instalment amount was the maximum allowed by the system according to the agreement. In this case, Jane, a personal banking customer service assistant, used this procedure to determine the capital value that could be borrowed, i.e., via a data entry procedure conditioned by the computer system.

We found another difference between Banks A and B with respect to data entry. At Bank A, the observed bankers reported that the system classified customers according to the AA, A, B, C, D or E codes and generated their financial profiles in connection with the system of other financial institutions. The bankers were able to intervene in this process to a small extent, for example, when Marcos was able to input his evaluation of the customer in a table using the 0-to-200 range that the system allowed. However, at Bank B, the corporate manager reported that these codes were generated using customer data that he input into the system. In this case, the customer rating (AA, A, B, C, D or E) that determined the transaction's risk was generated by the system based on information that the banker provided.

We can see that the data entry method of the bankers at Bank A appears to be more controlled by the limits that their system imposes, whereas at Bank B, employees have more responsibility for inputting data.

Regarding data entry, the employees must input their customers’ personal data, address the rate classifications that the system imposes based on the customer’s profile and simulate input values through trial and error.

When dealing with the rate classifications, in some cases, the bankers found only one alternative rate for a particular customer or had the prerogative to evaluate the customer and vary the rate within a range.

In all cases, the bankers’ data entry actions were conditioned by the system, and their forms of participation were more or less limited by that system.

4.2 The bankers interpreting financial mathematics in their relationship with the system

When providing customer service, the bankers sometimes needed to respond to the customer’s questions about loans and financing and interpret the outputs that the system provided, for example, in regard to variations in quotas or interest rates, as the following excerpt shows:

> A customer asked Jane if the rate was fixed or not and she said no, that it had a small variation because of the RR [reference rate] (Fieldbook, p. 58, line 13-16).

At Bank B, some personal loans and financing operations follow the Price Table depreciation and others follow the constant amortisation system (CAS). The basic characteristic of the CAS is that amortisations (repayments of borrowed capital) are constant with decreasing instalments, as the interest is applied to the outstanding balance from the previous period. CAS is used for real estate credit and for loans for constructing and improving real estate. The basic characteristic of the Price Table, a particular version of the French system (FS), is the fact that instalments are fixed. This system is used for payroll loan and consortium operations, as in the cases reported above.

In this case, because the customer expected fixed instalments, he was surprised by the small variations in these instalments and asked Jane, the personal banking customer service assistant at Bank B, about these variations. Jane explained what was causing the variations by interpreting the system's outputs (in this case, the instalment and rate amounts and charges).

The corporate manager explained how check discounting operations are performed. Roberto used fictitious amounts to explain how the system performs operations, as the following excerpt shows:

> [Roberto] began to show me how to perform the discounting operation. He said there are two types: 1. CH custody: D = 0 (meaning that the money goes in the same day). Ex.: R$ 10,000.00 (value of check) - I (interest) = net value. 2. MIT - Multiple instant turn. R$ 10,000.00 (as a limit). If the customer needs to use R$ 1,000.00 of the 10 [10,000.00], his limit drops to 9,000.00. He said the advantage of this for the customer is that if he does not need the money immediately, he will not pay interest on what he did not use and will pay only for what he is using (Fieldbook, p. 91, line 8-25).

Roberto explained that there are two check discounting methods. In the first case, which he called "CH custody", interest is deducted from the amount of the check to be cashed, and the net value (calculated by the system) is available to the customer the same day. In the second case, multiple instant turn (MIT), he said that the total value of the check(s) becomes a limit that the customer can use when needed. Using examples, Roberto explained how the system performs these operations; he noted that in the case of MIT, the fact that the customer does not have to pay interest on the unused amount represents a financial advantage, thus indicating the financial mathematics involved in the operation.

The bankers also viewed the system as inferring with their working assumptions. For example, at Bank B, there is a special line of credit for the housing section. Pedro, a high school student and bank intern, provides simulations of possible loans for customers and eventually finalises these loans. He showed the researcher various loan simulations and their respective interest rates.

By showing the researcher different spreadsheets in which the interest rates varied, Pedro inferred the assumptions that the system uses to generate these rates. He informed the researcher that the government provides subsidies for a portion of the value in some cases and wanted to show the researcher how this happens, as the following passage shows:

> Pedro called me to show me that, for a R$ 70,000.00 loan, when he increased his income, the conditions worsened. He explained that when financing amounts over R$ 80,000.00, the higher the income, the better, but the same is not true when financing R$ 70,000.00 or less. In the previous example, when the simulated income increased from R$ 1,800.00 to R$ 3,000.00, the initial amount required increased, and the rate rose from 5% pa. + RR to 8.16% pa. + RR. Pedro was asked if this occurred because of government subsidies, and he said yes (Fieldbook, p. 70-71, line 18-26, 1-2).

Pedro thus explained that when the government subsidy is included, the working assumptions of the system models change. That is, in general, the higher the customer's income, the better the conditions the bank offers for repaying the loan (for example, lower interest rates and fees) because a higher income represents a lower operational risk from the bank's point of view. However, when government subsidies are included, the rules change because the customer service logic also changes. That is, the lower the customer's income, the higher the subsidy the government offers to provide more for the citizens in greatest need of obtaining their own home. The example that Pedro showed in this last excerpt follows the rules of the federal programme "Minha Casa Minha Vida" (My Home, My Life). Pedro showed that in cases where property valued at R$ 70,000.00 or less is sought, the system model's assumptions change because these values identify people with more financial constraints. Thus, in such cases, when the reported income is higher, a greater down payment is required and interest rates are higher because the government subsidy rules prioritise borrowers with lower incomes.

Thus, while employees do not have access to the system's mathematical models, they interpret their outputs and infer the assumptions that generate them.

When we propose mathematical models, we consider specific assumptions for particular models. These assumptions are the prerequisites that must be met for the mathematical model to effectively solve the problem at hand. In other words, the assumptions must be considered before the mathematical model is constructed.

However, banking systems are pre-programmed with the mathematical models necessary for banking practice. Bankers, therefore, use these pre-constructed models and infer from them the assumptions that were used in their construction. They perform the inverse operation of those who programmed the system, i.e., they infer the assumption from the model.

In the cases analysed in this section, financial mathematics is involved in the banker's practice when bankers interpret system outputs to explain to customers how they work. Bankers identify the different ways the system works based on its outputs, inferring assumptions about the models the system uses and identifying the different ways that the system works based on the input. These ways the system works are the procedures that they system performs and the bankers identify as they interact with the system. Although the bankers have no access to the models programmed into these systems for customer service, they interpret the financial mathematics in the course of their relationship with the customers.

4.3 The financial analyses the bankers perform in conjunction with the system for customer service

Sometimes bankers guide their customers towards the best alternative, for example, in the case of loans or investments. This guidance requires coordination with the system. That is, the system lists the possibilities for such an operation; however, it is up to the banker to guide the customer in their financial decision, considering the customer's specific circumstances.

> [Marcos says that] it is very important [to be aware of what is being done by the system] for analysis, to guide the customer. For example, the bank offers two amortisation system methods: CAS and PRICE. The banker needs to know these operating principles to know what is best for the customer. He [Marcos] gave an example: a customer can apply for a loan, but says he wants to pay it off within three months. In this case,

> according to the evolution of the OB (outstanding balance), the type of spreadsheet type and the rate charged, the banker can guide the customer as to which type is best for him under these conditions (Fieldbook, p. 27-28, 22- line 26, 1-9).

Marcos had to know the construction principles of each type of amortisation spreadsheet to guide the customer in that particular loan situation, as the bank allowed the operation to be performed in two different ways. One difference between the CAS and the FS (which is based on the Price Table) is that in the first, amortisations (principal repayment, borrowed capital) are constant and instalments decrease, while in the second, the amortisations increase and the instalments are constant. Thus, in the second case, for the first instalments, the customer will have amortised a smaller portion of his debt, which is not of interest to customers who want to pay off their loans quickly. Thus, knowing these principles, Marcos can better guide his customers. The system cannot do this.

Regarding capital investment options, the guidance that Laura, the personal banking section manager at Bank B, gives her customers is essential, as the following excerpts show:

> She [Laura] showed me how to do [investment] applications for customers. According to the customer profile (conservative, moderate, bold, etc.) and the amount of capital that a person has to invest, she looks at the rates [on the system] for each application in the last period and in the last 12 months and advises the customer on the alternative that best fits his profile. Everything is seen in the system. For each type of application, the corresponding administration rate appears. The higher the amount the customer has to invest, the lower the management fee (Fieldbook, p. 87-88, line 23-26, 1-9).

We observed that Laura did not have to make calculations; she simply needed to guide the customer according to his/her profile. Although customers immediately want to know the investment rate, Laura explained that there are other variables associated with the decision. For example, an investment in Action Funds may offer a higher rate; however, the investment risk is also higher. Thus, if Laura knows that her customer has a more conservative profile, i.e., is not inclined to take risks, she offers another option that suits better that profile.

In such cases, although the bankers needed the system to complete these operations and did not need to perform calculations, the guidance that the bankers provided regarding loan, financing and investment alternatives was key for providing customer service. This guidance involved such aspects as customer profiling in the case of investments and determining the most appropriate options for customers in certain situations, including investments, loans, and financing. These are evaluations that the system cannot perform.

We observed that these bankers, both of whom were managers, used prior knowledge of the principles and procedures of financial mathematics to better guide their customers, studied their customer's profiles to guide them, and analysed the specific conditions of the situation at hand to guide decision making. We therefore consider that in such cases, bankers' activities involve financial analyses that go beyond the system's restrictions.

6 Discussion

The data presented and analysed in the previous section allow us to infer some financial mathematical characteristics from the observed banking practice. First, in this practice, we recognise that action mediated by the information technologies available, particularly the BIS, is a key characteristic (Wertsch, 1991). We can then establish some financial mathematical characteristics of this bankers-with-system practice based on the actions mediated by the systems and analyse the kind of learning the bankers engage in when they are embedded in this environment.

The data analysed in the previous section enable us to establish that *the information systems are central to the bankers' actions* and permeate the categories that were discussed. Moreover, we can say that the BISs structured these actions insofar as they defined the elements of the practice that were evidenced in the presented categories.

From the data, we can identify some of the actions that bankers perform with these systems. We have classified these *actions* into those *made possible by the systems* and those *that address the limitations of these systems*.

The following *actions* were *made possible by the systems*: the simulation of input values using trial and error (section 4.1); the inference of system's model's assumptions; the identification of the different ways the systems works based on inputs and outputs and the interpretation of the system's outputs to clarify them to customers (section 4.2). Analysing these actions we can say that the bankers could learn financial mathematics in different ways. For example, when using trial and error, instead of calculating previously the capital that the customer could ask for loan with the maximum fixed instalments required by the consignable margin, Jane learned to use the system inputting some values in the capital window till she could find the maximum instalment permitted by the rules.

The *actions that address the systems' limitations* were apparent when the bankers needed prior knowledge of principles and procedures of financial mathematics to better guide their customers, when they studied their customers' profiles to better guide them, and when they analysed the situation's specific conditions to guide the decision-making process (section 4.3). For all of these actions, the bankers provided customer service by using the system and performing actions that the system could not and that were specific to each customer or situation. These are examples on how the bankers learned to use their previous financial mathematics experiences together with the banking system to assist their clients.

We can say that the bankers' participation in the financial mathematics involved in these actions occurs through actions mediated by the objects that reify this practice. According to Wenger (1998, p. 206), "a computer program is an extreme example of a reification". In this case, the program's users participate in a practice using reified objects that originate from the programmers' practice. These reifications shape the users' experiences and modify their nature.

Considering our object of study, we can say that one characteristic of the financial mathematics that these bankers practice is that they respond to the reified routines of the systems. This mathematics learning is associated with the opportunities offered by such systems and the limitations that they impose.

Using Wertsch's (1991) theoretical constructs, we can say that bankers-with-systems-with-mathematics form a unit of banking practice characterised by the use of the mathematical models with which those systems are programmed; by performing operations that facilitate, streamline and avoid mistakes in this practice; and by the bankers' interpretations and financial analysis.

Therefore, in the bankers' practice, we observed participation in financial mathematics characterised by the bankers' interactions with the technological objects that reify this practice. These forms of participation do not involve many calculations or model constructions; rather, they involve interpretation and decision-making based on the outputs of the reified objects and the bankers' knowledge of financial mathematics. These findings can inform different policies related to teaching and learning financial mathematics in business courses.

We noted a way of dealing with mathematics that involves TmL (Hoyles et al., 2010) in the categories presented. That is, when technology is used to make better calculations and perform exhaustive operations that otherwise would demand a great amount of time and carry a high risk of error, it leaves the subject free to perform the reflective, interpretative and analytical part of decision-making. Thus, based on this kind of learning performed by the bankers, we can sign to different aims in educational contexts.

Although it was not a goal of the present study, our empirical data revealed some of the limitations of BISs. According to Wertsch (1998, p. 40), "the limitations imposed by cultural tools are typically recognised only in retrospect, through a process of comparison with the present perspective". Therefore, in addition to *establishing* some financial mathematics characteristics of bankers' practice through the actions described here – which was the purpose of our study - the present work also provided evidence of certain limitations of this cultural tool as currently used, which could be the subject of future studies.

7 Conclusions

The results of the present study suggest ways in which bankers-with-systems negotiate financial mathematics. Their actions are associated with and structured by BISs. We can say more generally that these actions are associated with ways of dealing with the reified aspects of this practice.

We found participation patterns in these bankers' routines that indicate that they negotiate the financial mathematics with the banking systems. In the first case, we observed actions in which the bankers' participation was qualitatively characterised by the negotiation of the financial mathematics made possible by the BISs and that enabled them to learn in this practice. In the second case, when dealing with a limitation of the BISs, the bankers negotiated the financial mathematics, combining their previous experience with financial mathematics concepts with the reified products of the system to deliver customer service. That is, their actions, although associated with the bankers-with-systems unit, produced results that were based on an analysis of the customer profile and the specific situation in which the operation was performed, something that the system alone cannot do. That is, we can say that another type of learning emerged, linking the experiences the bankers had inside and outside the bank. In both cases, TmL (Hoyles et al., 2010) was evident in the ways that these bankers-with-systems balanced participation and reification to negotiate the meanings of the specific financial mathematics of this practice.

The bankers' forms of participation are structured by the BISs and involve the observation, interpretation and analysis of the mathematical models used to conduct financial transactions with customers; such forms of participation are characteristic of the financial mathematics in the observed banking environments. These bankers, liberated from the calculations that technology can perform, can focus their attention on decision-making processes by solving problems involving specific customers' profiles and the specific situations of the operation they wish to perform, which the system cannot do. This can be a key to afford different foci in teaching and learning practices, more aligned to these types of participation required in rich-technology environments. That is, for example, presenting case studies based on workplace problems and preparing students to analyse decision-making processes based on specific contexts.

We understand that the observation of financial mathematics practiced by bankers in two banks does not provide enough evidence to permit widespread generalisations about all bankers. Therefore, studies in other banking environments in other cities and states and in countries other than Brazil can reveal actions not discussed here. However, the categories of analysis that we created to organise the observed actions could be applied to both of the banks we studied because they were somewhat similar in terms of their banking systems. Though distinct, these systems were programmed based on general financial market rules and were thus similar to some degree. We can say that observing other banks could reveal different actions, but the actions studied here do illustrate some characteristics of financial mathematics in bankers' practice, even though, in principle, they cannot be generalised.

We would like to conclude this discussion by proposing that these actions can inspire educational practices, especially in business courses.

References

Abor, J. (2017). *Entrepreneurial finance for MSMEs: a managerial approach for developing markets*. Cham: Springer.

Angrosino, M. (2005). Recontextualizing observation: ethnography, pedagogy, and the prospects for a progressive political agenda. In N. K. Denzin & Y. S Lincoln (Eds.), *The Sage Handbook of Qualitative Research* (pp. 729-745). Third Edition. London: Sage Publications.

Billet, S. (2011). Workplace curriculum: practice and propositions. In: F. Dochy et al. (Ed.), *Theories of Learning for the Workplace: building blocks for training and professional development programs* , (pp. 17-36). Oxon: Routledge.

Charmaz, K. (2005). Grounded Theory in the 21st century: applications for advancing social justice studies. In N. K. Denzin & Y. S Lincoln (Eds.), *The Sage Handbook of Qualitative Research (*pp. 507-535). Third Edition. London: Sage Publications.

Denzin, N. K. & Lincoln, Y. S. (2005). Introduction: the discipline and the practice of qualitative research. In N. K. Denzin & Y. S Lincoln (Eds.), *The Sage Handbook of Qualitative Research* (pp. 1-32). Third Edition. London: Sage Publications.

Drake, P. P. & Fabozzi, F. J. (2009). *Foundations and applications of the time value of money*. New Jersey: John Wiley & Sons.

FitzSimons, G. (2014). Commentary on vocational mathematics education: where mathematics education confronts the realities of people's work. *Educational Studies in Mathematics, 86*(2), 291-305.

Fontana, A. & Frey, J. (2005). The interview: from neutral stance to political involvment. In N. K. Denzin & Y. S. Lincoln (Eds.), *The Sage Handbook of Qualitative Research* (pp. 695-727). Third Edition. London: Sage Publications.

Hoyles, C., Noss, R., Kent, P. & Bakker, A. (2010.). *Improving mathematics at work: the need for techno-mathematical literacies*. New York: Routledge.

Lave, J. & Wenger, E. (1991). *Situated learning: legitimate peripheral participation*. New York: Cambridge University Press.

Lobato, J. (2006). Alternative perspectives on the transfer of learning: history, issues, and challenges for future research. *The Journal of the Learning Sciences, 15*(4), 431-449.

Schmidt, R. & Gibbs, P. (2009). The challenges of work-based learning in the changing context of the European Higher Education area. *European Journal of Education, 44*(3), 399-410.

Swanson, D. & Williams, J. (2014). Making abstract mathematics concrete in and out of school. *Educational Studies in Mathematics, 86*(2), 193-209.

Voigt, J. (1994). Negotiation of mathematical meaning and learning mathematics. *Educational Studies in Mathematics, 26*, 275-298.

Wake, G. (2014). Making sense of and with mathematics: the interface between academic mathematics and mathematics in practice. *Educational Studies in Mathematics, 86*(2), 271-290.

Wedege, T. (2010). Researching workers' mathematics at work. *Paper presented at Educational Interfaces Between Mathematics And Industry Conference*. Lisboa: Centro Internacional de Matemática, p. 565-574.

Wenger, E. (1998). *Communities of practice: learning, meaning, and identity*. New York: Cambridge University Press.

Wertsch, J. V. (1991). *Voices of the mind: a sociocultural approach to mediated action*. Cambridge: Harvard University Press.

Wertsch, J. V. (1998). *Mind as action*. New York: Oxford University Press.

Zevenbergen, R. J. (2011). Young workers and their dispositions towards mathematics: tensions of a mathematical habitus in the retail industry. *Educational Studies in Mathematics, 76*, 87-100.